Appendix (For Online Publication)

A Technical Proofs

Proof of Lemma 1

Proof. Under autarky, if the borrowing constraints are binding, $q_{t+1}^{i,A} R > r_t$ or equivalently $\mu_{t+1} < 1$. Combine equations (6) and (8) and use the definition of μ_{t+1} to get

$$\psi_t = 1 - \frac{\lambda}{\mu_{t+1}}. \tag{29}$$

Equalizing the investment demand and supply in sector A gives

$$\frac{\eta \mu_{t+1} w_t}{1 - \eta + \eta \mu_{t+1}} = M_t^{i,A} = \int_{\underline{\epsilon}_t}^{\infty} \frac{n_{j,t}}{\psi_t} dG(\epsilon_j) = \frac{\underline{\epsilon}_t^{-\frac{1-\theta}{\theta}}}{\psi_t} w_t, \;\Rightarrow\; \underline{\epsilon}_t^{-\frac{1-\theta}{\theta}} = \frac{\eta \mu_{t+1}}{1 - \eta(1 - \mu_{t+1})} \psi_t. \tag{30}$$

Combine equations (11), (29)-(30) to get

$$Y_t = \left(1 + \frac{1-\eta}{\eta \mu_{t+1}}\right)^{-\frac{\theta}{1-\theta}} \frac{(1 - \frac{\lambda}{\mu_{t+1}})^{\frac{1}{1-\theta}} \mathfrak{m}}{(1-\alpha)(1-\theta)}, \qquad \frac{\partial \ln \mu_{t+1}}{\partial \ln Y_t} = \frac{1-\theta}{\frac{\lambda}{\mu_{t+1} - \lambda} + \frac{\theta \frac{1-\eta}{\eta}}{\mu_{t+1} + \frac{1-\eta}{\eta}}} > 0 \tag{31}$$

$$\text{given } Y_t, \; \frac{\partial \ln \mu_{t+1}}{\partial \ln \mathfrak{m}} = -\frac{\partial \ln \mu_{t+1}}{\partial \ln Y_t} < 0, \qquad \frac{\partial \ln \mu_{t+1}}{\partial \ln \lambda} = \frac{1}{1 + \theta \frac{1-\eta}{1-\eta(1-\mu_{t+1})} \frac{\mu_{t+1} - \lambda}{\lambda}} > 0.$$

If the borrowing constraints are weakly binding, the sectoral investment is efficient and $\mu_{t+1} = 1$. Combine it with equation (31) to get the threshold value $\bar{Y}_A$. □

Proof of Proposition 1

Proof. Combine equations (1)-(4) to get (20) as the law of motion for wage under autarky.

According to lemma 1, for $w_t \geq \bar{w}_A \equiv (1-\alpha)\bar{Y}_A$, the sectoral investment is efficient $\mu_{t+1} = 1$. Combine it with (20) to get equation (19) as the law of motion for wage, which is concave

$$\mathbb{J}_t \equiv \frac{\partial w_{t+1}}{\partial w_t} = \alpha \frac{w_{t+1}}{w_t}, \quad \text{and} \lim_{w_t \to \infty} \mathbb{J}_t = \lim_{w_t \to \infty} \alpha \left(\frac{R}{\rho}\right)^{\alpha} w_t^{\alpha - 1} = 0. \tag{32}$$

$$\mathbb{H}_t \equiv \frac{\partial^2 w_{t+1}}{\partial w_t^2} = (\alpha - 1) \frac{\mathbb{J}_t}{w_t} < 0. \tag{33}$$

For $w_t \in (0, \bar{w}_A)$, the sectoral investment is inefficient, $\mu_{t+1} \in (\lambda, 1)$. The law of motion for wage is determined jointly by equations (20)-(21) and it has the slope

$$\mathbb{J}_t = \frac{w_{t+1}}{w_t} \left[\alpha + \frac{\alpha \eta (1 - \mu_{t+1})}{\frac{1 + \frac{\eta}{1-\eta}\lambda}{(1 - \frac{\lambda}{\mu_{t+1}})(1-\theta)} - 1} \right], \quad \lim_{w_t \to 0} \mathbb{J}_t = \infty. \tag{34}$$

Equations (32) and (34) ensure the existence of at least one steady state under autarky. The steady state is unique iff $\mathbb{J}_t \mid_{w_{t+1} = w_t} < 1$ always holds at the steady state.

