International Capital Flows and Aggregate Output*

Jürgen von Hagen† and Haiping Zhang‡

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Abstract

We develop a tractable multi-country overlapping-generations model and show that cross-country differences in financial development explain three recent empirical patterns of international capital flows. Domestic financial frictions in our model distort interest rates and aggregate output in the less financially developed countries. International capital flows help ameliorate the two distortions.

International capital mobility affects aggregate output in each country directly through affecting the size of domestic investment. In the meantime, it also affects aggregate output indirectly through affecting the size of domestic savings and the composition of domestic investment. Under certain conditions, the indirect effects may dominate the direct effects so that, despite "uphill" net capital flows, full capital mobility may raise output in the poor country as well as raise world output. Our results complement conventional neoclassical models by identifying the impacts of capital mobility on domestic savings and the composition of domestic investment.

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†University of Bonn, Indiana University and CEPR. Lennestrasse. 37, D-53113 Bonn, Germany. E-mail: vonhagen@uni-bonn.de

‡Corresponding author. School of Economics, Singapore Management University. 90 Stamford Road, Singapore 178903. E-mail: hpzhang@smu.edu.sg

1 Introduction

This paper analyzes the implications of the recent empirical patterns of international capital flows for aggregate output at the country and the world level. According to conventional neoclassical theory, capital should flow "downhill" from rich countries, where its marginal product is low, to poor countries, where its marginal product is high. As a result, international capital mobility improves the allocation of capital and increases world output. Recent empirical observations, however, suggest that net capital flows are "uphill" from poor to rich countries (Lane and Milesi-Ferretti, 2001, 2007a,b; Prasad, Rajan, and Subramanian, 2006, 2007). Furthermore, there are large differences between gross and net capital flows. More specifically, financial capital tends to flow from poor to rich countries, while foreign direct investment (FDI, hereafter) flows in the opposite direction (Ju and Wei, 2010). Finally, although it has had a negative net international investment position since 1986, the U.S. has continued to receive positive net investment income (Gourinchas and Rey, 2007; Hausmann and Sturzenegger, 2007; Higgins, Klitgaard, and Tille, 2007).

There is now a considerable literature offering explanations for these observations. This literature focuses on international differences in financial market development or in the severity of financial market imperfections, which distort the returns on financial capital and FDI away from their social returns (Antras and Caballero, 2009; Antras, Desai, and Foley, 2009; Aoki, Benigno, and Kiyotaki, 2009; Caballero, Farhi, and Gourinchas, 2008; Mendoza, Quadrini, and Rios-Rull, 2009; Smith and Valderrama, 2008). Ju and Wei (2008, 2010) show that such differences can explain two-way flows of financial capital and FDI and why net capital flows are "uphill", although the social return on capital is higher in poor countries.[1] While this literature does not commonly address the implications of international capital mobility for aggregate output, it seems intuitively plausible that, due to declining marginal productivity, "uphill" capital flows make the poor countries and the world economy poorer. Matsuyama (2004) and von Hagen and Zhang (2010) show that this may indeed be the case. The policy implications seem to be clear: The world would be better off without international capital movements between rich and poor countries.

In this paper, we develop a model of international capital flows addressing their implications explicitly and more generally than the literature has done so far. Our main contribution is to show that international capital mobility can increase output globally and in the poor countries, even if gross capital flows go in both directions and net capital flows are "uphill", and to identify the conditions under which this will be the case. There are two reasons for this. The first is that credit market imperfections depress the return

[1]Another strand of research focuses on the risk-sharing investors can achieve by diversifying their portfolios globally (Devereux and Sutherland, 2009; Tille and van Wincoop, 2008, 2010). These models fail to distinguish between financial capital flows and FDI flows.

on and, therefore, the level of savings. Allowing for international capital mobility provides domestic households with better returns on their savings. As a result, the level of savings increases and the amount of credit available to finance domestic investment with it. This is reminiscent of the financial-repression literature (Beim and Calomiris, 2001; McKinnon, 1973). The second reason is that financial frictions distort the allocation of capital across different sectors within a country. This is in line with Barlevy (2003); Hsieh and Klenow (2009); Jeong and Townsend (2007); Levine (1997); Midrigan and Xu (2009) who show that financial frictions bias investment decisions in the direction of less productive projects. International capital flows can generate output gains by triggering a reallocation of investment towards more productive sectors. In this regard, our analysis is reminiscent of the recent trade literature (Melitz, 2003) arguing that international trade leads to the reallocation of market shares from less to more productive firms.

The remainder of paper is organized as follows. Section 2 sets up the model under international financial autarky (IFA) and shows how financial frictions distort interest rates and aggregate output. Section 3 shows the patterns of international capital flows and analyzes the implications on aggregate output. Section 4 addresses the output implications of partial capital mobility where FDI is restricted. Section 5 concludes and the appendix collects relevant proofs.

2 The Model under International Financial Autarky

2.1 The Model Setting

The world economy consists of $N \geq 2$ countries, which are fundamentally identical except in the level of financial development as specified later. In the following, variables in country $i \in \{1, 2, ..., N\}$ are denoted with the superscript i. There is a tradable final good, which is taken as the numeraire, and there are two types of nontradable intermediate goods, A and B. The price of the intermediate good $k \in \{A, B\}$ in period t is denoted by $v_t^{i,k}$. In this section, we assume that international capital flows are not allowed.

Individuals live for two periods and there is no population growth. In each country, the size of each generation is normalized to one. Each generation consists of two types of agents, *entrepreneurs* and *households*, of mass η and $1 - \eta$, respectively, who only differ in their production opportunities.[2] Individuals are endowed with one unit of labor when

[2]Matsuyama (2004) assumes that individuals are identical ex ante. Due to credit rationing, a fraction of individuals randomly become entrepreneurs ex post and this fraction is endogenously determined. As shown in von Hagen and Zhang (2010), such an assumption is essential for the symmetry-breaking property of financial globalization, but FDI cannot be addressed. In order to analyze the joint determination of financial capital and FDI flows, we follow the assumption of Antras and Caballero (2009) by assuming that entrepreneurs account for a fixed fraction of population.

young and $\epsilon \geq 0$ units of labor when old, which they supply inelastically to aggregate production. Thus, aggregate labor supply is $L = 1 + \epsilon$ in each period.

Young individuals can produce intermediate goods using final goods as the only input. The production takes one period to complete. The rate of transformation from final to intermediate goods is constant and normalized to one in both sectors. Thus, the price of intermediate good k in period $t+1$, $v_{t+1}^{i,k}$, equals the gross rate of return on the investment of a unit of the final good made in sector k and period t. Entrepreneurs can produce both intermediate goods, while households can produce only intermediate good A.

Final goods are produced instantaneously using the amounts $M_t^{i,A}$ and $M_t^{i,B}$ of intermediate goods and labor L_t in a Cobb-Douglas technology. All inputs are rewarded with their respective marginal products. To summarize,

$$Y_t^i = \left[\frac{\left(\frac{M_t^{i,A}}{1-\gamma}\right)^{1-\gamma} \left(\frac{M_t^{i,B}}{\gamma}\right)^{\gamma}}{\alpha} \right]^{\alpha} \left(\frac{L}{1-\alpha}\right)^{1-\alpha}, \text{where } \alpha \in (0,1),\ \gamma \in (0,1], \tag{1}$$

$$\omega_t^i L = (1-\alpha) Y_t^i \quad v_t^{i,A} M_t^{i,A} = (1-\gamma)\alpha Y_t^i, \quad v_t^{i,B} M_t^{i,B} = \gamma \alpha Y_t^i. \tag{2}$$

Y_t^i and ω_t^i denote the aggregate output of final goods and the wage rate, respectively. α denotes the joint factor share of the intermediate goods and γ denotes the relative factor share of intermediate good B. There is no uncertainty in the economy.

Assumption 1. $\eta \in (0, \gamma)$.

Assumption 1 ensures that aggregate entrepreneurial net worth is smaller than the socially efficient investment size in sector B; therefore, entrepreneurs wish to borrow from households. In equilibrium, entrepreneurs only produce intermediate good B.

Individuals have log-linear preferences over consumption in the two periods of life,

$$U_t^{i,j} = (1-\beta) \ln c_{y,t}^{i,j} + \beta \ln c_{o,t+1}^{i,j}, \tag{3}$$

where $c_{y,t}^{i,j}$ and $c_{o,t+1}^{i,j}$ denote individual j's consumption when young and old, respectively; $j \in \{e, h\}$ denotes entrepreneurs or households, and $\beta \in (0,1]$ is the relative weight of utility from consumption when old.

A household born in period t receives a labor income ω_t^i, consumes $c_{y,t}^{i,h}$, and saves $s_t^i = \omega_t^i - c_{y,t}^{i,h}$ when young. It saves by investing $i_t^{i,h}$ units of final goods in its own production of intermediate good A and by lending d_t^i units of final goods to entrepreneurs at the gross loan rate R_t^i. In period $t+1$, it receives a labor income, $\epsilon \omega_{t+1}^i$, the return on its own production, $v_{t+1}^{i,A} i_t^{i,h}$, and the return on its loan to entrepreneurs, $R_t^i d_t^i$. The household's no-arbitrage condition is

$$R_t^i = v_{t+1}^{i,A}. \tag{4}$$

In period $t+1$, the household consumes its total wealth $c^{i,e}_{o,t+1} = v^{i,A}_{t+1} i^{i,h}_t + R^i_t d^i_t + \epsilon\omega^i_{t+1} = R^i_t s^i_t + \epsilon\omega^i_{t+1}$ before exiting from the economy. Its lifetime budget constraint is $c^{i,h}_{y,t} + \frac{c^{i,h}_{o,t+1}}{R^i_t} = W^{i,h}_t$, where $W^{i,h}_t \equiv \omega^i_t + \frac{\epsilon\omega^i_{t+1}}{R^i_t}$ denotes the present value of its lifetime wealth. Given the log-linear utility function (3), the household's optimal consumption-savings choices are

$$c^{i,h}_{y,t} = (1-\beta)W^{i,h}_t \quad \text{and} \quad c^{i,h}_{o,t+1} = R^i_t \beta W^{i,h}_t, \tag{5}$$

$$s^i_t = \omega^i_t - c^{i,h}_{y,t} = \beta\omega^i_t - (1-\beta)\frac{\epsilon\omega^i_{t+1}}{R^i_t}. \tag{6}$$

An entrepreneur born in period t receives a labor income ω^i_t, consumes $c^{i,e}_{y,t}$, and invests $i^{i,e}_t$ units of final goods in the production of intermediate good B. His investment is financed with his own savings, $n^i_t = \omega^i_t - c^{i,e}_{y,t}$, and loans from households, $z^i_t = i^{i,e}_t - n^i_t$. Subsequently, we call n^i_t the entrepreneur's equity. In period $t+1$, he receives the project revenue $v^{i,B}_{t+1} i^{i,e}_t$ and a labor income $\epsilon\omega^i_{t+1}$. After repaying the debt, he consumes the rest, $c^{i,e}_{o,t+1} = v^{i,B}_{t+1} i^{i,e}_t - R^i_t z^i_t + \epsilon\omega^i_{t+1}$, before exiting from the economy.

Due to credit market frictions, the entrepreneur can borrow only up to a fraction of his future project revenues,

$$R^i_t z^i_t = R^i_t (i^{i,e}_t - n^i_t) \leq \theta^i v^{i,B}_{t+1} i^{i,e}_t. \tag{7}$$

Following Matsuyama (2004, 2007), we use $\theta^i \in [0,1]$ as a measure of financial development or the severity of credit market imperfections in country i. It captures a wide range of institutional factors and is higher in countries with more sophisticated financial and legal systems, better creditor protection, and more liquid asset market, etc. Countries are ranked in terms of the level of financial development and there exists at least one country with a level of financial development lower than another country.

Assumption 2. $\forall\ i \in \{1, 2, ..., N-1\}$, $0 \leq \theta^i \leq \theta^{i+1} \leq 1$, *and,* $\exists \imath$ *s.t.* $0 \leq \theta^\imath < \theta^{\imath+1} \leq 1$.

Define the equity rate as the rate of return to an entrepreneur's equity,

$$\Gamma^i_t \equiv \frac{v^{i,B}_{t+1} i^{i,e}_t - R^i_t z^i_t}{n^i_t} = v^{i,B}_{t+1} + (v^{i,B}_{t+1} - R^i_t)(\lambda^i_t - 1) \geq R^i_t, \tag{8}$$

where $\lambda^i_t \equiv \frac{i^{i,e}_t}{n^i_t}$ denotes the investment-equity ratio. For a unit of equity invested, the entrepreneur borrows the amount $(\lambda^i_t - 1)$ in period t. In period $t+1$, the entrepreneur receives the net return from his leveraged investment, $(v^{i,B}_{t+1} - R^i_t)(\lambda^i_t - 1)$ in addition to the marginal product of his equity, $v^{i,B}_{t+1}$. In equilibrium, the equity rate should be no less than the loan rate; otherwise, the entrepreneur would rather lend than borrow. Inequality (8) thus marks the participation constraint for the entrepreneur. If $R^i_t < v^{i,B}_{t+1}$, the entrepreneur borrows to the limit defined by (7); after repaying the debt in period $t+1$, he gets $(1-\theta^i)v_{t+1}^{i,B}i_t^{i,e}$ and the equity rate is $\Gamma_t^i = \frac{(1-\theta^i)v_{t+1}^{i,B}i_t^{i,e}}{n_t^i} = \frac{(1-\theta^i)v_{t+1}^{i,B}}{1-\frac{\theta^i v_{t+1}^{i,B}}{R_t^i}}$. If $R_t^i = v_{t+1}^{i,B}$, the entrepreneur does not borrow to the limit; after repaying the debt in period $t+1$, he gets $v_{t+1}^{i,B}n_t^{i,e}$ and the equity rate is $\Gamma_t^i = v_{t+1}^{i,B}$.

