Online Appendix for “Upstream Financial Flows, Intangible Investment, and Allocative Efficiency”

—- Restoring Allocative Efficiency with Tax and Subsidy

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In our model, the commitment and enforcement problems in the credit relationship give rise to limited and heterogeneous pledgeability, which distorts aggregate allocation along two dimensions. First, due to limited pledgeability, agents cannot borrow against their entire project revenue. The constraint on aggregate credit demand pushes the loan rate below the social rate of return, \( \psi_t = \frac{\alpha}{q_t} \Phi_t < 1 \), which reflects an intertemporal distortion. Second, due to heterogeneous pledgeability, only tangible capital can serve as collateral for loans.\(^1\) The unit-cost differential \( u_I = 1 > u_{T,t} = 1 - \frac{1-\eta}{1-a_t} \) induces entrepreneurs to under-invest in the intangibles, \( a_t < \eta \), which reflects an intratemporal distortion.

If the financial sector cannot address these commitment and enforcement problems in the credit relationship, the government may subsidize intangible investment, which reduces the unit-cost differential. Thus, entrepreneurs shift their investment towards the intangibles, which improves allocative efficiency. The subsidies may be financed in three ways, i.e., taxing tangible investment in the current period, taxing investment revenue in the next period, and taxing the labor income. As shown below, the first two options do not involve income redistribution across individuals in the net term, while the third option does.

**Scheme I: Subsidize the Intangibles and Tax the Tangibles**

Let \( \pi_{T,t} \) and \( \pi_{I,t} \) denote respectively the tax rates on tangible and intangible investments in period \( t \), with the negative value denoting the subsidy rate. The government uses the entire tax revenues to fund the subsidies, \( \pi_{T,t}a_tM_t + \pi_{T,t}(1-a_t)M_t = 0 \), or equivalently, \( \pi_{T,t} = -\pi_{T,t}\frac{1-a_t}{a_t} \).

As proved in our paper, entrepreneurs choose the same intangible share of investment in equilibrium \( a_{j,t} = a_t \), implying that the subsidy an entrepreneur receives is equal to the tax it pays \( \pi_{T,t}a_tM_{j,t} + \pi_{T,t}(1-a_t)m_{j,t} = 0 \). Thus, this scheme does not involve income redistribution across individuals in the net term, while the third option does.

\(^{1}\)In appendix A.3 of our paper, we consider the case where both tangible and intangible capitals serve as collateral for loans, while the intangibles have a smaller pledgeability than the tangibles, i.e., \( \lambda_t < \lambda \). The unit-cost differential \( u_{I,t} = 1 - \frac{\eta}{a_t} \lambda_t > u_{T,t} = 1 - \frac{1-\eta}{1-a_t} \lambda_t \) leads to under-investment in the intangibles, \( a_t < \eta \).
across individuals in equilibrium. The unit costs of the two types of investments become

\[
\psi_{T,t} = 1 + \frac{1 - \eta}{1 - a_t} \psi_t \quad \text{and} \quad \psi_{I,t} = 1 - \frac{1 - \eta}{1 - a_t}.
\]

(1)

Agent \(j\) equalizes the internal rates of return on intangible and tangible investments,

\[
\frac{q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,t+1}}}{\psi_{I,t}} = \frac{q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,t+1}} - \psi_{T,t}}{1 - \psi_{T,t} - 1},
\]

(2)

\[
\Rightarrow \frac{u_{T,t}}{u_{I,t}} = \frac{1 + \psi_{T,t} - \frac{1 - \eta}{1 - a_t} \psi_t}{1 - \frac{1 - \eta}{1 - a_t} (1 - \lambda)} = \frac{1 - \eta}{1 - a_t} (1 - \lambda).
\]

(3)

Put \(a_t = \eta\) into equation (3) and solve for the rates that restores allocative efficiency,

\[
\psi_{T,t} = \frac{\eta (\frac{1}{\psi_t} - 1)}{\eta - 1} > 0 \quad \text{and} \quad \psi_{I,t} = \frac{1}{\psi_t} - 1 < 0
\]

(4)

Given the level of financial development \(\lambda\), \(\psi_{T,t}\) is negatively related to the normalized interest rate \(\psi_t\). Intuitively, the lower the initial income level, the smaller the mass of entrepreneurs, the lower the aggregate credit demand, the lower the normalized interest rate, the larger the rate-of-return wedge and the leverage effect, the larger the over-investment in the tangibles, the higher the tax and subsidy rates needed to restore within-firm allocative efficiency.