- If there is a steady state with $w_t > \bar{w}_A$, according to equation (32), $\mathbb{J}_t \mid_{w_{t+1} = w_t} = \alpha < 1$ so that the uniqueness condition is satisfied;

- if there is a steady state with $w_t \in (0, \bar{w}_A)$, according to equation (34), $\mathbb{J}_t \mid_{w_{t+1}=w_t}$ may exceed unity. If so, the uniqueness condition is violated and multiple steady states arise.

In the following, I derive the threshold values in the parameter space for the uniqueness of steady state by keeping $\mathbb{J}_t \mid_{w_{t+1}=w_t} < 1$ in the interval of $w_t \in (0, \bar{w}_A)$.

Let $\hat{\mu} \in (\lambda, 1)$ denote the relative sectoral MRK at a steady state where the borrowing constraints are binding. According to equation (22), the uniqueness condition is rewritten as

$$\hat{\mu}^2 - \mathbb{B}\hat{\mu} + \mathbb{C} > 0 \quad \text{where} \quad \mathbb{B} \equiv 1 + \lambda - \frac{1}{\rho\eta(1-\theta)}\left(\theta + \frac{\lambda\eta}{1-\eta}\right), \quad \mathbb{C} \equiv \lambda\left(\frac{1}{\rho\eta} + 1\right). \tag{35}$$

$$\hat{\mu}^2 - \mathbb{B}\hat{\mu} + \mathbb{C} = \begin{cases} \frac{\lambda}{\rho(1-\eta)(1-\theta)}\left(\lambda + \frac{1-\eta}{\eta}\right) > 0, & \text{for } \hat{\mu} \to \lambda; \\ \frac{\lambda}{\rho(1-\eta)(1-\theta)}\frac{1-(1-\eta)\theta}{\eta} + \frac{\theta}{\rho\eta(1-\theta)} > 0, & \text{for } \hat{\mu} \to 1. \end{cases} \tag{36}$$

Let $\check{\theta} \equiv 1 - \frac{\frac{\lambda\eta}{1-\eta}+1}{\rho\eta(1-\lambda)+1}$, $\check{\lambda} \equiv \frac{1-\frac{\theta}{\rho\eta(1-\theta)}}{1+\frac{1}{(1-\theta)\rho(1-\eta)}}$, and $\hat{\theta} \equiv \check{\theta} \mid_{\lambda=0} = \frac{1}{1+\frac{1}{\rho\eta}}$. Since (35) holds at the two boundary points, there are three cases where it holds for the entire interval of $\hat{\mu} \in (\lambda, 1)$.

1. Case 1: $\frac{\mathbb{B}}{2} > 1$. However, given $\lambda \in (0, 1)$, $\frac{\mathbb{B}}{2} > 1$ does not hold.
2. Case 2: $\frac{\mathbb{B}}{2} < \lambda$. In this case, $\theta \in (\check{\theta}, 1)$ or equivalently $\lambda \in (\check{\lambda}, 1)$. In particular, for $\theta \in (\hat{\theta}, 1)$, condition (35) holds for the entire space of $\lambda \in (0, 1)$.
3. Case 3: $\frac{\mathbb{B}}{2} \in (\lambda, 1)$ and $\mathbb{B}^2 - 4\mathbb{C} < 0$. In this case, $\theta \in (0, \check{\theta})$, $\lambda \in (0, \check{\lambda})$, and

$$\mathfrak{A}\lambda^2 - 2\mathfrak{B}\lambda + \mathfrak{C} < 0, \quad \text{where } \mathfrak{A} \equiv \left(1 - \frac{1}{\rho(1-\eta)(1-\theta)}\right)^2, \quad \mathfrak{C} \equiv \left(1 - \frac{\theta}{\rho\eta(1-\theta)}\right)^2, \tag{37}$$
$$\mathfrak{B} \equiv \left(1 - \frac{1}{\rho(1-\eta)(1-\theta)}\right)\left(\frac{\theta}{\rho\eta(1-\theta)} - 1\right) + 2\left(\frac{1}{\eta\rho} + 1\right).$$

- For $\rho(1-\eta)(1-\theta) = 1$, (37) holds if $\lambda \in (\underline{\lambda}_A, 1)$, where $\underline{\lambda}_A \equiv \frac{\mathfrak{C}}{2\mathfrak{B}} = \frac{\left(1-\frac{\theta}{\rho\eta(1-\theta)}\right)^2}{4\left(\frac{1}{\rho\eta}+1\right)}$.
- For $\rho(1-\eta)(1-\theta) \neq 1$, (37) holds if $\lambda \in (\underline{\lambda}_A, 1)$, where $\underline{\lambda}_A \equiv \frac{\mathfrak{B}-\sqrt{\mathfrak{B}^2-\mathfrak{A}\mathfrak{C}}}{\mathfrak{A}}$.[28]