The entrepreneur's lifetime budget constraint is $c_{y,t}^{i,e} + \frac{c_{o,t+1}^{i,e}}{\Gamma_t^i} = W_t^{i,e}$, where $W_t^{i,e} \equiv \omega_t^i + \frac{\epsilon\omega_{t+1}^i}{\Gamma_t^i}$ denotes the present value of his lifetime wealth. Given the log-linear utility function (3), the entrepreneur's optimal consumption-savings choices are,

$$c_{y,t}^{i,e} = (1-\beta)W_t^{i,e} \text{ and } c_{o,t+1}^{i,e} = \Gamma_t^i\beta W_t^{i,e}, \tag{9}$$

$$n_t^i = \omega_t^i - c_{y,t}^{i,e} = \beta\omega_t^i - (1-\beta)\frac{\epsilon\omega_{t+1}^i}{\Gamma_t^i}. \tag{10}$$

Aggregate output of intermediate goods A and B in period $t+1$ is

$$M_{t+1}^{i,A} = (1-\eta)i_t^{i,h} \text{ and } M_{t+1}^{i,B} = \eta i_t^{i,e}. \tag{11}$$

Credit and the final goods market clearing require

$$(1-\eta)d_t^i = \eta z_t^i, \Rightarrow (1-\eta)(s_t^i - i_t^{i,h}) = \eta(i_t^{i,h} - n_t^i), \tag{12}$$

$$C_t^i + I_t^i = Y_t^i, \tag{13}$$

where $C_t^i \equiv \eta(c_{y,t}^{i,e} + c_{o,t}^{i,e}) + (1-\eta)(c_{y,t}^{i,h} + c_{o,t}^{i,h})$ and $I_t^i \equiv \eta i_t^{i,e} + (1-\eta)i_t^{i,h}$ denote aggregate consumption and aggregate investment in country i and period t.

Definition 1. *Given the level of financial development θ^i, a market equilibrium in country $i \in \{1,2,...,N\}$ under IFA is a set of allocations of households, $\{i_t^{i,h}, s_t^i, c_{y,t}^{i,h}, c_{o,t}^{i,h}\}$, entrepreneurs, $\{i_t^{i,e}, n_t^i, c_{y,t}^{i,e}, c_{o,t}^{i,e}\}$, and aggregate variables, $\{Y_t^i, M_t^{i,A}, M_t^{i,B}, \omega_t^i, v_t^{i,A}, v_t^{i,B}, R_t^i, \Gamma_t^i\}$, satisfying equations* (1)-(2), (4)-(12),

For notational convenience, we define some auxiliary parameters, $\rho \equiv \frac{\alpha}{1-\alpha}$, $m \equiv \frac{(1-\beta)\epsilon}{(1+\epsilon)\rho}$, $\mathcal{Q} \equiv \frac{(1+\epsilon)\rho}{\beta}(1+m)$, $\bar{\theta} \equiv 1 - \frac{\eta}{\gamma}$, $\mathcal{A}^i \equiv 1 - \gamma\frac{\bar{\theta}-\theta^i}{1-\eta}$, $\mathcal{B}^i \equiv 1 + \gamma\frac{\bar{\theta}-\theta^i}{\eta}$.

According to equations (6) and (10), iff $\epsilon > 0$ and $\beta < 1$, individual savings are interest elastic, $\frac{\partial s_t^i}{\partial R_t^i} > 0$ and $\frac{\partial n_t^i}{\partial \Gamma_t^i} > 0$, and, by definition, $m > 0$; the interest elasticities of savings, $\frac{\partial \ln s_t^i}{\partial \ln R_t^i} = \frac{1}{\frac{\omega_t^i R_t^i}{\omega_{t+1}^i}\frac{\beta}{(1-\beta)\epsilon}-1}$ and $\frac{\partial \ln n_t^i}{\partial \ln \Gamma_t^i} = \frac{1}{\frac{\omega_t^i \Gamma_t^i}{\omega_{t+1}^i}\frac{\beta}{(1-\beta)\epsilon}-1}$, are positively (negatively) correlated with ϵ (β) and so is m. Iff either $\epsilon = 0$ or $\beta = 1$, individual savings are interest inelastic and $m = 0$. In this sense, m is positively correlated with the interest elasticities of savings through the factor of $(1-\beta)\epsilon$. As shown below, if $\theta^i \geq \bar{\theta}$, the borrowing constraint is slack and $\mathcal{Q}$ is equal to the steady-state social rate of return; if $\theta^i \in [0, \bar{\theta})$, the borrowing constraint is strictly binding with $0 < \mathcal{A}^i < 1 < \mathcal{B}^i$ and $\frac{\partial \mathcal{A}^i}{\partial \theta^i} > 0 > \frac{\partial \mathcal{B}^i}{\partial \theta^i}$.

Let $\chi_t^i \equiv \frac{v_t^{i,A}}{v_t^{i,B}}$ denote the relative intermediate goods price and let $\Psi_t^i \equiv \frac{v_{t+1}^{i,A}M_{t+1}^{i,A} + v_{t+1}^{i,B}M_{t+1}^{i,B}}{I_t^i}$ denote the social rate of return to aggregate investment. Given the Cobb-Douglas aggregate production function, it is trivial to prove that $\Psi_t^i = \frac{v_{t+1}^{i,A}}{1-\gamma(1-\chi_{t+1}^i)}$. Let $\psi_t^i \equiv \frac{R_t^i}{\Psi_t^i}$ denote the relative loan rate. Using the household no-arbitrage condition (4), we can show that the relative intermediate goods price and the relative loan rate are positively related,

$$\psi_t^i = 1 - \gamma(1 - \chi_{t+1}^i). \tag{14}$$

As shown below, the relative intermediate goods price and the relative loan rate reflect two different aspects of the distortions caused by the financial frictions. Finally, we define the indicator of production efficiency $\Lambda_t^i = \frac{(\chi_{t+1}^i)^\gamma}{1 - \frac{\gamma}{1+m}\frac{\bar{\theta}-\theta^i}{1-\eta}}$.

2.2 Equilibrium under Financial Autarky

In period $t+1$, aggregate revenue of intermediate goods $v_{t+1}^{i,A} M_{t+1}^{i,A} + v_{t+1}^{i,B} M_{t+1}^{i,B} = \rho L \omega_{t+1}^i$ is distributed to households and entrepreneurs as the return to their savings, $(1-\eta)s_t^i R_t^i + \eta n_t^i \Gamma_t^i = \rho L \omega_{t+1}^i$. Using equations (6) and (10) to substitute away s_t^i and n_t^i, we get

$$(1-\eta)R_t^i + \eta \Gamma_t^i = \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Q}. \tag{15}$$

Let X_{IFA}^i denote the steady-state value of variable X_t^i under IFA. If the borrowing constraints are binding, the model solutions are as follows,

$$I_t^i = \frac{\beta \omega_t^i}{m+1}\left[1 - \frac{m(1-\mathcal{A}^i)(\mathcal{B}^i - 1)}{(m+\mathcal{A}^i)(m+\mathcal{B}^i)}\right], \tag{16}$$

$$\Gamma_t^i = \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Q}\left(1 + \frac{\mathcal{B}^i - 1}{m+1}\right), \tag{17}$$

$$R_t^i = \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Q}\left(1 - \frac{1-\mathcal{A}^i}{m+1}\right), \tag{18}$$

$$\Psi_t^i = \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Q}\left[1 + \frac{m(1-\mathcal{A}^i)(\mathcal{B}^i - 1)}{(m+1)(m+\mathcal{A}^i\mathcal{B}^i)}\right], \tag{19}$$

$$\psi_t^i = \psi_{IFA}^i = 1 - \frac{(1-\mathcal{A}^i)\mathcal{B}^i}{m+\mathcal{B}^i}, \tag{20}$$

$$\chi_{t+1}^i = \chi_{IFA}^i = 1 - \frac{1}{\gamma}\frac{(1-\mathcal{A}^i)\mathcal{B}^i}{m+\mathcal{B}^i}, \tag{21}$$

$$\Lambda_t^i = \Lambda_{IFA}^i = \frac{(\chi_{IFA}^i)^\gamma}{1 - \frac{\gamma}{1+m}\frac{\bar{\theta}-\theta^i}{1-\eta}}, \tag{22}$$

$$\omega_{t+1}^i = \left(\frac{\Lambda_{IFA}^i}{\mathcal{Q}}\omega_t^i\right)^\alpha, \tag{23}$$

$$\frac{\partial \ln \Lambda_{IFA}^i}{\partial \theta^i} = \frac{m(\mathcal{B}^i - 1) + \mathcal{B}^i(1-\mathcal{A}^i)(\frac{1}{\gamma} - 1)}{\chi_{IFA}^i(\mathcal{B}^i + m)(\mathcal{A}^i + m)}\frac{\partial \mathcal{A}^i}{\partial \theta^i} - \frac{m(1-\mathcal{A}^i)}{\chi_{IFA}^i(\mathcal{B}^i + m)^2}\frac{\partial \mathcal{B}^i}{\partial \theta^i}. \tag{24}$$

The relative price of intermediate goods, χ_{t+1}^i, the relative loan rate, ψ_t^i, and the efficiency indicator Λ_t^i are all time-invariant. Aggregate output is proportional to the wage rate, $Y_t^i = \frac{(1+\epsilon)\omega_t^i}{(1-\alpha)}$. Thus, the model dynamics can be characterized by the dynamics of wages.

In view of equation (23) and with $\alpha \in (0,1)$, it is straightforward to show that there is a unique and stable steady state with the wage at $w^i_{IFA} = \left(\frac{\Lambda^i_{IFA}}{\mathcal{Q}}\right)^{\rho}$.

Lemma 1 summarizes the unconstrained case where the borrowing constraint is slack.

Lemma 1. *For $\theta^i \in [\bar{\theta}, 1]$, the borrowing constraints are slack and there exists a unique and stable non-zero steady state in country i with the wage at $\omega^i_{IFA} = \mathcal{Q}^{-\rho}$.*

The private and social rates of return coincide, $R^i_t = \Gamma^i_t = \Psi^i_t = v^{i,A}_{t+1} = v^{i,B}_{t+1} = \frac{\omega^i_{t+1}}{\omega^i_t}\mathcal{Q}$, and the relative loan rate is $\psi^i_{IFA} = 1$. In the steady state, $R^i_{IFA} = \Gamma^i_{IFA} = \Psi^i_{IFA} = \mathcal{Q}$.

Aggregate investment $\frac{\beta\omega^i_t}{1+m}$ is allocated in the two sectors, proportional to their respective factor shares, $M^{i,A}_{t+1} = (1-\gamma)\frac{\beta\omega^i_t}{1+m}$ and $M^{i,B}_{t+1} = \gamma\frac{\beta\omega^i_t}{1+m}$. The relative intermediate goods price and the efficiency indicator are $\chi^i_{IFA} = \Lambda^i_{IFA} = 1$.

If the borrowing constraints are binding, the depressed demand for credit keeps the loan rate lower than the social rate of return. Entrepreneurs benefit from the inefficiently low loan rate in the sense that the equity rate is higher than the social rate of return, i.e., $R^i_t < \Psi^i_t < \Gamma^i_t$. Thus, financial frictions distort the interest rates, and this affects the income distribution between households and entrepreneurs.

Furthermore, if either $m > 0$ or $\gamma < 1$, the binding borrowing constraints keep the efficiency indicator $\Lambda^i_{IFA} < 1$, so that output is less than the efficient level.[3] This output distortion results from two different effects, one operating through the composition of investment and the other through the level of savings.

The Investment Composition Effect

In order to highlight this effect, we assume, first, that savings are interest inelastic ($m = 0$), and, second, that both intermediate goods are used in aggregate production ($\gamma < 1$). If the borrowing constraint is binding, the entrepreneurs' borrowing and, therefore, their investment in sector B are inefficiently low, while investment in sector A is inefficiently high. This distortion in the composition of investment reduces production efficiency so that output is lower than in the unconstrained case. A rise in θ^i raises the entrepreneurs' borrowing capacity and increases their investment in sector B, which improves production efficiency. According to equation (24), $\frac{\partial\Lambda^i_{IFA}}{\partial\theta^i} > 0$, and Λ^i_{IFA} reaches its maximum of one, when the borrowing constraint is weakly binding at $\theta^i = \bar{\theta}$. The distortion in the composition of investment can be measured by the relative intermediate goods price. According to equation (21), this relative price also rises in θ^i and reaches its maximum of one, for $\theta^i = \bar{\theta}$. Thus, the higher the relative intermediate goods price, the smaller the output distortion.

The Savings Effect

[3] With $m = 0$ and $\gamma = 1$, savings are interest inelastic and only intermediate good B is used in aggregate production. Thus, financial frictions only distort interest rates, while aggregate output is efficient.

In order to highlight this effect, we assume, first, that savings are interest elastic ($m > 0$), and, second, that only intermediate good B is used in aggregate production ($\gamma = 1$). If the borrowing constraint is binding, the loan rate is below and the equity rate is above the social rate of return, so that households save less and entrepreneurs save more than the efficient level. Overall, aggregate savings are inefficiently low and so are aggregate investment. This distortion in aggregate savings keeps output lower than in the unconstrained case. A rise in θ^i raises the entrepreneurs' borrowing capacity, which pushes up the loan rate and pushes down the equity rate in equilibrium. Overall, aggregate savings rise, which raises aggregate investment and output. According to equation (24), $\frac{\partial \Lambda^i_{IFA}}{\partial \theta^i} > 0$, and Λ^i_{IFA} reaches its maximum of one, when the borrowing constraint is weakly binding at $\theta^i = \bar{\theta}$. The distortion in aggregate savings can be measured by the relative loan rate, which rises in θ^i, according to equation (20), and reaches its maximum of one, for $\theta^i = \bar{\theta}$. Thus, the higher the relative loan rate, the smaller the output distortion.

Proposition 1 summarizes the case where the borrowing constraints are binding.

Proposition 1. *For $\theta^i \in [0, \bar{\theta})$, the borrowing constraints are binding and there exists a unique and stable non-zero steady state in country i with the wage at $\omega^i_{IFA} = \left(\frac{\Lambda^i_{IFA}}{Q}\right)^\rho$.*

There is a wedge between the private and social rates of return, $R^i_t < \Psi^i_t < \Gamma^i_t$. In the steady state, the loan rate rises and the equity rate falls in the level of financial development.

If either $m > 0$ or $\gamma < 1$, aggregate output is below the efficient level and rises in the level of financial development. In that case, the relative intermediate goods price and the relative loan rate reflect the output distortion.

3 Full Capital Mobility

Under full capital mobility, individuals are allowed to lend and make direct investments globally. Without loss of generality, we assume $\theta^i \in [0, \bar{\theta}]$ so that the borrowing constraints are binding in all countries. All countries are initially in the steady state under IFA before capital mobility is introduced at period $t = 0$.