**Does the optimal tax-subsidy scheme fully eliminate the unit-cost differential?** In the absence of the tax-subsidy scheme, within-firm investment composition is inefficient \(a_t < \eta\) and the unit-cost differential exists \(\frac{u_{T,t}}{u_{I,t}} = \frac{1 - \eta}{1 - \psi_t} (1 - \lambda) < 1 - \lambda < 1\). The optimal scheme restores allocative efficiency \(a_t = \eta\). Although the unit-cost differential becomes smaller, it still exists \(\frac{u_{T,t}}{u_{I,t}} = 1 - \lambda < 1\). Intuitively, the optimal tax-subsidy scheme aims directly at the intratemporal distortion, without addressing the fundamental causes of heterogeneous pledgeability. As long as the borrowing constraints are binding, the intertemporal distortion in terms of the rate-of-return wedge still exists and so is the unit-cost differential.\(^2\)

**Model Dynamics under the Optimal Tax-Subsidy Scheme** Under autarky, domestic investment is fully financed by domestic saving, \(M_t = w_tL\), while the optimal scheme ensures allocative efficiency \(\Phi_t = 1\). Then, the law of motion for wage becomes \(w_{t+1} = \left(\frac{w_t}{p}\right)^\alpha\), independent of whether the borrowing constraints are binding.

In the left panel of figure 1, the solid (dashed) curve shows the law of motion for wage under autarky in the absence (presence) of the optimal tax-subsidy scheme. Suppose that there

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\(^2\)The optimal scheme equalizes the marginal revenues of two types of capital, \(q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,t+1}} = q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,t+1}}\). As long as \(\psi_t < 1\), agent \(j\) borrows to the limit; after paying off the debt in the next period, the agent gets a lower unit return from the tangibles than from the intangibles, \(q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,t+1}} - \lambda p_{t+1} < q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,t+1}} = q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,t+1}}\). When initiating the project, agents equalize the two internal rates of return as specified by equation (2). Thus, the unit cost of tangibles must be lower than that of intangibles in equilibrium.
is no tax/subsidy before period 0 and country H is initially at the autarkic steady state A with inefficient allocation $\Phi_A < 1$ and the wage rate $w_A = \left(\frac{\Phi_A}{\rho}\right)^\alpha$. From period $t = 0$ on, the optimal scheme is introduced to restore allocative efficiency. Period-0 efficiency gain $\Phi_0 = 1 > \Phi_A$ leads to a higher aggregate income in period 1, $w_1 = \left(\frac{w_A}{\rho}\right)^\alpha > w_A$, which stimulates domestic investment along the intensive and the extensive margins over time. Eventually, country H converges to the new steady state B with the wage rate $w_B = \rho^{-\rho} > w_A$. The right panel of figure 1 shows the impulse responses of wage over time, with the time periods on the horizontal axis. Along the convergence path, aggregate consumption rises with aggregate income $C_{o,t+1} = \alpha Y_{t+1}$, while the rise in the normalized interest rate reduces the gap between the loan rate and the equity rate, implying that consumption inequality among agents in the same generation declines over time.

In the left panel of figure 2, the dashed curve shows the optimal tax rate $\pi_{T,t}$ as a decreasing function of aggregate income, according to equation (4). Along the convergence path, the government gradually reduces the tax-subsidy rates.
The Constant Tax-Subsidy Rate Scheme

Next, let us consider an alternative scheme with the constant tax and subsidy rates that brings country H from steady state A to B.

Combine \( a_t = \eta \) with equations (1) and (4) to get the unit cost of total investment

\[
u_t = a_t u_{T,t} + (1 - a_t) u_{I,t} = 1 - \frac{(1 - \eta)\lambda}{\psi_t}.
\]

(5)

As the tax-subsidy scheme does not change the format of the borrowing constraints, the unit cost of investment has the same form as in the no-tax case and so do the following conditions.

\[
u_t = \frac{1 - \theta}{m} = \delta_t = \frac{\theta}{\psi_t}, \Rightarrow \ u_t = \left( \frac{1 - \theta}{m} \right)^{1-\theta}.
\]

(6)

In steady state B, \( w_B = \rho^{-\rho} \). Combine it with equations (5) and (6) to get

\[
\psi_B = \frac{\lambda(1-\eta)}{1 - \left( \frac{1-\theta}{\rho m} \right)^{1-\eta}}.
\]