Let $\hat{\lambda}_A \equiv \underline{\lambda}_A \mid_{\theta=0}$. The left panel of figure 1 shows $\underline{\lambda}_A$ as a function of θ and has a horizontal intercept at $\hat{\lambda}_A$. Given $\theta \in (0, \hat{\theta})$, I derive the threshold values in the $\{\lambda, Z\}$ space for the uniqueness of the autarkic steady state.

Detour: the Mapping between ψ_A and Z

Consider the case of the unique steady state under autarky. As the steady-state value of variable ψ_t, ψ_A is a function of parameters as defined below.

- If the borrowing constraints are binding at the steady state, $\mu_A \in (\lambda, 1)$ and $\psi_A = 1 - \frac{\lambda}{\mu_A} < 1 - \lambda$. Combine them with equations (20)-(21) to get

$$Z = \psi_A^{\frac{1}{\rho(1-\theta)}}\left[1 + \frac{(1-\eta)(1-\psi_A)}{\eta\lambda}\right]^{1-\frac{\theta}{\rho(1-\theta)}}\left(\frac{\lambda}{1-\psi_A}\right)^{1-\eta}. \tag{38}$$

[28]For $\lambda = 0$, $\mathfrak{A}\lambda^2 - 2\mathfrak{B}\lambda + \mathfrak{C} = \mathfrak{C} \geq 0$; for $\lambda = \check{\lambda}_A$, $\mathfrak{A}\lambda^2 - 2\mathfrak{B}\lambda + \mathfrak{C} = -4\lambda\rho\frac{(1-\eta)^2}{\eta}[(1-\lambda)\rho\eta + 1] < 0$. Thus, $\underline{\lambda}_A \in (0, \check{\lambda}_A)$.

- If the borrowing constraints are slack at the steady state, $\mu_A = 1$ and the individual agent's choices are indeterminant. I consider an equilibrium where agents who can meet the MIR still invest the entire labor income and choose the same leverage ratio $\psi_t > 1-\lambda$. In the steady state, combine $\mu_A = 1$ with equations (11), (20), and (30) to get

$$Z = \psi_A^{\frac{1}{\rho(1-\theta)}} \eta^{-\left[1-\frac{\theta}{\rho(1-\theta)}\right]} \tag{39}$$

Figure 9 shows that, under autarky, **m**ultiple steady states arise for $\{\lambda, \psi_A\}$ in region M, while the steady state is **u**nique and the borrowing constraints are **b**inding (**s**lack) for $\{\lambda, \psi_A\}$ in region UB (US). The right panel of figure 1 is a mapping of figure 9 by using equations (38)-(39) to convert the borders of three regions from the $\{\lambda, \psi_A\}$ space to the $\{\lambda, Z\}$ space. In the following, I characterize the borders of three regions in figure 9.

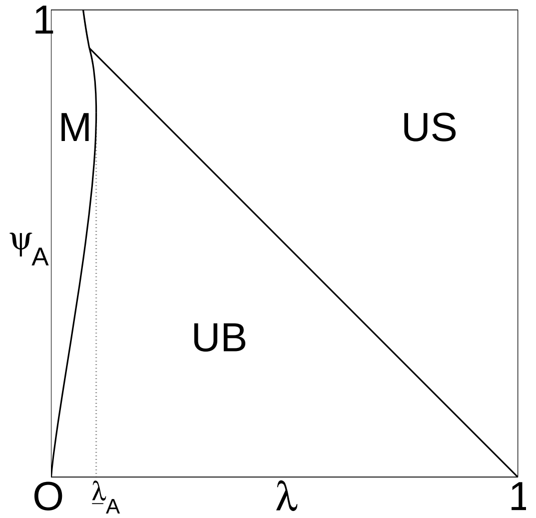

Figure 9: Multiple Steady States under Autarky: $\theta \in (0, \hat{\theta})$

If the borrowing constraints are weakly binding at the steady state, $\psi_A = 1 - \lambda$. It defines the border between region US and UB. Combine it with (39) to get $\tilde{\lambda}_A$ as defined in (23).