Let Φ^i_t and Ω^i_t denote the aggregate outflows of financial capital and FDI from country i in period t, respectively, with negative values indicating capital inflows. With capital mobility, net credit supply in country i is $(1-\eta)(s^i_t - i^{i,A}_t) - \Phi^i_t$, and aggregate equity capital invested in country i is $\eta n^i_t - \Omega^i_t$. Assuming that entrepreneurs borrow in the country where they invest in the production of intermediate goods, FDI flows raise aggregate credit demand in the host country and reduce it in the source country. With these changes, the analysis in section 2 carries through due to the linearity of intermediate goods production and the borrowing constraints. Financial capital flows equalize loan rates and FDI flows equalize equity rates in all countries. Credit and equity markets clear in each country as well at the world level. To summarize,

$$\sum_{i=1}^{N}\Phi_t^i=\sum_{i=1}^{N}\Omega_t^i=0,\qquad R_t^i=R_t^*,\qquad \Gamma_t^i=\Gamma_t^*,$$
$$(1-\eta)(s_t^i-i_t^{i,A})=(\lambda_t^i-1)(\eta n_t^i-\Omega_t^i)+\Phi_t^i,\quad M_{t+1}^{i,B}=\lambda_t^i(\eta n_t^i-\Omega_t^i).$$

The remaining conditions for market equilibrium are same as under IFA.

At the world level, aggregate revenue of intermediate goods in period $t+1$ is distributed to households and entrepreneurs as the return to their savings,

$$(1-\eta)R_t^*\sum_{i=1}^{N}s_t^i+\eta\Gamma_t^*\sum_{i=1}^{N}n_t^i=\sum_{i=1}^{N}(v_{t+1}^{i,A}M_{t+1}^{i,A}+v_{t+1}^{i,B}M_{t+1}^{i,B})=\rho L\sum_{i=1}^{N}\omega_{t+1}^i.$$

Using equations (6) and (10) to substitute away s_t^i and n_t^i, we get

$$(1-\eta)R_t^*+\eta\Gamma_t^*=\frac{\omega_{t+1}^w}{\omega_t^w}\mathcal{Q},\quad \text{where}\quad \omega_t^w\equiv\frac{\sum_{i=1}^{N}\omega_t^i}{N}. \tag{25}$$

Let X_{FCM} denote the steady-state value of variable X under full capital mobility. Define $\wp_{IFA}^i\equiv\frac{\Gamma_{IFA}^i}{\mathcal{Q}}=1+\frac{\gamma}{1+m}\frac{\bar{\theta}-\theta^i}{\eta}$ and $\mathcal{Z}_{FCM}^i\equiv\frac{(\chi_{FCM}^i-\chi_{IFA}^i)\Gamma_{IFA}^i}{(\chi_{FCM}^i-\chi_{IFA}^i)+\frac{1-\theta^i}{(1-\eta)\wp_{IFA}^i}}$. The model solutions under full capital mobility are,

$$\Gamma_t^i=\frac{\omega_{t+1}^w}{\omega_t^w}\left(\Gamma_{IFA}^i-\mathcal{Z}_{FCM}^i\right), \tag{26}$$

$$R_t^i=\frac{\omega_{t+1}^w}{\omega_t^w}\left(R_{IFA}^i+\frac{\eta}{1-\eta}\mathcal{Z}_{FCM}^i\right), \tag{27}$$

$$\chi_{t+1}^i=\chi_{FCM}^i=\frac{(1-\theta^i)R_{FCM}^i}{\Gamma_{FCM}^i}+\theta^i, \tag{28}$$

$$\psi_{FCM}^i=1-\gamma(1-\chi_{FCM}^i), \tag{29}$$

$$\Phi_t^i=(1-\eta)\beta\omega_t^i\left[1-\frac{\omega_{t+1}^i}{\omega_t^i}\frac{R_{IFA}^i}{R_t^*}\right], \tag{30}$$

$$\Omega_t^i=\eta\beta\omega_t^i\left[1-\frac{\omega_{t+1}^i}{\omega_t^i}\frac{\Gamma_{IFA}^i}{\Gamma_t^*}\right], \tag{31}$$

$$\Omega_t^i+\Phi_t^i=\beta\omega_t^i\left\{1-\frac{\omega_{t+1}^i}{\omega_t^i}\left[\eta\frac{\Gamma_{IFA}^i}{\Gamma_t^*}+(1-\eta)\frac{R_{IFA}^i}{R_t^*}\right]\right\}, \tag{32}$$

$$\omega_{t+1}^i=\left[\frac{(1-\theta^i)R_t^*}{\Gamma_t^*}+\theta^i\right]^{\gamma\rho}(\frac{1}{R_t^*})^{\rho}. \tag{33}$$

Lemma 2. *Under full capital mobility, the relative intermediate goods price and the relative loan rate are time-invariant. There exists a unique and stable steady state.*

3.1 Steady-State Patterns of Capital Flows

In the steady state under full capital mobility, the interest rates and capital flows are,

$$\Gamma^i_{FCM} = \Gamma^i_{IFA} - \mathcal{Z}^i_{FCM}, \tag{34}$$

$$R^i_{FCM} = R^i_{IFA} + \frac{\eta}{1-\eta}\mathcal{Z}^i_{FCM}, \tag{35}$$

$$\Phi^i_{FCM} = (1-\eta)\beta\omega^i_{FCM}\left(1 - \frac{R^i_{IFA}}{R^*_{FCM}}\right) = \eta\beta\omega^i_{FCM}\frac{\mathcal{Z}^i_{FCM}}{R^*_{FCM}}, \tag{36}$$

$$\Omega^i_{FCM} = \eta\beta\omega^i_{FCM}\left(1 - \frac{\Gamma^i_{IFA}}{\Gamma^*_{FCM}}\right) = -\eta\beta\omega^i_{FCM}\frac{\mathcal{Z}^i_{FCM}}{\Gamma^*_{FCM}}, \tag{37}$$

$$\Phi^i_{FCM} + \Omega^i_{FCM} = \eta\beta\omega^i_{FCM}\mathcal{Z}^i_{FCM}\frac{(\Gamma^*_{FCM} - R^*_{FCM})}{\Gamma^*_{FCM}R^*_{FCM}}. \tag{38}$$

Proposition 2. *In the steady state under full capital mobility, there exists a threshold value of the country index $\hat{N}$ such that the world interest rates are $R^*_{FCM} \in (R^{\hat{N}}_{IFA}, R^{\hat{N}+1}_{IFA}]$ and $\Gamma^*_{FCM} \in [\Gamma^{\hat{N}+1}_{IFA}, \Gamma^{\hat{N}}_{IFA})$. In country $i \in \{1, 2, ..., \hat{N}\}$, the relative intermediate goods price and the relative loan rate are higher than under financial autarky, $\chi^i_{FCM} > \chi^i_{IFA}$ and $\psi^i_{FCM} > \psi^i_{IFA}$, the gross and net capital flows are $\Phi^i_{FCM} > 0 > \Omega^i_{FCM}$ and $\Phi^i_{FCM} + \Omega^i_{FCM} > 0$; the opposite applies for country $i \in \{\hat{N}+1, \hat{N}+2, ..., N\}$. The relative intermediate goods price and the relative loan rate increase in the level of financial development, i.e., $\chi^{i+1}_{FCM} > \chi^i_{FCM}$ and $\psi^{i+1}_{FCM} > \psi^i_{FCM}$ for $\theta^{i+1} > \theta^i$. Gross international investment returns sum up to zero in each country, $\Phi^i_{FCM}R^*_{FCM} + \Omega^i_{FCM}\Gamma^*_{FCM} = 0$.*

Countries $i = \hat{N}+1, \hat{N}+2, ..., N$, which have relatively high levels of financial development, import financial capital, export FDI, and receive net capital inflows. Since the rate of return on their foreign assets (FDI outflows) exceed the interest rate paid for their foreign liabilities (financial capital inflows), $\Gamma^*_{FCM} > R^*_{FCM}$, they receive positive net international investment incomes, $\Phi^i_{FCM}(R^*_{FCM} - 1) + \Omega^i_{FCM}(\Gamma^*_{FCM} - 1) > 0$, despite their negative international investment positions, $\Phi^i_{FCM} + \Omega^i_{FCM} < 0$. Thus, our model results are compatible with the three empirical evidences noted above.

3.2 Steady-State Levels of Output

We now turn to the implications of full capital mobility for steady-state output of final goods at the country and the global level. Output in each country may be affected through three different ways. The first is the familiar effect of the international reallocation of investment and we call it the *investment reallocation effect*. The second and third are the *investment composition effect* and the *savings effect* explained in section 2.2. To simplify the exposition and without loss of generality, we categorize all countries into two groups. Group S consists of the less financially developed countries indexed with

$s \in \{1, 2, ..., \hat{N}\}$, and group N of the more financially developed countries indexed with $n \in \{\hat{N}+1, \hat{N}+2, ..., N\}$, where $\hat{N}$ is defined in Proposition 2.

The Investment Reallocation Effect

Suppose that savings are interest inelastic ($m = 0$) and that only intermediate good B is used in aggregate production ($\gamma = 1$). Under IFA, output is efficient in the steady state at $Y^i_{IFA} = \frac{Q-\rho}{1-\alpha}$ and identical for all countries, according to Lemma 1. Under full capital mobility, net capital flows from country group S to country group N raise (reduce) aggregate investment in group N (S), according to Proposition 2. Thus, steady-state output of final goods rises (falls) in group N (S). In the meantime, since the production function of final goods is concave with intermediate good B at the country level, the cross-country reallocation of investment widens cross-country output gap, which reduces world output. The size of the investment reallocation effect depends on net capital flows.

If $\gamma < 1$ and (or) $m > 0$, full capital mobility also affects output at the country and the global level indirectly through the investment composition effect and (or) the savings effect, besides directly through the investment reallocation effect. Proposition 3 provides the sufficient conditions for the net effect on output.

Proposition 3. *Define* $\mathcal{N}^i \equiv \left[\frac{\gamma(1-\theta^i)}{\eta(1-\eta)}\left(\theta^i - \frac{m(1-\gamma)\eta}{\gamma}\right) - m^2\right](\bar{\theta} - \theta^i)$ *as a function of* θ^i. *In the case of* $m > 0$ *and* $\gamma < 1$, *full capital mobility raises steady-state output in the countries in group N with* θ^n *satisfying* $\mathcal{N}^n \geq 0$ *and in the countries in group S with* θ^s *satisfying* $\mathcal{N}^s \leq 0$.

For countries in group N with $\theta^n = \bar{\theta}$, *full capital mobility raises steady-state output,* $Y^n_{FCM} > Y^n_{IFA}$; *there exists a threshold value* $\hat{\theta}^S$ *such that, for countries in group S with* $\theta^s \in [0, \hat{\theta}^S)$, *full capital mobility raises steady-state output,* $Y^s_{FCM} > Y^s_{IFA}$.

In the following, we elaborate on the output implications of full capital mobility in the presence of the investment composition effect and the savings effect, respectively.

The Investment Composition Effect

As in subsection 2.2, we assume $m = 0$ and $\gamma < 1$ in order to highlight this effect. Consider a country in group S. According to Proposition 2, both financial capital outflows and FDI inflows raise the loan rate. Thus, households reduce investment in sector A and lend more to entrepreneurs, which improves the composition of investment, reflected by the rise in the relative intermediate goods price. Steady-state output tends to rise. The opposite happens to countries in group N. The size of the investment composition effect depends on gross capital flows.

For both groups of countries, output is affected in opposite ways through the investment composition effect and the investment reallocation effect. Lemma 3 summarizes the net effect on output at the country level.

Lemma 3. *In the case of* $m = 0$ *and* $\gamma < 1$*, the positive* investment reallocation *effect dominates the negative* investment composition *effect for all countries in group N so that full capital mobility strictly raises steady-state output,* $Y^n_{FCM} > Y^n_{IFA}$*. For countries in group S, there exists a threshold value* $\hat{\theta}^S_{IC}$ *such that, if* $\theta^s \in [0, \hat{\theta}^S_{IC})$*, the positive* investment composition *effect dominates the negative* investment reallocation *effect so that full capital mobility strictly raises steady-state output,* $Y^s_{FCM} > Y^s_{IFA}$*.*

For a country in group S with $\theta^s < \hat{\theta}^S_{IC}$, the initial output distortion is sufficiently severe. Under full capital mobility, two-way capital flows imply that gross flows are much larger than net flows. Thus, the positive investment composition effect dominates the negative investment reallocation effect so that steady-state output rises. In general, the overall impact of full capital mobility on steady-state world output depend on the distribution of the levels of financial development across countries. For example, if the levels of financial development in all countries in group S are below $\hat{\theta}^S_{IC}$, full capital mobility raises steady-state world output, given that the net output effects are strictly positive for all countries in group N.

The Savings Effect

As in subsection 2.2, we assume $m > 0$ and $\gamma = 1$ in order to highlight this effect. Consider a country in group S. According to Proposition 2, due to financial capital outflows and FDI inflows, the loan rate rises and the equity rate falls, which raises household savings and reduces entrepreneurs' savings. Overall, aggregate savings $(1-\eta)s^i_t + \eta n^i_t$ rise, which tends to raise domestic investment, $I^i_t = (1-\eta)s^i_t + \eta n^i_t - (\Phi^i_t + \Omega^i_t)$, reflected by the rise in the relative loan rate. Steady-state output tends to rise. The opposite happens to countries in group N. The size of the savings effect depends on gross capital flows.

Once again, for both groups of countries, output is affected in opposite ways through the savings effect and the investment reallocation effect. Lemma 4 summarizes the net effect on output at the country level.