Plug it into equations (4) to get the tax rates that restore allocative efficiency in steady state B.

\[
\pi_{T,B} = \frac{\eta \left( \frac{1}{\psi_B} - 1 \right)}{\chi - 1 + \eta} > 0 \quad \text{and} \quad \pi_{I,B} = -\frac{(1 - \eta) \left( \frac{1}{\psi_B} - 1 \right)}{\chi - 1 + \eta} < 0.
\]

(7)

Suppose that country H is initially at the autarkic steady state A. From period 0 on, the tangibles are taxed at the constant rate \( \pi_{T,B} \) and the intangibles are subsidized at the constant rate \( \pi_{I,B} \). Given \( w_B > w_A \), the tax rates that restore allocative efficiency in steady state B are lower than those in steady state A \( \pi_{T,B} < \pi_{T,A} \), as shown in the left panel of figure 2. In the middle panel of figure 2, the solid curve shows \( a_t \) as an increasing function of the income level \( w_t \in (0, w_A) \) under no tax-subsidy scheme, the flat dashed line shows \( a_t = \eta \) under the optimal scheme, and the dash-dotted curve shows \( a_t \) as an increasing function of the income level under the scheme with the constant tax rates.

In period 0, the constant-rate scheme partially improves rather than fully restores allocative efficiency, \( a_A < a_0 < a_B \). As the gap of allocative efficiency under the two schemes is fairly small, the law of motion for wage under the constant-rate scheme lies slightly below the one under the optimal scheme, as shown in the right panel of figure 2. The constant-rate scheme brings country H from steady state A to B, while the convergence time is slightly longer than under the optimal scheme.

Scheme II: Subsidize Intangibles and Tax Investment Revenues

Instead of taxing tangible investment in the current period, the government may issue one-period bonds to fund the subsidy on the intangibles, \( B_i^g = -\pi_t a_t M_t \), and redeem the bonds with the tax on the project revenues in the next period, \( r_t B_i^g = \pi_{t+1} q_{t+1} \Phi_i M_t \), where \( \pi_{t+1} \) denotes the tax rate on the project revenue. The two rates satisfy the condition \( \pi_t = -\frac{\pi_{t+1} + \Phi_i q_{t+1} M_t}{a_t \psi_t} \). Same as scheme I mentioned above, this scheme does not involve cross-agent income redistribution in the net term, because the subsidy an entrepreneur receives is equal to the present value of the taxes it pays in the next period, \( \pi_t a_t m_{j,t} + \frac{\pi_{t+1} q_{t+1} \Phi_i m_{j,t}}{r_t} = 0 \).
By reducing the price of tangible capital \( p_t = (1 - \pi_t + 1)q_t \frac{\partial K_t}{\partial K_t} \), the tax on the project revenue reduces the collateral value of tangibles. The unit costs of two types of investments are:

\[
u_{t,1} = 1 + \pi_{t,1} = 1 - \frac{\pi_{t,1}}{1 - \eta} \psi_t, \quad \nu_{t,1}^T = 1 - \frac{(1 - \pi_{t,1})(1 - \eta)}{1 - \eta} \frac{\lambda}{\psi_t}.
\] (8)

As shown in equation (2), agent \( j \) chooses intangible and tangible investments optimally so that their internal rates of return equalize,

\[
\frac{\nu_{t,1}^T}{\nu_{t,1}} = 1 - \frac{(1 - \pi_{t,1})(1 - \eta) \lambda}{1 - \pi_{t,1} \psi_t} = \frac{a_t}{1 - a_t} \frac{1 - \eta}{\eta} (1 - \lambda).
\] (9)

Put \( a_t = \eta \) in equation (9) to solve for the optimal tax rates that restore allocative efficiency,

\[
\frac{1 - (1 - \pi_{t,1}) \lambda}{1 - \frac{\pi_{t,1}}{\eta} \psi_t} = 1 - \lambda, \quad \Rightarrow \quad \pi_{t,1} = \frac{(1 - \eta) \psi_t}{\lambda - 1 + \eta} \text{ and } \pi_{t,1} = -\frac{\lambda - 1}{\lambda - 1 + \eta}.
\] (10)

Combine \( a_t = \eta \) with equations (8) and (10) to get the unit cost of total investment,

\[
u_t = a_t \nu_{t,1} + (1 - a_t) \nu_{t,1}^T = 1 - \frac{(1 - \eta) \lambda}{\psi_t} - \frac{(1 - \psi_t^I) \eta \lambda}{\psi_t^I},
\] (11)

**Lemma 1.** Starting from steady state \( A \), country \( H \) converges to steady state \( B \) along the same path under scheme I and II. Along the convergence path, the dynamics of the subsidy rate on the intangibles \( \pi_{t,1} \) are the same under the two schemes; as long as the borrowing constraints are binding, the normalized interest rate is higher under scheme II than under scheme I.