If multiple steady states arise under autarky, the borrowing constraints are either binding $\psi_H < 1 - \lambda$ or slack $\psi_H > 1 - \lambda$ at the high-income, stable steady state H. See the right panel of figure 2. In each scenario, I characterize the boundary case where the law of motion for wage is tangent with the 45° line at steady state M with $w_M \in (0, \bar{w}_A)$.

Let μ_M denote the steady-state value of μ_{t+1} at point M. Let condition (35) holds with equality. μ_M is a function of λ and other parameters.

$$\mu_M = \frac{\mathbb{B} \pm \sqrt{\mathbb{B}^2 - 4\mathbb{C}}}{2}, \quad \text{where } \mathbb{B} \equiv 1 + \lambda - \frac{1}{\rho\eta(1-\theta)}\left(\theta + \frac{\lambda\eta}{1-\eta}\right), \quad \mathbb{C} \equiv \lambda\left(\frac{1}{\rho\eta} + 1\right). \tag{40}$$

Given $w_M \in (0, \bar{w}_A)$, use equations (20)-(21) to get

$$w_M = \left(1 - \frac{\lambda}{\mu_M}\right)^{\frac{1}{1-\theta}} \left(1 + \frac{\frac{1}{\eta} - 1}{\mu_M}\right)^{-\frac{\theta}{1-\theta}} \frac{\mathfrak{m}}{1-\theta} = \left(\frac{R}{\rho\eta} \frac{\mu_M^{\eta-1}}{1 + \frac{\frac{1}{\eta}-1}{\mu_M}}\right)^{\rho}. \tag{41}$$

- If the borrowing constraints are binding at steady state H, $w_H \in (0, \bar{w}_A)$ and $\mu_H < 1$. Use equations (20)-(21) to get

$$w_H = \psi_H^{\frac{1}{1-\theta}} \left(1 + \frac{\frac{1}{\eta} - 1}{\mu_H}\right)^{-\frac{\theta}{1-\theta}} \frac{\mathfrak{m}}{1-\theta} = \left(\frac{R}{\rho\eta} \frac{\mu_H^{\eta-1}}{1 + \frac{\frac{1}{\eta}-1}{\mu_H}}\right)^{\rho}, \text{ and } \psi_H = 1 - \frac{\lambda}{\mu_H}. \tag{42}$$

Combine equations (41) and (42) to get

$$\psi_H = \left(1 - \frac{\lambda}{\mu_M}\right) \left[\frac{\eta + \frac{1-\eta}{\mu_M}}{\eta + \frac{(1-\eta)(1-\psi_H)}{\lambda}}\right]^{(1-\theta)\rho-\theta} \left(\frac{\mu_M \lambda}{1 - \psi_H}\right)^{(1-\theta)(1-\eta)\rho}. \tag{43}$$

The border between region UB and M in figure 9 is characterized by $\{\lambda, \psi_A\}$ satisfying equations (40) and (43).

- If the borrowing constraints are slack at steady state H, $w_H > \bar{w}_A$ and $\mu_H = 1$. Combine $\mu_H = 1$ and $\psi_H > 1 - \lambda$ with (42) to get

$$w_H = \psi_H^{\frac{1}{1-\theta}} \eta^{\frac{\theta}{1-\theta}} \frac{\mathfrak{m}}{1-\theta} = \left(\frac{R}{\rho}\right)^{\rho}. \tag{44}$$

Combine equations (41) and (44) to get

$$\psi_H = \left(1 - \frac{\lambda}{\mu_M}\right) \left(\eta + \frac{1-\eta}{\mu_M}\right)^{(1-\theta)\rho-\theta} \mu_M^{(1-\theta)(1-\eta)\rho}. \tag{45}$$

The border between region US and M in figure 9 is characterized by $\{\lambda, \psi_A\}$ satisfying equations (40) and (45).

□

Proof of Proposition 2

Proof. The asterisk superscript applies to variables at the world level.
Step 1: equilibrium conditions under trade integration

Let $\varsigma_t^f \equiv \frac{Y_t^f - V_t^f}{V_t^f}$ denote the sectoral export-to-domestic-absorbtion ratio, with the negative value for the case of imports. With no international capital flows,[29] trade is balanced every period. Besides, under free trade, the relative sectoral price in country i is aligned to the world level, $\chi_t = \chi^*$ and so is the relative sectoral MRK, $\mu_t = \mu^*$. Use equations (2) to get

$$p_t^A \varsigma_t^A V_t^A + p_t^B \varsigma_t^B V_t^B = 0, \ \Rightarrow \ \chi^* \frac{\eta}{1-\eta} = \frac{V_t^A}{V_t^B} = -\chi^* \frac{\varsigma_t^B}{\varsigma_t^A}, \ \Rightarrow \ \varsigma_t^B = -\frac{\eta}{1-\eta} \varsigma_t^A. \tag{46}$$