Lemma 4. *In the case of* $m > 0$ *and* $\gamma = 1$*, if* $\eta \in (0, 0.5)$*, define* $\kappa \equiv \frac{1-\sqrt{1-4m^2(1-\eta)\eta}}{2} < \frac{1}{2}$ *and there are three scenarios:*

1. *for* $m \in (0, 1)$*, full capital mobility raises steady-state output in the countries in group S with* $\theta^s \in (0, \kappa)$*,* $Y^s_{FCM} > Y^s_{IFA}$*, and in the countries in group N with* $\theta^n \in (\kappa, \bar{\theta})$*,* $Y^n_{FCM} > Y^n_{IFA}$*;*

2. *for* $m \in (1, \frac{1}{2\sqrt{\eta(1-\eta)}})$*, full capital mobility raises steady-state output in the countries in group S with* $\theta^s \in (0, \kappa) \cup (1-\kappa, \bar{\theta})$*,* $Y^s_{FCM} > Y^s_{IFA}$*, and in the countries in group N with* $\theta^n \in (\kappa, 1-\kappa)$*,* $Y^n_{FCM} > Y^n_{IFA}$*;*

3. *for* $m > \frac{1}{2\sqrt{\eta(1-\eta)}}$*, full capital mobility raises steady-state output in the countries in group S with* $\theta^s \in (0, \bar{\theta})$*,* $Y^s_{FCM} > Y^s_{IFA}$*.*

If $\eta \in (0.5, 1)$, there are two scenarios:

1. *for $m \in (0, 1)$, full capital mobility raises steady-state output in the countries in group S with $\theta^s \in (0, \kappa)$, $Y^s_{FCM} > Y^s_{IFA}$, and in the countries in group N with $\theta^n \in (\kappa, \bar{\theta})$, $Y^n_{FCM} > Y^n_{IFA}$;*

2. *for $m > 1$, full capital mobility raises steady-state output in the countries in group S with $\theta^s \in (0, \bar{\theta})$, $Y^s_{FCM} > Y^s_{IFA}$.*

As before, the overall effect of full capital mobility on steady-state world output depends critically on the difference between the levels of financial development in the two groups of countries. The larger that difference, the more likely it is that output effects are positive in the group of the less financially developed countries and at the world level.

Three Effects Combined

Finally, we consider the case of $m > 0$ and $\gamma < 1$ where both the *investment composition* effect and the *savings* effect are at work together with the *investment reallocation* effect. Lemma 5 summarizes the net impact on output at the country level.

Lemma 5. *In the case of $m > 0$ and $\gamma < 1$, for every pair of $\theta^N \leq \bar{\theta}$ and $\theta^S < \theta^N$ such that $\theta^N(1-\theta^N) \geq \theta^S(1-\theta^S)$, there exists a non-empty set of values for m such that full capital mobility raises steady-state output in the countries in group N with $\theta^n(1-\theta^n) \geq \theta^N(1-\theta^N)$ and in the countries in group S with $\theta^s \leq \theta^S$.*

The intuition behind Lemma 5 is that, if the difference in the level of financial development is large between group S and N, capital flows may raise global output. Since proposition 3 gives only a sufficient condition, more scenarios may be derived such that full capital mobility raises global output, which, however, are more complicated. We discuss some scenarios in a numerical example in subsection 3.4.

3.3 Welfare Effects of International Capital Flows

As shown above, international capital flows affect wages and rates of return in all countries. Since the wage and rate-of-return effects can go in opposite directions, international capital flows change the distribution of income between households and entrepreneurs, within the same generation as well as among generations, at the national level as well as between countries. This complicates the analysis of their welfare effects.[4] Nevertheless, full capital mobility may raise global output in the presence of domestic financial frictions as shown above. In that case, it is feasible to develop an international transfer scheme which assures

[4]In von Hagen and Zhang (2010) show that, in a model setting only with the investment reallocation effect, full capital mobility reduce welfare at the global level by changing the distribution of income and consumption across countries and among generations.

that no individual is worse off under full capital mobility than under IFA. In this sense, if international capital flows raise global output, they may also raise global welfare.

3.4 A Numerical Example

In this section, we show numerically the output implications of full capital mobility in a two-country version of our model economy. The world economy consists of country N with $\theta^N = \bar{\theta}$ and country S with $\theta^S \in [0, \bar{\theta})$. In the benchmark case, we set the population share of entrepreneurs at $\eta = 10\%$, the share of labor income in aggregate output, $1 - \alpha = 64\%$, and the share of utility from consumption when old at $\beta = 0.4$.

The Investment Composition Effect

We first shut down the savings effect by assuming that individuals do not have labor endowment when old, $\epsilon = 0$, and hence, $m = 0$. Let the two intermediate goods have equal factor shares in aggregate production, $\gamma = 0.5$, and hence, $\bar{\theta} = 1 - \frac{\eta}{\gamma} = 0.8$.

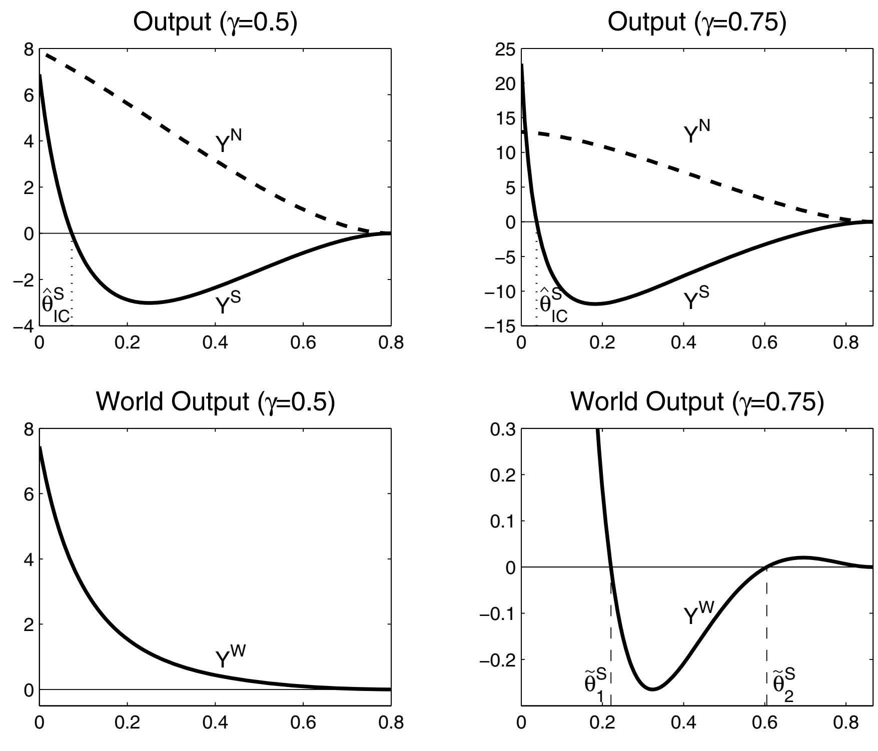

Figure 1: Steady-State Output Patterns only with the Investment Composition Effect

The left panels of figure 1 compare steady-state output under full capital mobility versus under IFA at the country and the global level. The horizontal axes denote $\theta^S \in [0, \bar{\theta})$ and the vertical axes denote the percentage differences under two scenarios. The upper-left panel shows that output rises strictly in country N and output rises in country S if θ^S is below a threshold value, confirming the results in Lemma 3. Since the output gains in country N exceed the losses in country S with larger values of θ^S, full capital mobility strictly raises world output as shown in the bottom-left panel.

In the case of $\gamma = 1$, the size of sector A, $1 - \gamma$, shrinks to zero, so that financial frictions do not distort output under IFA. Thus, there is no investment composition effect under full capital mobility and steady-state world output is strictly lower than under IFA, due to the investment reallocation effect. This suggests that the potential for world output gains depends on the relative size of two sectors. To illustrate that intuition, we set $\gamma = 0.75$ and keep other parameter values unchanged. The right panels of figure 1 show the percentage differences of steady-state output under two scenarios. The output patterns at the country level are qualitatively similar as in the case of $\gamma = 0.5$. However, for intermediate values of θ^S, the output decline in country S exceeds the output rise in country N so that world output is lower than under IFA.

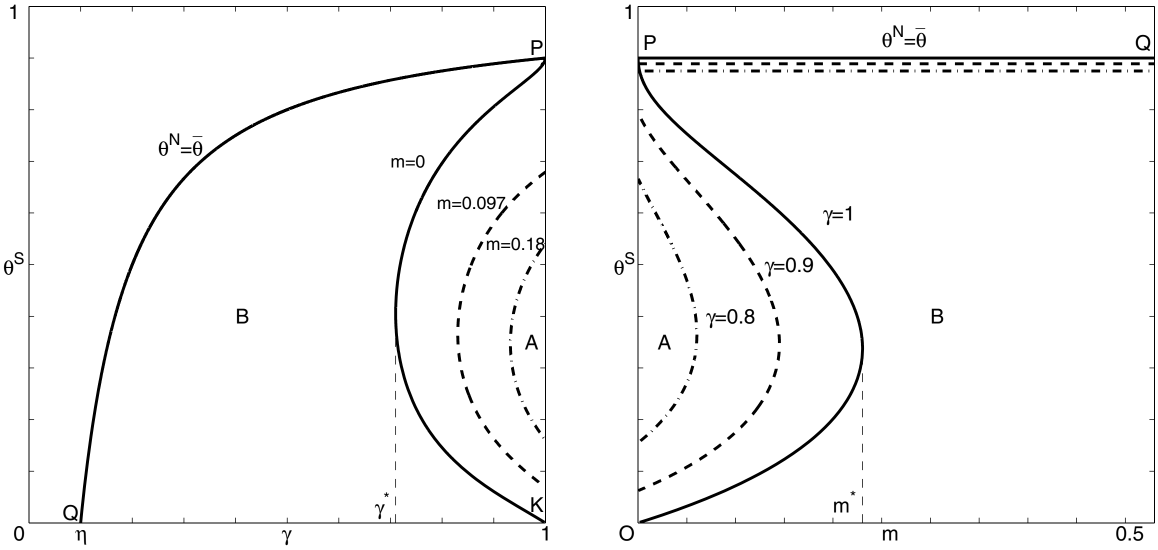

Figure 2: Threshold Values under Full Capital Mobility

The left panel of figure 2 shows the relationship between γ, and the threshold values for world output gains, $\tilde{\theta}^S$ which is illustrated in the bottom-right panel of figure 1. Here, the vertical axis denotes $\theta^S \in [0, 1]$, while the horizontal axis denotes $\gamma \in [0, 1]$. Curve PQ represents the threshold value $\bar{\theta} \equiv 1 - \frac{\eta}{\gamma}$ as a function of γ. We focus on the case of $0 \leq \theta^S < \theta^N = \bar{\theta}$, i.e., the region below curve PQ. Curve PK shows the threshold value $\tilde{\theta}^S$ as a function of γ. The region between curve PK and the right boundary is defined as region A, where world output is smaller, while the region between curve PK, PQ, and the horizontal axis is defined as region B, where world output is larger than under IFA.

Intuitively, if θ^S close to θ^N, on the one hand, the output distortion in country S is small under IFA and so is the investment composition effect under full capital mobility; on the other hand, the cross-country interest rate differentials are small under IFA and so are net capital flows as well as the investment reallocation effect under full capital mobility. Overall, the investment composition effect still dominates the investment reallocation effect so that world output is higher. By the similar logic, if θ^S close to 0, although the cross-country interest rate differentials are large under IFA and so are net capital flows as well as the investment reallocation effect, the output distortion in country S is also large under IFA and so is the investment composition effect under full capital mobility. Overall, world output may still be higher. The key factor here is the output distortion under IFA which depends critically on γ. For γ converges to one, the investment composition effect declines gradually and it is more likely that full capital mobility reduces world output.

The Savings Effect

Next, we set $\gamma = 1$ to shut down the investment composition effect. Consider two alternative cases where the labor endowment when old are $\epsilon \in \{1, 0.2\}$ and accordingly, $m \in \{0.53, 0.18\}$. Thus, savings are interest elastic in the two cases.

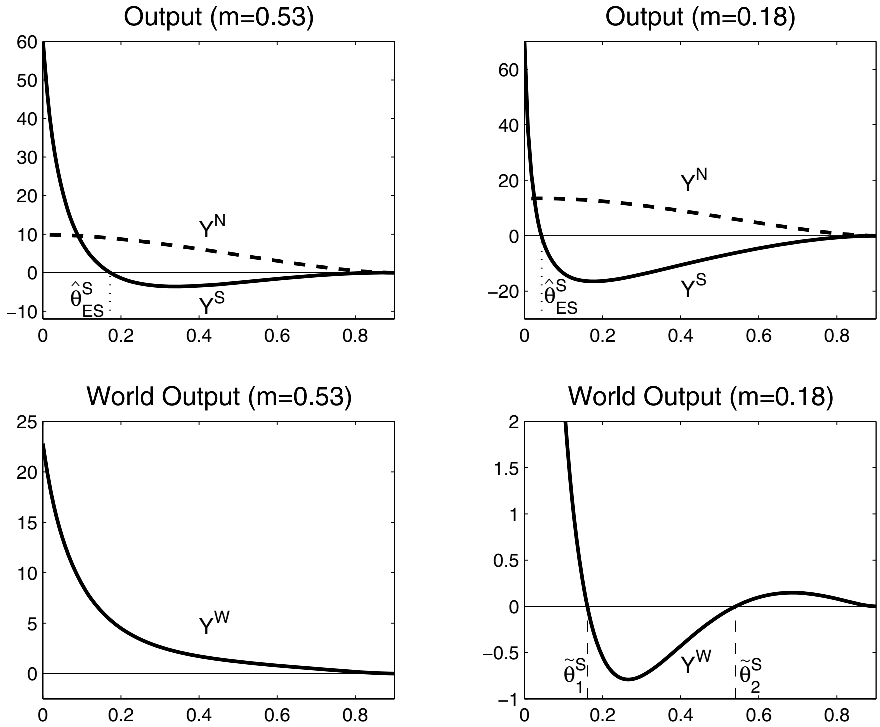

Figure 3: Steady-State Output Patterns only with the Savings Effect

The left and right panels of figure 3 compares steady-state output under full capital mobility versus under IFA at the country and the global level for $m \in \{0.53, 0.18\}$, respectively. The axis scalings are same as in figure 1. Here, the output patterns at the country level confirm the results in Lemma 4, which are also qualitatively same as in the presence of only the investment composition effect.

The world output implications depend critically on m. Intuitively, a higher ϵ leads to a higher interest elasticity of savings as well as a higher m. Thus, the output distortion under IFA is more severe and the saving effect is larger under full capital mobility. Hence, it is more likely that full capital mobility raises world output. The right panel of figure

2 shows the relationship between m and the threshold values for world output gains, $\tilde{\theta}^S$ which is illustrated in the bottom-right panel of figure 3. Solid line PQ represents the threshold value $\bar{\theta}$ which is independent of m. We focus on the case of $0 \leq \theta^S < \theta^N = \bar{\theta}$, i.e., the region below line PQ. Curve PO shows the threshold value $\tilde{\theta}^S$ as a function of m. The region between curve PO and the left vertical boundary is defined as region A, where world output is smaller, while the region to the right of curve PO and below line PQ is defined as region B, where world output is larger under full capital mobility than under IFA. Obviously, the higher m, the more likely it is that full capital mobility raises world output.