**Proof.** By restoring allocative efficiency \( a_t = \eta \) and \( \Phi_t = 1 \), the two schemes give rise to the same law of motion for wage \( w_{t+1} = \left( \frac{w_t}{\rho} \right)^{\alpha} \). It implies that country \( H \) converges from steady state \( A \) to \( B \) along the same path under the two schemes.

Let \( \psi_t^I \) and \( \psi_t^{II} \) denote the normalized interest rate under scheme I and II, respectively. Under autarky, if the borrowing constraints are binding, equation (6) specifies \( u_t \) as a function of the income level. Given the initial income level, \( u_t \) takes the same value under both schemes.

\[
1 - \frac{(1 - \eta) \lambda}{\psi_t^I} = u_t = 1 - \frac{(1 - \psi_t^{II}) \eta \lambda}{\psi_t^{II}}, \quad (12)
\]

\[
(1 - \eta) \lambda \left( \frac{1}{\psi_t^I} - \frac{1}{\psi_t^{II}} \right) = \frac{(1 - \psi_t^{II}) \eta \lambda}{\psi_t^{II}} > 0, \quad \Rightarrow \psi_t^{II} > \psi_t^I. \tag{13}
\]

As entrepreneurs cannot borrow against their entire project revenue, the constraint on aggregate credit demand leads to the rate-of-return wedge, \( \psi_A < 1 \). As scheme I does not change the borrowing constraints for individual agents, the aggregate credit demand remains unchanged in period 0 and so does the normalized interest rate, \( \psi_0 = \psi_A. \)

\[3\text{Scheme I affects aggregate credit demand } \frac{D_t}{\Delta t} = \frac{\lambda K_t}{\psi_t} = \frac{(1 - \eta) \lambda M_t}{\psi_t}, \text{ in two opposite ways. First, it induces entrepreneurs to reduce the tangible fraction of investment } 1 - a_0 < 1 - A_0, \text{ which tends to reduce aggregate credit demand. Second, by restoring allocative efficiency, it raises the social rate of return and the price of tangible capital } p_t = q_1 > q_0 \Phi_A > q_A \Phi_A \frac{1 - M_A}{1 - A_A} = p_A, \text{ which tends to raise aggregate credit demand. As the second effect dominates the first effect, the loan rate rises in period 0, } r_0 > r_A. \text{ As the social rate of return and the loan rate rise in equal proportions, the normalized interest rate remains unchanged, } \psi_0 = \frac{\psi_0}{\psi_0} = \frac{\psi_A}{\psi_A} = \psi_A.\]
Under scheme II, the subsidy can be treated as a “loan” provided by the government at the market interest rate, because the subsidy an entrepreneur receives is equal to the present value of the tax it pays in the next period. By using its exclusive power of taxation to overcome the commitment and enforcement problems in the credit relationship, the government allows individual agents to finance their current investment against a larger fraction of their future project revenue than before, which pushes up the normalized interest rate in period 0.\(^4\)

The higher the \(\psi_t\), the smaller the gap between the equity rate and the loan rate, the smaller the consumption inequality among individual agents in the same generation. Along the convergence path, the dynamics of aggregate income are identical under the two schemes, while the rate-of-return gap is lower under scheme II and so is the consumption inequality among agents in the same generation.

### Scheme III: Subsidize the Intangibles and Tax Labor Income

Under scheme III, the subsidy on intangible investment is financed by the labor income tax, \(\pi_{I,t} a_t M_t + \pi_{L,t} w_t L = 0\), where \(\pi_{L,t}\) denote the tax rate on labor income in the current period. Under autarky, domestic investment is financed by domestic saving and subsidies, \(M_t = (1 - \pi_{L,t}) w_t L + \pi_{L,t} w_t L\) and hence, \(\pi_{L,t} = -a_t \pi_{I,t}\). The unit costs of two types of investments are

\[
\begin{align*}
  u_{I,t} &= 1 + \pi_{I,t}, \\
  u_{T,t} &= 1 - \frac{1 - \eta}{1 - a_t} \frac{1}{\psi_t}.
\end{align*}
\]