Three Effects Combined

Figure 2 shows the world output implications in the presence of the investment composition effect and the savings effect together with the investment reallocation effect. The left panel plots the threshold values $\tilde{\theta}^S$ in the space of (γ, θ^S) under the alternative values of $\epsilon \in \{0, 0.1, 0.2\}$ which correspond to $m \in \{0, 0.097, 0.18\}$, while the right panel plots the threshold values $\tilde{\theta}^S$ in the space of (m, θ^S) under the alternative values of $\gamma \in \{0.8, 0.9, 1\}$. As before, region A (B) indicates the parameter combinations where full capital mobility reduces (raises) world output.

Consider the left panel of figure 2. A rise in m from 0 to 0.097 makes savings interest elastic and financial frictions distort output through the savings effect. Since the presence of the savings effect reduces the size of region A, it is more likely that the investment composition effect dominates the investment reallocation effect and full capital mobility raises world output. Similarly, if γ declines from 1 to 0.9, financial frictions distort output through the investment composition effect. According to the right panel of figure 2, since the presence of the investment composition effect reduces the size of region A, it is more likely that the savings effect dominates the investment reallocation effect and full capital mobility raises world output.

4 Partial Capital Mobility

In this section, we compare the world output implications under full capital mobility versus under two cases of partial capital mobility, i.e., *free mobility of financial capital* where individuals are allowed to lend abroad but not to make direct investment abroad, and *free mobility of FDI* where entrepreneurs are allowed to make direct investment abroad but individuals are not allowed to lend abroad. In either cases, the steady-state patterns of capital flows, interest rates, relative prices are similar as under full capital mobility. See Appendix B for details. Here, we only focus on the output implications.

Under free mobility of financial capital, since capital flows are one-way and the net

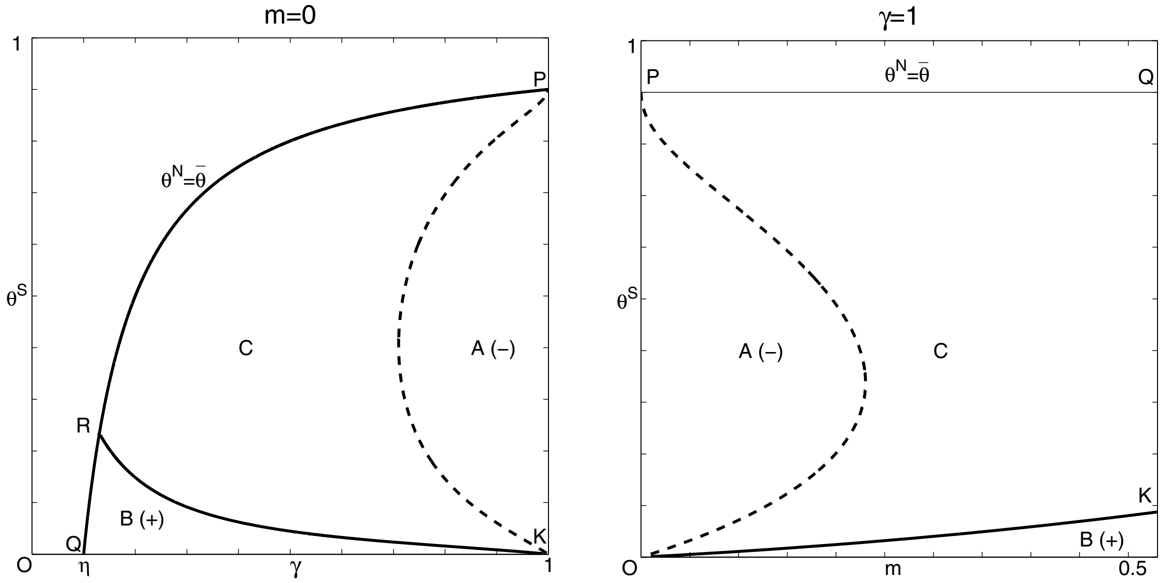

Figure 4: Threshold Values under Free Mobility of Financial Capital

flows coincide with the gross flows, the investment reallocation effect always dominates the investment composition effect and the savings effect so that "uphill" financial capital flows reduces (raises) steady-state output in the countries in group S (N), which widens the output gap between the two groups. Given $\theta^N = \bar{\theta}$, the left and the right panels of figure 4 show the threshold values under two scenarios of capital mobility in the space of (γ, θ^S) and (m, θ^S), respectively. The dashed curve refers to the threshold values under full capital mobility as shown in figure 2, while the solid curve splitting region B and C refers to the threshold values under free mobility of financial capital. Full capital mobility raises world output for the parameter values in region B and C, while partial capital mobility raises world output only for the parameter values in region B. Intuitively, by restricting FDI flows, capital flows become one-way and net flows coincide with gross flows so that world output is more likely lower.

Under free mobility of FDI, since the investment reallocation effect works in the same direction as the investment composition effect and the savings effect, FDI flows strictly raises (reduces) steady-state output in the countries in group S (N). Given $\theta^N = \bar{\theta}$, the left and the right panels of figure 5 show the threshold values under two scenarios of capital mobility in the space of (γ, θ^S) and (m, θ^S), respectively. The dashed curves refer to the threshold values under full capital mobility as shown in figure 2, while the solid curves RK and OK to the threshold values under free mobility of FDI. Full capital mobility raises world output for the parameter values in region B and C, while partial capital mobility raises world output for the parameter values in region B and D. Intuitively, for the parameter values in region C, θ^S is close to θ^N so that FDI flows not only make steady-state output higher in country S than in country N but also widen the cross-

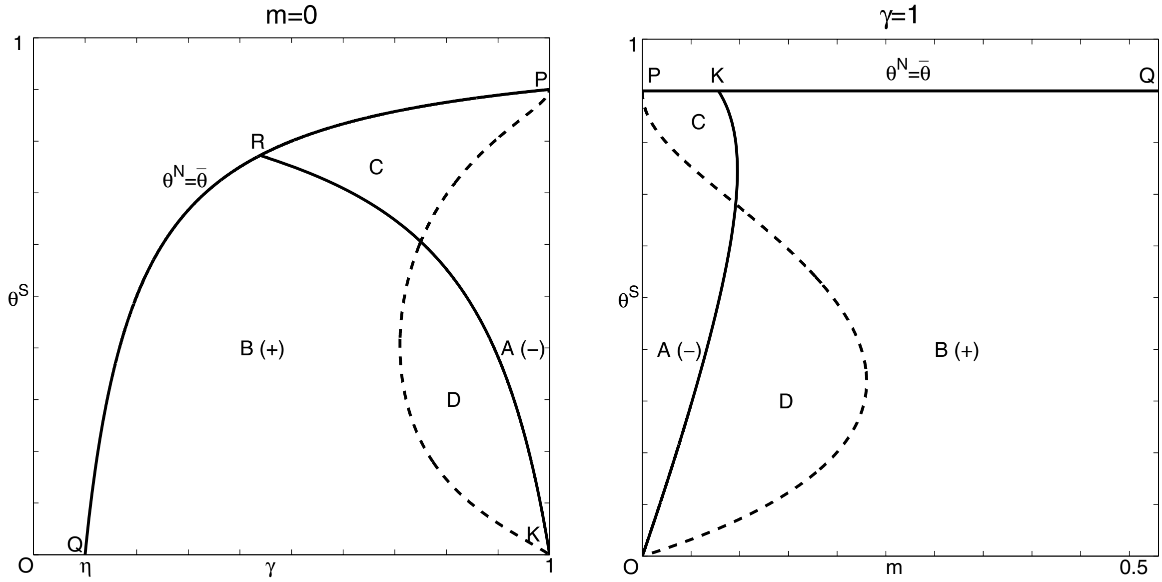

Figure 5: Threshold Values under Free Mobility of FDI

country output gap, which reduces world output; for the parameter values in region D, θ^S is much smaller than θ^N so that FDI flows reduce the cross-country output gap, which raises world output.

5 Conclusion

We develop a tractable multi-country model where domestic financial frictions distort interest rates. Given the cross-country differences in financial development, the interest rate differentials drive international capital flows and the theoretical predictions are consistent with the empirical patterns in the recent past.

We also address the output implications of international capital mobility. Financial frictions in our model distort the composition of domestic investment and the size of domestic savings under IFA. Under full capital mobility, on the one hand, net capital flows directly generate cross-country resource reallocation and affect aggregate output, on the other hand, financial capital and FDI flows indirectly trigger the change in the size of domestic savings and in the composition of domestic investment. Under certain conditions, the indirect effects may dominate the direct effect so that, despite "uphill" net capital flows, full capital mobility may raise aggregate output in the poor country as well as raise world output. Our results complement conventional neoclassical models by showing the impacts of capital mobility on investment composition and aggregate savings.

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A Proofs

Proof of Lemma 2

Proof. The proof consists of three steps. First, we prove that equation (26) is the solution to the equity rate under full capital mobility. Define $\Delta\chi^i_{t+1} \equiv \chi^i_{t+1} - \chi^i_{IFA}$. If the borrowing constraints are binding, it holds under IFA and under full capital mobility,

$$\chi^i_{t+1} = \frac{R^i_t(1-\theta^i)}{\Gamma^i_t} + \theta^i, \quad \Rightarrow \quad \frac{\Delta\chi^i_{t+1}}{1-\theta^i} = \frac{R^i_t}{\Gamma^i_t} - \frac{R^i_{IFA}}{\Gamma^i_{IFA}}. \tag{39}$$

According to equation (15), $(1-\eta)R^i_{IFA} + \eta\Gamma^i_{IFA} = \mathcal{Q}$. Substituting R^i_t and R^i_{IFA} with Γ^i_t and Γ^i_{IFA} using equation (25) and $R^i_{IFA} = \frac{1}{(1-\eta)}(\mathcal{Q} - \eta\Gamma^i_{IFA})$, we solve the equity rate from equation (39). Plug the solution to the equity rate in equation (25) to solve R^i_t.

Second, we prove that χ^i_{t+1} is constant under full capital mobility. Let us assume that χ^i_{t+1} is time variant and so is the auxiliary variable Z^i_{t+1} defined in equation (26). According to equation (26), the equity rate equalization in country i and N implies that

$$\Gamma^i_{IFA} - \mathcal{Z}^i_{t+1} = \Gamma^N_{IFA} - \mathcal{Z}^N_{t+1}, \tag{40}$$

$$\Delta\chi^i_{t+1} = \frac{1-\theta^i}{1-\theta^N}\Delta\chi^N_{t+1} + \left(\frac{1}{p^N_{IFA}} - \frac{1}{\wp^i_{IFA}}\right)\frac{1-\theta^i}{1-\eta}, \tag{41}$$

$$\frac{\partial\Delta\chi^i_{t+1}}{\partial\Delta\chi^N_{t+1}} = \frac{1-\theta^i}{1-\theta^N} > 0. \tag{42}$$

Using equations (26), (31), and (41), we rewrite the condition, $\sum_{i=1}^N \Omega^i_t = 0$, into

$$\sum_{i=1}^N \omega^i_{t+1}\Delta\chi^i_{t+1}\frac{\wp^i_{IFA}(1-\eta)}{1-\theta^i} = 0 \tag{43}$$

Given the Cobb-Douglas production function, $\omega^i_{t+1} = (\chi^i_{t+1})^{\gamma\rho}(R^i_t)^{-\rho}$. Combining it with the loan rate equalization, $R^i_t = R^*_t$, we simplify equation (43) as

$$\sum_{i=1}^N \mathcal{K}^i_{t+1} = 0, \quad \text{where } \mathcal{K}^i_{t+1} \equiv (\Delta\chi^i_{t+1} + \chi^i_{IFA})^{\gamma\rho}\Delta\chi^i_{t+1}\frac{\wp^i_{IFA}(1-\eta)}{1-\theta^i}, \tag{44}$$

$$\frac{\partial\mathcal{K}^i_{t+1}}{\partial\Delta\chi^i_{t+1}} = (\chi^i_{t+1})^{\gamma\rho-1}(\chi^i_{t+1} + \gamma\rho\Delta\chi^i_{t+1})\frac{\wp^i_{IFA}(1-\eta)}{1-\theta^i} > 0. \tag{45}$$

Using equations (41) to substitute $\Delta\chi^i_{t+1}$ with $\Delta\chi^N_{t+1}$, the left-hand side of equation (44) becomes a monotonically increasing function of $\Delta\chi^N_{t+1}$,

$$\frac{\partial\sum_{i=1}^N \mathcal{K}^i_{t+1}}{\partial\Delta\chi^N_{t+1}} = \sum_{i=1}^N \frac{\partial\mathcal{K}^i_{t+1}}{\partial\Delta\chi^i_{t+1}}\frac{\partial\Delta\chi^i_{t+1}}{\partial\Delta\chi^N_{t+1}} > 0. \tag{46}$$

Thus, there exits a unique solution of $\Delta\chi^N_{t+1}$ which is time-invariant. Using equations (41), we can then solve $\Delta\chi^i_{t+1}$ for $i \in \{1, 2, ..., N-1\}$, accordingly.

Finally, we prove the existence of a unique and stable steady state under full capital mobility. χ^i_{t+1} is time-invariant and so is $\mathcal{Z}^i_{t+1}$. Let $R^i_{FCM} \equiv R^i_{IFA} + \frac{\eta}{1-\eta}\mathcal{Z}^i_{FCM}$ which is same across countries, $R^i_{FCM} = R^*_{FCM}$. Thus, the loan rate depends on the dynamics of the world-average wages, according to equation (27). So is the wage in country i,

$$\omega^i_{t+1} = \left(\frac{\omega^w_{t+1}}{\omega^w_t} R^*_{FCM}\right)^{-\rho} (\chi^i_{FCM})^{\rho\gamma}.$$

The dynamics of the world-average wages are

$$\omega^w_{t+1} = \frac{\sum_{i=1}^N \omega^i_{t+1}}{N} = \left(\frac{\omega^w_{t+1}}{\omega^w_t} R^*_{FCM}\right)^{-\rho} \frac{\sum_{i=1}^N (\chi^i_{FCM})^{\rho\gamma}}{N},$$

$$\omega^w_{t+1} = \left(\frac{\omega^w_t}{R^*_{FCM}}\right)^{\alpha} \left[\frac{\sum_{i=1}^N (\chi^i_{FCM})^{\rho\gamma}}{N}\right]^{1-\alpha}$$

Given $\alpha \in (0, 1)$, the phase diagram of the world-average wage is concave. Thus, there exists a unique and stable steady state. Proportional to the wage, aggregate output in country i is determined by the world output dynamics. □

Proof of Proposition 2

Proof. According to equation (18), the steady-state loan rate in country i monotonically increases in θ^i under IFA, which together with equation (36) and the world credit market clearing condition, $\sum_{t=1}^N \Phi^i_{FCM} = 0$, implies that there exists a threshold value of the country index $\hat{N}$ such that $Z^{\hat{N}}_{FCM} > 0 \geq Z^{\hat{N}+1}_{FCM}$. Thus, the world loan rate is $R^*_{FCM} \in (R^{\hat{N}}_{IFA}, R^{\hat{N}+1}_{IFA}]$. According to equations (15) and (25), it holds in the steady state that $(1-\eta)R^i_j + \eta\Gamma^i_j = \mathcal{Q}$, where $j \in \{IFA, FCM\}$ denotes the scenario of IFA and full capital mobility. Thus, $\Gamma^*_{FCM} \in [\Gamma^{\hat{N}+1}_{IFA}, \Gamma^{\hat{N}}_{IFA})$.

Given that $\mathcal{Z}^i_{FCM}$ monotonically increases in $\Delta\chi^i_{FCM}$ and $R^*_{FCM} \in (R^{\hat{N}}_{IFA}, R^{\hat{N}+1}_{IFA})$, it is obvious that full capital mobility raises the relative prices in country $i \in \{1, 2, ..., \hat{N}\}$, $\chi^i_{FCM} > \chi^i_{IFA}$ and $\psi^i_{FCM} > \psi^i_{IFA}$. The gross equity premium is by definition, $\frac{\Gamma^i_{FCM}}{R^i_{FCM}} = \frac{1-\theta^i}{\chi^i_{FCM}-\theta^i}$, and its cross-country equalization implies $\frac{1-\chi^i_{FCM}}{1-\theta^i} = \frac{1-\chi^{i+1}_{FCM}}{1-\theta^{i+1}} = \frac{\chi^{i+1}_{FCM}-\chi^i_{FCM}}{\theta^{i+1}-\theta^i} > 0$. Given $\theta^{i+1} > \theta^i$, it holds that $\chi^{i+1}_{FCM} > \chi^i_{FCM}$. According to equation (29), we get $\psi^{i+1}_{FCM} > \psi^i_{FCM}$. Similar as under IFA, the relative prices under full capital mobility monotonically increase in θ^i. According to equations (36) and (37), the changes in the interest rates imply that in country $i \in \{1, 2, ..., \hat{N}\}$, $\Phi^i_{FCM} > 0 > \Omega^i_{FCM}$. Since $\Gamma^*_{FCM} > R^*_{FCM}$, the steady-state net capital flows have the same sign as $\mathcal{Z}^i_{FCM}$, according to equation (38). Thus, $\Phi^i_{FCM} + \Omega^i_{FCM} > 0$ in country $i \in \{1, 2, ..., \hat{N}\}$. The opposite applies to country $i \in \{\hat{N}+1, \hat{N}+2, ..., N\}$.

According to equations (36) and (37), the gross international investment returns are

$$R^*_{FCM}\Phi^i_{FCM} + \Gamma^*_{FCM}\Omega^i_{FCM} = \rho\omega^i_{FCM}(1-\eta)(\mathcal{Z}^i_{FCM} - \mathcal{Z}^i_{FCM}) = 0.$$

□

Proof of Proposition 3

Proof. According to equations (15), (25), (33), the steady-state relative prices, interest rates, and wages have the same relationship under full capital mobility and under IFA,

$$\omega_j^i = (\chi_j^i)^{\gamma\rho}(R_j^i)^{-\rho}, \quad \chi_j^i = \frac{R_j^i}{\Gamma_j^i}(1-\theta^i) + \theta^i, \quad \eta\Gamma_j^i + (1-\eta)R_j^i = \mathcal{Q}. \tag{47}$$

where $j \in \{IFA, FCM\}$ refers to the scenarios of IFA and full capital mobility, respectively. Under full capital mobility, $\omega_{FCM}^i = (\chi_{FCM}^i)^{\gamma\rho}(R_{FCM}^i)^{-\rho}$, $R_{FCM}^i = R_{FCM}^{i+1}$ and $\chi_{FCM}^i < \chi_{FCM}^{i+1}$ jointly imply that $\omega_{FCM}^i < \omega_{FCM}^{i+1}$, or equivalently, $Y_{FCM}^i < Y_{FCM}^{i+1}$.

We define two auxiliary variables, $r_j^i \equiv \frac{R_j^i}{\mathcal{Q}}$ and $\wp_j^i \equiv \frac{\Gamma_j^i}{\mathcal{Q}}$ by normalizing the interest rates with $\mathcal{Q}$. According to equations (47), the steady-state wage under IFA as well as under full capital mobility w_j^i is a function of the normalized equity rate $\wp_j^i$,

$$\omega_j^i = \frac{1}{\mathcal{Q}^\rho}\left[\frac{(1-\theta^i)r_j^i}{\wp_j^i} + \theta^i\right]^{\gamma\rho}(r_j^i)^{-\rho} \quad \text{and} \quad r_j^i = \frac{1-\eta\wp_j^i}{1-\eta}. \tag{48}$$

Given θ^i, if full capital mobility affects the equity rate in country i, the wage and hence output in this country change accordingly. Define $\mathcal{T}_j^i \equiv \frac{\partial\omega_j^i}{\partial\wp_j^i}$ as the first derivative of ω_j^i with respect to $\wp_j^i$,

$$\mathcal{T}_j^i \equiv \frac{\partial\omega_j^i}{\partial\wp_j^i} = \frac{\rho\omega_j^i\mathcal{N}_j^i}{[(1-\theta^i)r_j^i + \theta^i\wp_j^i]\wp_j^i r_j^i}, \tag{49}$$

$$\mathcal{N}_j^i \equiv \theta^i\left[\frac{(1-r_j^i)^2}{\eta} + \frac{1}{1-\eta}\right] - r_j^i\left[r_j^i - (1-\theta^i)\frac{(1-\gamma)}{1-\eta}\right]. \tag{50}$$

Thus, $\mathcal{T}_j^i$ has the same sign as $\mathcal{N}_j^i$.

By definition, $\frac{\partial\mathcal{A}^i}{\theta^i} = \frac{\gamma}{1-\eta} > 0$. Thus, the minimum value of $\mathcal{A}^i$ is $\mathcal{A}_{min}^i = \frac{1-\gamma}{1-\eta}$ for $\theta^i = 0$. Accordingly, the minimum value of the normalized loan rate is

$$r_{min}^i = \frac{m + A_{min}^i}{m+1} > A_{min}^i = \frac{1-\gamma}{1-\eta} > (1-\theta^i)\frac{(1-\gamma)}{1-\eta} \quad \Rightarrow \quad r_j^i - (1-\theta^i)\frac{(1-\gamma)}{1-\eta} > 0.$$

Thus, the formula in the second square bracket on the right hand side of equation (50) is positive. If $\mathcal{N}_j^i \geq 0$, a marginal decline in r_j^i keeps $\mathcal{T}_j^i > 0$; if $\mathcal{N}_j^i \leq 0$, a marginal rise in r_j^i keeps $\mathcal{T}_j^i < 0$.

According to equations (17)-(18), $\wp_{IFA}^i = \frac{m+B^i}{m+1}$ and $r_{IFA}^i = \frac{m+A^i}{m+1}$. Evaluate $\mathcal{T}_j^i$ in the steady state under IFA by substituting $\wp_{IFA}^i$ and r_{IFA}^i into equation (49)-(50), we get

$$\mathcal{T}_{IFA}^i = \frac{\partial\omega_{IFA}^i}{\partial\wp_{IFA}^i} = \rho\omega_{IFA}^i(1+m)\frac{\frac{(\bar\theta-\theta^i)}{1-\eta}\left[\frac{\gamma(1-\theta^i)}{\eta(1-\eta)}\left(\theta^i - \frac{m(1-\gamma)\eta}{\gamma}\right) - m^2\right]}{(m+A^i)(m+B^i)\left[m + B\left(1 - \frac{(\bar\theta-\theta^i)}{1-\eta}\right)\right]}, \tag{51}$$

$$\mathcal{N}_{IFA}^i = \left[\frac{\gamma(1-\theta^i)}{\eta(1-\eta)}\left(\theta^i - \frac{m(1-\gamma)\eta}{\gamma}\right) - m^2\right]\frac{(\bar\theta-\theta^i)}{(1-\eta)}\frac{1}{(1+m)^2}, \tag{52}$$

and $\mathcal{T}^i_{IFA}$ has the same sign as $\mathcal{N}^i_{IFA}$.

We take the following approach to provide the sufficient conditions on the output implications of full capital mobility. Consider countries in group N. For any country with θ^n making $\mathcal{N}^n_{IFA} \geq 0$, full capital mobility reduces the steady-state loan rate so that $\mathcal{N}^n_{FCM} > \mathcal{N}^n_{IFA} \geq 0$. Thus, $\mathcal{T}^n_{FCM} > 0$ and $\mathcal{T}^n_{IFA} \geq 0$. As full capital mobility raises the steady-state equity rate for country n, we get $\omega^n_{FCM} > \omega^n_{IFA}$ or $Y^n_{FCM} > Y^n_{IFA}$. Consider countries in group S. For any country with θ^s making $\mathcal{N}^s_{IFA} \leq 0$, full capital mobility raises the steady-state loan rate so that $\mathcal{N}^s_{FCM} < \mathcal{N}^s_{IFA} \leq 0$. Thus, $\mathcal{T}^n_{FCM} < 0$ and $\mathcal{T}^n_{IFA} \leq 0$. As full capital mobility reduces the steady-state equity rate for country s, we get $\omega^s_{FCM} > \omega^s_{IFA}$ or $Y^s_{FCM} > Y^s_{IFA}$.

It is trivial to prove the general results for $\theta^i = 0$ and $\theta^i = \bar{\theta}$ in this approach. Consider a country in group N. If $\theta^n = \bar{\theta}$, $\mathcal{N}^n_{IFA} = 0$ so that full capital mobility strictly raises its steady-state output, $Y^n_{FCM} > Y^n_{IFA}$. Consider a country in group S. If $\theta^s = 0$, $\mathcal{N}^s_{IFA} \leq 0$ so that full capital mobility strictly raises its steady-state output, $Y^s_{FCM} > Y^s_{IFA}$.

□

Proof of Lemma 3

Proof. In the case of $m = 0$ and $\gamma \in (0, 1)$, equation (52) is simplified as

$$\mathcal{T}^i_{IFA} = \rho\omega^i_{IFA} \frac{\frac{(\bar{\theta}-\theta^i)\theta^i}{(1-\eta)^2}}{A^i B^i \left(1 - \frac{(\bar{\theta}-\theta^i)}{1-\eta}\right)}. \tag{53}$$

For $\theta^i \in (0, \bar{\theta})$, it holds that $\mathcal{T}^i_{IFA} > 0$.

Consider country n. For $\theta^n \in (\theta^{\hat{N}}, \bar{\theta})$, $\mathcal{N}^n_{IFA} \geq 0$. According to the approach mentioned above, it strictly holds that $\omega^n_{FCM} > \omega^n_{IFA}$ and hence $Y^n_{FCM} > Y^n_{IFA}$. Thus, full capital mobility strictly raises the steady-state output of each country in group N.

Consider country s. For $\theta^s \in (0, \theta^{\hat{N}}]$, it holds that $\mathcal{T}^i_{IFA} > 0$. If θ^s is slightly lower than $\theta^{\hat{N}+1}$, it is likely that $\mathcal{T}^i_{FCM} > 0$. In this case, by reducing the steady-state equity rate, full capital mobility reduces the steady-state aggregate output, $Y^s_{FCM} < Y^s_{IFA}$. In contrast, for θ^s close to 0, despite $\mathcal{T}^i_{IFA} > 0$, the rise in the steady-state loan rate may keep $\mathcal{T}^i_{FCM} < 0$. Thus, by reducing the steady-state equity rate, full capital mobility raises the steady-state aggregate output, $Y^s_{FCM} > Y^s_{IFA}$. There exists a threshold value $\hat{\theta}^S_{IC}$ such that for $\theta^s \in [0, \hat{\theta}^S_{IC})$, $Y^s_{FCM} > Y^s_{IFA}$, and for $\theta^s \in (\hat{\theta}^S_{IC}, \theta^{\hat{N}}]$, the opposite applies. □

Proof of Lemma 4

Proof. In the case of $m > 0$ and $\gamma = 1$, equation (52) is simplified as

$$\mathcal{T}^i_{IFA} = \rho\omega^i_{IFA}(1+m)\frac{\frac{(\bar{\theta}-\theta^i)}{1-\eta}\left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - m^2\right]}{(m+A^i)(m+B^i)\left[m + B\left(1 - \frac{(\bar{\theta}-\theta^i)}{1-\eta}\right)\right]}, \tag{54}$$

$$\mathcal{N}^i_{IFA} = \left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - m^2\right]\frac{(\bar{\theta}-\theta^i)}{(1-\eta)}\frac{1}{(1+m)^2}. \tag{55}$$

For $\theta^i \in [0, \bar{\theta})$, the sign of $\mathcal{T}^i_{IFA}$ depends on that of $\mathcal{N}^i_{IFA}$, or, that of $\left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - m^2\right]$.

Figure 6 shows all possible cases on the relative size of $\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)}$ and m^2 where the three panels in the first row show the cases with $\eta \in (0, 0.5)$, the two panels in the second row show the cases with $\eta \in (0, 5, 1)$, and the horizontal axis shows $\theta^i \in (0, \bar{\theta})$.