As shown in equation (2), agent \(j\) chooses the two types of investment optimally so that their internal rates of return equalize,

\[
\frac{u_{T,t}}{u_{I,t}} = \frac{1 - \frac{1 - \eta}{1 - a_t} \frac{1}{\psi_t}}{1 + \pi_{I,t}} = \frac{a_t}{1 - a_t} \frac{1 - \eta}{\eta} (1 - \lambda).
\]

Put \(a_t = \eta\) in equation (15) to solve for the tax rates that restore allocative efficiency,

\[
\pi_{I,t} = -\frac{1}{\psi_t} - 1 < 0 \quad \text{and} \quad \pi_{L,t} = -a_t \pi_{I,t} = \eta \frac{1}{\psi_t} - 1 > 0.
\]

Combine \(a_t = \eta\) with equations (8) and (16) to get the unit cost of total investment,

\[
u_t = a_t u_{I,t} + (1 - a_t) u_{T,t} = 1 - \frac{(1 - \eta) \lambda}{\psi_t} - \pi_{L,t}.
\]

\(^4\)Scheme II affects the credit market in two opposite ways. First, taxing the project revenue in the next period reduces the price of tangible capital \(p_{t+1} = (1 - \pi_{t+1}) q_{t+1} \Phi_t \frac{1 - \eta}{1 - a_t}\), which weakens the individual’s borrowing capacity and reduces aggregate credit demand. Second, the one-period bonds issued by the government boosts up the aggregate demand in the credit market. Under scheme II, the second effect dominates the first effect and hence, the aggregate credit demand rises in the net term, leading to a higher normalized interest rate in period 0.
Under autarky, the following conditions hold,

\[ M_t = \int_{E_t}^{\infty} \frac{(1 - \pi_{L,t})w_t(1 - \theta)\varepsilon_j}{u_t} dG(\varepsilon_j) = \frac{(1 - \pi_{L,t})w_t L}{u_t} \varepsilon_{1-\theta} = w_t L, \quad (18) \]

\[ \frac{(1 - \pi_{L,t})w_t(1 - \theta)}{u_t m} = \frac{\varepsilon_t^{-1} = \left( \frac{u_t}{1 - \pi_{L,t}} \right)^{\frac{\theta}{1 - \theta}}}{\frac{w_t(1 - \theta)}{m} = \left( \frac{u_t}{1 - \pi_{L,t}} \right)^{\frac{1}{1 - \theta}}} \quad (19) \]

\[ \left[ \frac{w_t(1 - \theta)}{m} \right]^{1-\theta} = \frac{u_t}{1 - \pi_{L,t}} = 1 - \frac{(1 - \eta)\lambda}{(1 - \pi_{L,t})\psi_t}. \quad (20) \]

**Lemma 2.** Starting from steady state A, country H converges along the same path to steady state B under scheme III and I. Along the convergence path, if the borrowing constraints are binding, the normalized interest rate is higher under scheme III than under scheme I.

**Proof.** By restoring allocative efficiency \( a_t = \eta \) and \( \Phi_t = 1 \), the two schemes give rise to the same law of motion for wage \( w_{t+1} = \left( \frac{w_t}{\rho} \right)^\alpha \). It implies that country H converges from steady state A to B along the same path under the two schemes.

Under autarky, if the borrowing constraints are binding, equations (6) hold under scheme I. Let \( \psi^I_t \) and \( \psi^{III}_t \) denote the normalized interest rate under scheme I and III, respectively. Given the initial income level \( w_0 = w_A \), combine equations (20) with (6) and (16),

\[ 1 - \frac{(1 - \eta)\lambda}{\psi^I_0} = u^I_0 = \left[ \frac{w_0(1 - \theta)}{m} \right]^{1-\theta} = \frac{u_t}{1 - \pi_{L,t}} = 1 - \frac{(1 - \eta)\lambda}{(1 - \pi_{L,t})\psi^{III}_t}, \]

\[ \psi^I_0 = (1 - \pi_{L,t})\psi^{III}_0 < \psi^{III}_t, \quad \psi^{III}_t = \psi^I_0 + \frac{\eta \lambda (1 - \psi^I_0)}{1 - \lambda + \eta \lambda} > \psi^I_0 \]

\( \Box \)

Under scheme III, only a fraction of agents become entrepreneurs and receive the subsidy, while all agents pay the labor income tax. Thus, scheme III involves income redistribution from households to entrepreneurs.