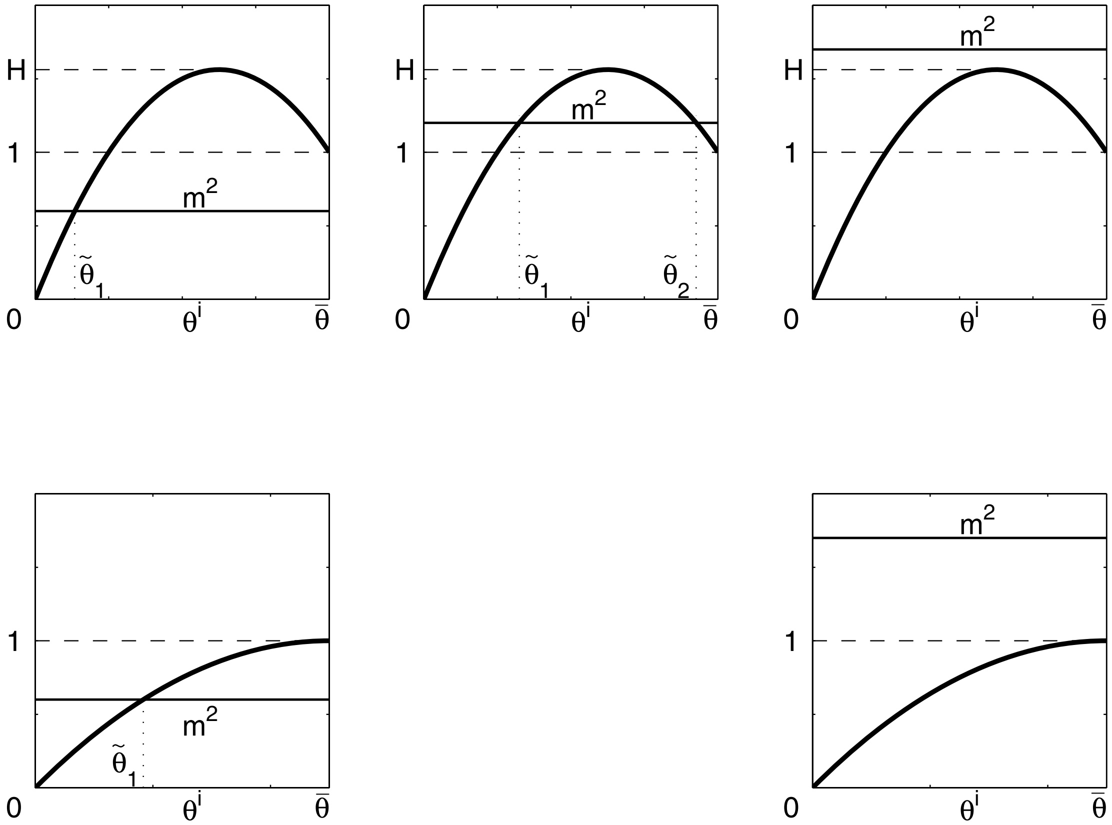

Figure 6: Threshold Values under Various Scenarios

Given $\eta \in (0, 0.5)$, $\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} \in (0, \frac{1}{4\eta(1-\eta)})$ is a hump-shaped function of $\theta^i \in (0, \bar{\theta})$. Point H denotes its highest value $\frac{1}{4\eta(1-\eta)} > 1$. Define $\kappa \equiv \frac{1-\sqrt{1-4m^2(1-\eta)\eta}}{2}$.

- If $m \in (0, 1)$, there exists a threshold value $\tilde{\theta}_1 = \kappa$ such that, for $\theta^i \in (0, \tilde{\theta}_1)$, $\mathcal{N}^i_{IFA} < 0$ and, for $\theta^i \in (\tilde{\theta}_1, \bar{\theta})$, the opposite applies.
- If $m \in (1, \frac{1}{2\sqrt{\eta(1-\eta)}})$, there exists two threshold values $\tilde{\theta}_1 = \kappa$ and $\tilde{\theta}_2 = 1 - \kappa$ such that for $\theta^i \in (\tilde{\theta}_1, \tilde{\theta}_2)$, $\mathcal{N}^i_{IFA} > 0$ and, for $\theta^i \in (0, \tilde{\theta}_1) \cup (\tilde{\theta}_2, \bar{\theta})$, the opposite applies.

- If $m > \frac{1}{2\sqrt{\eta(1-\eta)}}$, for $\theta^i \in (0, \bar{\theta})$, it holds that $\mathcal{N}^i_{IFA} < 0$.

Given $\eta \in (0.5, 1)$, $\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} \in (0, 1)$ is a monotonically increasing function of $\theta^i \in (0, \bar{\theta})$.

- If $m \in (0, 1)$, there exists a threshold value $\tilde{\theta}_1 = \kappa$ such that, for $\theta^i \in (0, \tilde{\theta}_1)$, $\mathcal{N}^i_{IFA} > 0$ and, for $\theta^i \in (\tilde{\theta}_1, \bar{\theta})$, the opposite applies.
- If $m > 1$, for $\theta^i \in (0, \bar{\theta})$, $\mathcal{N}^i_{IFA} < 0$.

Using the approach mentioned in the proof of Proposition 3, we can prove Lemma 4. □

B Partial Capital Mobility

B.1 Free Mobility of Financial Capital

Financial capital flows equalize the loan rate globally and the credit markets clear at the country and at the world level,

$$R^i_t = R^*_t, \quad (1-\eta)(s^i_t - i^{i,A}_t) = (\lambda^i_t - 1)\eta n^i_t + \Phi^i_t, \quad \text{and} \quad \sum_{i=1}^{N} \Phi^i_t = 0.$$

The remaining conditions for market equilibrium are same as under IFA. The model solutions are

$$\Gamma^i_t = \frac{\omega^i_{t+1}}{\omega^i_t}\Gamma^i_{IFA} \tag{56}$$

$$R^i_t = \frac{\omega^i_{t+1}}{\omega^i_t}R^i_{IFA} + \frac{\omega^i_{t+1}}{\omega^i_t}\mathcal{Z}^i_{t+1}, \quad \text{where} \quad \mathcal{Z}^i_{t+1} \equiv \frac{(\chi^i_{t+1} - \chi^i_{IFA})\Gamma^i_{IFA}}{1-\theta^i}, \tag{57}$$

$$\Phi^i_t = (1-\eta)\beta\omega^i_t\left(1 - \frac{\omega^i_{t+1}}{\omega^i_t}\frac{R^i_{IFA}}{R^i_t}\right) \tag{58}$$

$$\omega^i_{t+1} = \left(\frac{\Lambda^i_t}{\mathcal{Q}}\omega^i_t\right)^{\alpha}, \quad \text{where} \quad \Lambda^i_t \equiv \frac{(\chi^i_{t+1})^{\gamma}(1-\theta^i)(m+1)}{(\chi^i_{t+1}-\theta^i)(m+B^i)}, \tag{59}$$

$$\frac{\partial \ln \Lambda^i_t}{\partial \chi^i_{t+1}} = -\frac{\chi^i_{t+1}(1-\gamma)+\gamma\theta^i}{\chi^i_{t+1}(\chi^i_{t+1}-\theta^i)} < 0 \tag{60}$$

Let X_{FCF} denote the steady-state value of variable X under free mobility of financial capital.

Lemma 6. *There exists a unique and stable steady state under free mobility of financial capital.*

Proof. Combining equations (57) and (59), we rewrite the dynamic equation of wages,

$$\ln \omega^i_{t+1} = -\rho \ln R^*_t + \gamma\rho \ln\left(\frac{\omega^i_t}{\omega^i_{t+1}}R^*_t\frac{(1-\theta^i)}{\Gamma^i_{IFA}} + \theta^i\right). \tag{61}$$

Define $\mathcal{W}^i \equiv \frac{\partial \ln \omega_{t+1}^i}{\partial \ln \omega_t^i}$. The first and the second derivatives of ω_{t+1}^i with respect to ω_t^i are

$$\frac{\partial \omega_{t+1}^i}{\partial \omega_t^i} = \frac{\omega_{t+1}^i}{\omega_t^i} \frac{\rho\gamma}{\rho\gamma + 1 + \frac{\theta^i}{\chi_{t+1}^i - \theta^i}} \in \left(0, \frac{\omega_{t+1}^i}{\omega_t^i}\right), \quad \Rightarrow \mathcal{W}^i \in (0,1)$$
$$\frac{\partial^2 \omega_{t+1}^i}{\partial (\omega_t^i)^2} = -\left(1 - \mathcal{W}^i\right) \left(\mathcal{W}^i\right)^2 \frac{\omega_{t+1}^i}{(\omega_t^i)^2} \left(1 + \frac{1}{\rho\gamma}\right).$$

Since $\mathcal{W}^i \in (0,1)$, we get $\frac{\partial^2 \omega_{t+1}^i}{\partial (\omega_t^i)^2} < 0$. Thus, the phase diagram of wages is a concave function under free mobility of financial capital if the borrowing constraints are binding.

According to equation (61), for $\omega_t^i = 0$, the phase diagram has a positive intercept on the vertical axis at $\omega_{t+1}^i = (R_t^*)^{-\rho}(\theta^i)^{(\gamma\rho)}$. Define a threshold value $\bar{\omega}_t^i = \Gamma_{IFA}^i (R_t^*)^{-\frac{1}{1-\alpha}}$. For $\omega_t^i \in (0, \bar{\omega}_t^i)$, the phase diagram of wages is monotonically increasing and concave. For $\omega_t^i > \bar{\omega}_t^i$, aggregate saving and investment in sector B is so high that the intratemporal relative price is equal to one, or equivalently, $R_t^i = v_{t+1}^{i,B}$. Thus, the borrowing constraints are slack and the phase diagram is flat with $\omega_{t+1}^i = \bar{\omega}_{t+1}^i = (R_t^*)^{-\rho}$. Given $R_t^* < \mathcal{Q} < \Gamma_{IFA}^i$, we get $\bar{\omega}_{t+1}^i < \bar{\omega}_t^i$. In other words, the kink point is below the 45 degree line.

Thus, the phase diagram of wages crosses the 45 degree line once and only once from the left, and the intersection is in its concave part. Thus, the model economy has a unique and stable steady state under free mobility of financial capital. □

In the steady state, the interest rates and financial capital flows are

$$\Gamma_{FCF}^i = \Gamma_{IFA}^i, \tag{62}$$
$$R_{FCF}^i = R_{IFA}^i + \mathcal{Z}_{FCF}^i, \quad \text{where} \quad \mathcal{Z}_{FCF}^i \equiv \frac{(\chi_{FCF}^i - \chi_{IFA}^i)\Gamma_{IFA}^i}{1-\theta^i}, \tag{63}$$
$$\Phi_{FCF}^i = (1-\eta)\beta\omega_{FCF}^i \frac{\mathcal{Z}_{FCF}^i}{R_{FCF}^*}, \tag{64}$$

Proposition 4. *In the steady state under free mobility of financial capital, there exists a threshold value of the country index $\tilde{N}$ such that the world loan rate is $R_{IFA}^{\tilde{N}} < R_{FCF}^* \leq R_{IFA}^{\tilde{N}+1}$ and the equity rate in each country is $\Gamma_{FCF}^i = \Gamma_{IFA}^i$. In country $i \in \{1, 2, ..., \tilde{N}\}$, free mobility of financial capital leads to financial capital outflows, $\Phi_{FCF}^i > 0$, so that the relative intermediate goods price and the relative loan rate rise, $\chi_{PCM}^i > \chi_{IFA}^i$ and $\psi_{FCF}^i > \psi_{IFA}^i$, and aggregate output falls, $Y_{FCF}^i < Y_{IFA}^i$; the opposite applies for country $i \in \{\tilde{N}+1, \tilde{N}+2, ..., N\}$.*

The relative intermediate goods price and the relative loan rate increase in the level of financial development, i.e., $\chi_{FCF}^{i+1} > \chi_{FCF}^i$ and $\psi_{FCF}^{i+1} > \psi_{FCF}^i$ for $\theta^{i+1} > \theta^i$.

Proof. Following the proof of proposition 2, there exists a threshold value of the country index $\tilde{N}$ such that $R_{FCF}^* \in (R_{FCF}^{\tilde{N}}, R_{FCF}^{\tilde{N}+1}]$. In the steady state, free mobility of financial capital raises the relative prices and equation (60) implies that free mobility of financial capital reduces aggregate output in country $i \in \{1, 2, ..., \tilde{N}\}$, i.e., $\chi^i_{FCF} > \chi^i_{IFA}$, $\psi^i_{FCF} > \psi^i_{IFA}$, and $Y^i_{FCF} < Y^i_{IFA}$. The opposite applies to country $i \in \{\tilde{N}+1, \tilde{N}+2, ..., N\}$.

In the steady state, $\Gamma^i_{FCF} = \Gamma^i_{IFA}$ and equation (17) imply that $\Gamma^i_{FCF} > \Gamma^{i+1}_{FCF}$. The loan rate equalization implies that

$$\Gamma^i_{FCF}\left(1 - \frac{1-\chi^i_{FCF}}{1-\theta^i}\right) = \Gamma^{i+1}_{FCF}\left(1 - \frac{1-\chi^{i+1}_{FCF}}{1-\theta^{i+1}}\right), \quad \Rightarrow \quad \frac{1-\chi^i_{FCF}}{1-\theta^i} > \frac{1-\chi^{i+1}_{FCF}}{1-\theta^{i+1}}$$

$$1-\theta^i > 1-\theta^{i+1}, \quad \Rightarrow \quad 1-\chi^i_{FCF} > 1-\chi^{i+1}_{FCF}$$

Thus, the steady-state relative prices rise in θ^i, i.e., $\chi^{i+1}_{FCF} > \chi^i_{FCF}$. □

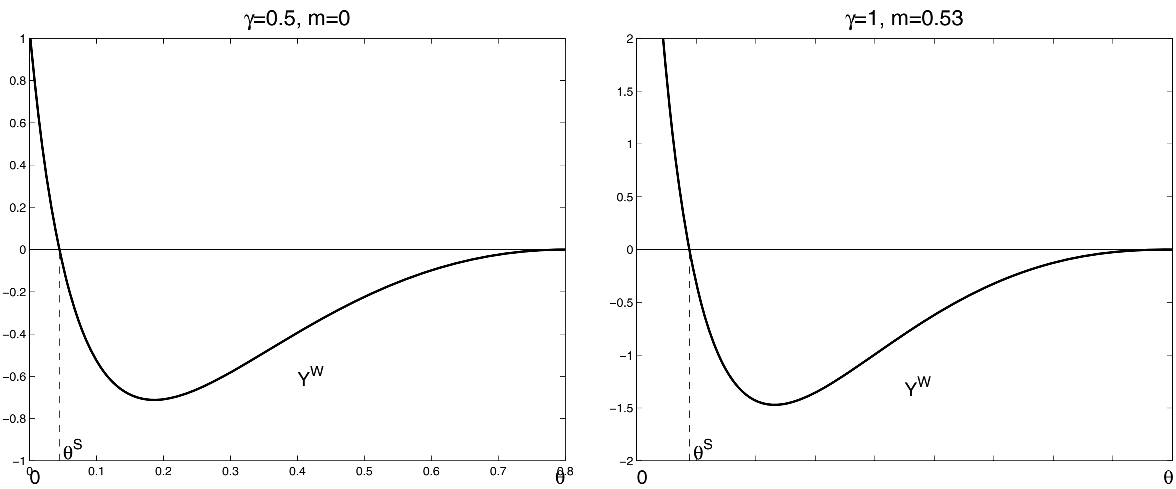

Figure 7: Free Mobility of Financial Capital and World Output

For the illustration purpose, we compare the steady-state world output under free mobility of financial capital versus under IFA in two cases, given $\theta^N = \bar{\theta}$ and $\theta^S \in [0, \bar{\theta}]$. In the first case, we focus on the investment composition effect by setting $\gamma = 0.5$ and $m = 0$ while keeping the values of other parameters same as in the benchmark case; in the second case, we focus on the savings effect by setting $\gamma = 1$ and $m = 0.53$ while keeping the values of other parameters same as in the benchmark case. The left and the right panels of figure 7 show the percentage differences of world output in the two cases, respectively and the axis scalings are same as in the bottom-left panel of figure 1. In each case, there exists a threshold value $\tilde{\theta}^S$ such that for $\theta^S \in (0, \tilde{\theta}^S)$, free mobility of financial capital raises world output; otherwise, the opposite applies. See section 4 for detailed discussion on the threshold values.

B.2 Free Mobility of FDI

FDI flows equalize the equity rate globally and the credit markets clear at the country and at the world level,

$$\Gamma_t^i = \Gamma_t^*, \quad (1-\eta)(s_t^i - i_t^{i,A}) = (\lambda_t^i - 1)(\eta n_t^i - \Omega_t^i), \quad \text{and} \quad \sum_{i=1}^{N} \Omega_t^i = 0.$$

The remaining conditions for market equilibrium are same as under IFA. The model solutions are

$$R_t^i = \frac{\omega_{t+1}^i}{\omega_t^i} R_{IFA}^i \tag{65}$$

$$\Gamma_t^i = \frac{\omega_{t+1}^i}{\omega_t^i}\Gamma_{IFA}^i - \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Z}_{t+1}^i, \quad \text{where} \quad \mathcal{Z}_{t+1}^i \equiv \frac{(\chi_{t+1}^i - \chi_{IFA}^i)\Gamma_{IFA}^i}{\chi_{t+1}^i - \theta^i} \tag{66}$$

$$\Omega_t^i = \eta\beta\omega_t^i \left(1 - \frac{\omega_{t+1}^i}{\omega_t^i}\frac{\Gamma_{IFA}^i}{\Gamma_t^i}\right) \tag{67}$$

$$\omega_{t+1}^i = \left(\frac{\Lambda_t^i}{\mathcal{Q}}\omega_t^i\right)^\alpha, \quad \text{where} \quad \Lambda_t^i \equiv \frac{(\chi_{t+1}^i)^\gamma (m+1)}{m + A^i}, \tag{68}$$

$$\frac{\partial \ln \Lambda_t^i}{\partial \chi_{t+1}^i} = \frac{\gamma}{\chi_{t+1}^i} > 0 \tag{69}$$

Let X_{FDI} denote the steady-state value of variable X under free mobility of FDI.

Lemma 7. *There exists a unique and stable steady state under free mobility of FDI.*

Proof. Combining equations (66) and (68), we rewrite the dynamic equation of wages,

$$(1+\rho)\ln\omega_{t+1}^i = -\rho \ln R_{IFA}^i + \rho\ln\omega_t^i + \gamma\rho\ln\left(\frac{\omega_{t+1}^i}{\omega_t^i}R_{IFA}^i\frac{(1-\theta^i)}{\Gamma_t^*} + \theta^i\right). \tag{70}$$

Define $\mathcal{W}^i \equiv \frac{\partial \ln \omega_{t+1}^i}{\partial \ln \omega_t^i}$. The first and the second derivatives of ω_{t+1}^i with respect to ω_t^i are

$$\frac{\partial \omega_{t+1}^i}{\partial \omega_t^i} = \frac{\omega_{t+1}^i}{\omega_t^i}\frac{1}{1 + \frac{1}{\rho(1-\gamma) + \rho\gamma\frac{\theta^i}{\chi_{t+1}^i}}} \in \left(0, \frac{\omega_{t+1}^i}{\omega_t^i}\right), \quad \Rightarrow \mathcal{W}^i \in (0,1)$$

$$\frac{\partial^2 \omega_{t+1}^i}{\partial(\omega_t^i)^2} = -\left(1-\mathcal{W}^i\right)^2 \frac{\omega_{t+1}^i}{(\omega_t^i)^2}\rho\left[1 - \gamma + \frac{\gamma\theta^i}{\chi_{t+1}^i}\mathcal{W}^i + \frac{\gamma(\theta^i)^2}{(\chi_{t+1}^i)^2}(1-\mathcal{W}^i)\right]$$

Since $\mathcal{W}^i \in (0,1)$, we get $\frac{\partial^2\omega_{t+1}^i}{\partial(\omega_t^i)^2} < 0$. Thus, the phase diagram of wages is a concave function under free mobility of FDI if the borrowing constraints are binding.

Define a threshold value $\underline{\omega}_t^i = R_{IFA}^i(\Gamma_t^*)^{-\frac{1}{1-\alpha}}$. For $\omega_t^i \in (0, \underline{\omega}_t^i)$, aggregate saving and investment in sector A is so low that $\chi_{t+1}^i = \psi_{t+1}^i = 1$ and $R_t^i = v_{t+1}^{i,A} = v_{t+1}^{i,B} = \Gamma_t^i$. Thus, the borrowing constraints are not binding and the phase diagram is flat with $\omega_{t+1}^i = \underline{\omega}_{t+1}^i = (\Gamma_t^*)^{-\rho}$. For $\omega_t^i > \underline{\omega}_t^i$, the phase diagram of wages is monotonically increasing and concave. Given $R_t^* < \mathcal{Q} < \Gamma_{IFA}^i$, we get $\bar{\omega}_{t+1}^i > \bar{\omega}_t^i$. so that the kink point on the phase diagram is above the 45 degree line.

Thus, the phase diagram of wages crosses the 45 degree line once and only once from the left, and the intersection is in its concave part. Thus, the model economy has a unique and stable steady state under free mobility of FDI. □

In the steady state, the interest rates and financial capital flows are

$$R_{FDI}^i = R_{IFA}^i, \tag{71}$$

$$\Gamma_{FDI}^i = \Gamma_{IFA}^i - \mathcal{Z}_{FDI}^i, \quad \text{where} \quad \mathcal{Z}_{FDI}^i \equiv \frac{(\chi_{FDI}^i - \chi_{IFA}^i)\Gamma_{IFA}^i}{\chi_{t+1}^i - \theta^i} \tag{72}$$

$$\Phi_{FDI}^i = (1-\eta)\beta\omega_{FDI}^i \frac{\mathcal{Z}_{FDI}^i}{R_{FDI}^*}, \tag{73}$$

Proposition 5. *In the steady state under free mobility of FDI, there exists a threshold value of the country index $\tilde{N}$ such that the world equity rate is $\Gamma_{IFA}^{\tilde{N}} > \Gamma_{FDI}^* \geq \Gamma_{IFA}^{\tilde{N}+1}$ and the loan rate in each country is $R_{FDI}^i = R_{IFA}^i$. In country $i \in \{1, 2, ..., \tilde{N}\}$, free mobility of FDI leads to FDI inflows, $\Phi_{FDI}^i > 0$, so that the relative intermediate goods price and the relative loan rate rise, $\chi_{FDI}^i > \chi_{IFA}^i$ and $\psi_{FDI}^i > \psi_{IFA}^i$, and aggregate output rises, $Y_{FDI}^i > Y_{IFA}^i$; the opposite applies for country $i \in \{\tilde{N}+1, \tilde{N}+2, ..., N\}$.*

The relative intermediate goods price and the relative loan rate increase in the level of financial development, i.e., $\chi_{FDI}^{i+1} > \chi_{FDI}^i$ and $\psi_{FDI}^{i+1} > \psi_{FDI}^i$ for $\theta^{i+1} > \theta^i$.

Proof. Following the proof of proposition 2, there exists a threshold value of the country index $\tilde{N}$ such that $\Gamma_{FDI}^* \in (\Gamma_{FDI}^{\tilde{N}}, \Gamma_{FDI}^{\tilde{N}+1}]$. In the steady state, free mobility of FDI raises the relative prices and equation (69) implies that free mobility of FDI raises aggregate output in country $i \in \{1, 2, ..., \tilde{N}\}$, i.e., $\chi_{FDI}^i > \chi_{IFA}^i$, $\psi_\Gamma^i > \psi_{IFA}^i$, and $Y_\Gamma^i > Y_{IFA}^i$. The opposite applies to country $i \in \{\tilde{N}+1, \tilde{N}+2, ..., N\}$.

In the steady state, $R_{FDI}^i = R_{IFA}^i$ and equation (17) imply that $R_{FDI}^i < R_{FDI}^{i+1}$. The equity rate equalization implies that

$$\frac{R_{FDI}^i}{1 - \frac{1-\chi_{FDI}^i}{1-\theta^i}} = \frac{R_{FDI}^{i+1}}{1 - \frac{1-\chi_{FDI}^{i+1}}{1-\theta^{i+1}}}, \qquad \Rightarrow \qquad \frac{1-\chi_{FDI}^i}{1-\theta^i} > \frac{1-\chi_{FDI}^{i+1}}{1-\theta^{i+1}}$$

$$1-\theta^i > 1-\theta^{i+1}, \qquad \Rightarrow \qquad 1-\chi_{FDI}^i > 1-\chi_{FDI}^{i+1}$$

Thus, the steady-state relative prices rise in θ^i, i.e., $\chi_{FDI}^{i+1} > \chi_{FDI}^i$. □

Figure 8 shows the percentage differences of the steady-state output at the country and the global level under free mobility of FDI versus under IFA in two cases, given $\theta^N = \bar{\theta}$ and $\theta^S \in [0, \bar{\theta}]$. The left panels highlights the investment composition effect by setting $\gamma = 0.9$ and $m = 0$, while the right panel highlights the savings effect by setting $\gamma = 1$

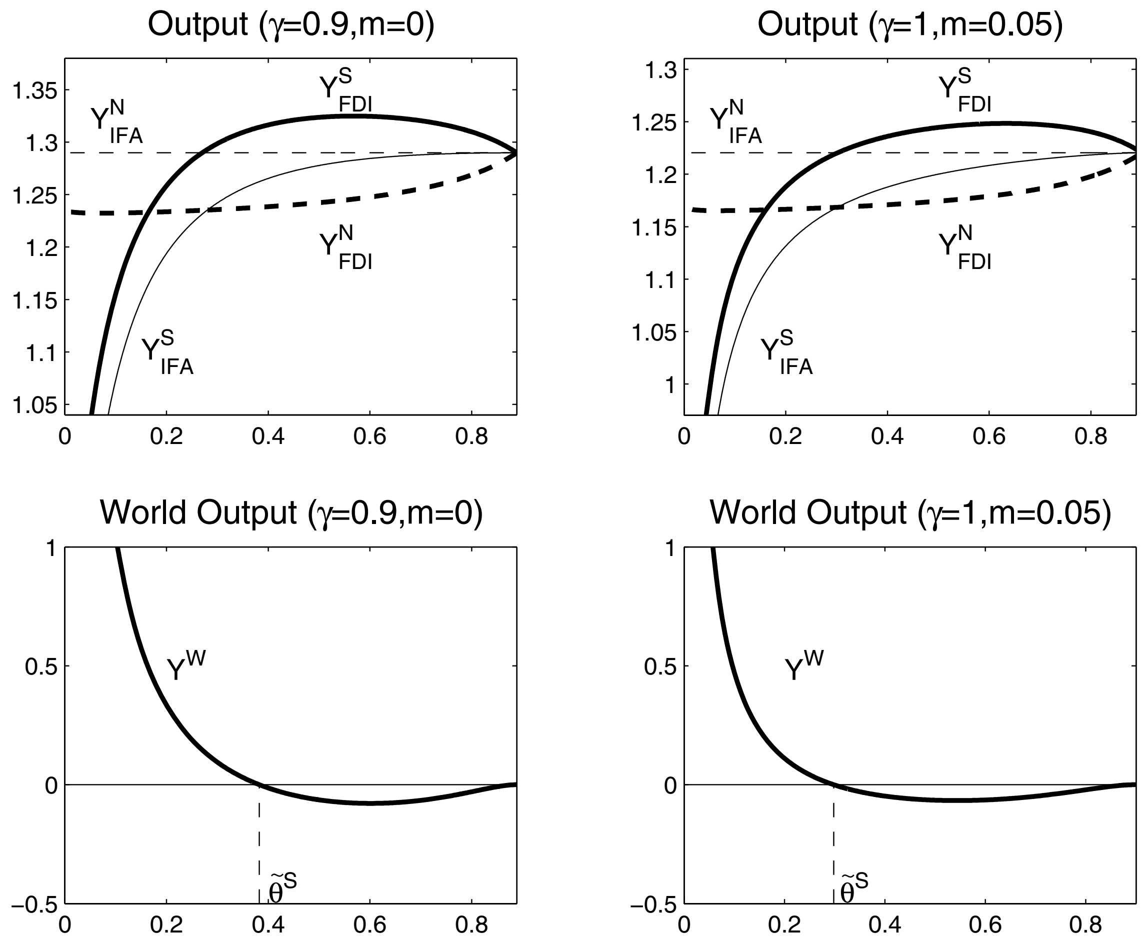

Figure 8: Free Mobility of FDI and Output

and $m = 0.05$. The values of other parameters same as in the benchmark case and the axis scalings are same as in figure 7. For country S, FDI inflows not only raise the size of domestic investment, but also improve the composition of domestic investment and raise the size of domestic savings. Different from the scenarios of full capital mobility or free mobility of financial capital, the investment reallocation under free mobility of FDI works in the same direction as the investment composition effect and the savings effect under free mobility of FDI. The opposite applies for country N. Thus, free mobility of FDI raises (reduces) steady-state output in country S (N), as shown in the upper panels of figure 8. In particular, for θ^S close to θ^N, FDI flows not only lead to output reversal in the sense that steady-state output in country S may exceed that in country N but also widen the cross-country gap, which reduces world output. As shown in the bottom panels of figure 8, there exists a threshold value $\tilde{\theta}^S$ such that for $\theta^S \in (0, \tilde{\theta}^S)$, free mobility of FDI raises world output. See section 4 for detailed discussion on the threshold values.