If country i specializes fully in sector f, $Y_t^f = 0$ and $\varsigma_t^f = -1$. Combine it with equation (46) to get the range for $\varsigma_t^A \in [-1, \frac{1}{\eta} - 1]$. From period $t = 1$ on, the market clearing condition for good f in country i is $V_t^f(1 + \varsigma_t^f) = Y_t^f$. Combine it with equations (1)-(2) to get

$$\frac{q_{t+1}^A RM_t^A}{q_{t+1}^B RM_t^B} = \frac{M_t^A}{\mu^* M_t^B} = \frac{\eta}{1-\eta} \frac{(1 + \varsigma_{t+1}^A)}{(1 + \varsigma_{t+1}^B)}, \quad \text{and} \quad \frac{L_{t+1}^A}{L_{t+1}^B} = \frac{\eta}{1-\eta} \frac{(1 + \varsigma_{t+1}^A)}{(1 + \varsigma_{t+1}^B)} \tag{47}$$

[29]Matsuyama (2004) shows that financial globalization may lead to income divergence among financially underdeveloped countries. Zhang (2015) extends Matsuyama (2004)'s model and points out that wealth inequality is a factor equally important in determining the possibility of symmetry breaking.

Domestic investment is financed by domestic saving, $M_t^A + M_t^B = w_t$, and the aggregate labor supply is constant at $L_{t+1}^A + L_{t+1}^B = 1$. Combine them with equations (46)-(47) to get the sectoral demands for investment in period t and for labor in period $t+1$

$$M_t^A = \frac{\eta(1+\varsigma_{t+1}^A)w_t\mu^*}{1-\eta(1+\varsigma_{t+1}^A)(1-\mu^*)} \quad \text{and} \quad M_t^B = \frac{[1-\eta(1+\varsigma_{t+1}^A)]w_t}{1-\eta(1+\varsigma_{t+1}^A)(1-\mu^*)}, \tag{48}$$

$$L_{t+1}^A = \eta(1+\varsigma_{t+1}^A) \quad \text{and} \quad L_{t+1}^B = 1-\eta(1+\varsigma_{t+1}^A). \tag{49}$$

For $w_t \geq \hat{w}_T$, the mass of entrepreneurs is so high that the aggregate credit demand pushes the interest rate above the rate of return in sector B. Households lend out their entire labor income and do not invest in sector B. Thus, country i specializes fully in sector A, i.e., $M_t^B = L_{t+1}^B = 0$ and $\varsigma_{t+1}^A = \frac{1}{\eta} - 1$. For $w_t \in (0, \hat{w}_T)$, the mass of entrepreneurs is so low that the total debt capacity of entrepreneurs is less than the labor income of households. Besides lending in the credit market, households also invest in sector B. Thus, both sectors are active in country i,[30] i.e., $M_t^f > 0$ and $L_{t+1}^f > 0$, where $f \in \{A, B\}$.

If $\chi^* = 1$, $\mu^* = (\chi^*)^{\frac{1}{\alpha}} = 1$ and the borrowing constraints are slack. The law of motion for wage $w_{t+1} = \left(\frac{R}{\rho}w_t\right)^\alpha$ is concave. Next, I focus on the case of $\chi^* < 1$ and hence $\mu^* = (\chi^*)^{\frac{1}{\alpha}} < 1$.

Derive first the condition under which sector B is active, i.e., $M_t^B > 0$ or equivalently $\varsigma_{t+1}^A \in (-1, \frac{1}{\eta} - 1)$. The positive investment in sector B implies $r_t = q_{t+1}^B R$, while $\mu_{t+1} = \mu^* < 1$ implies $q_{t+1}^A R > q_{t+1}^B R$. The binding borrowing constraints imply $\psi_t = \psi^* = 1 - \frac{\lambda}{\mu^*}$. Combine the investment demand and supply in sector A with (11) to get

$$\frac{\eta(1+\varsigma_{t+1}^A)\mu^*}{1-\eta(1+\varsigma_{t+1}^A)(1-\mu^*)}w_t = M_t^A = \int_{\underline{\epsilon}_t}^{\infty} \frac{n_{j,t}}{\psi_t} dG(\epsilon_j) = \frac{\underline{\epsilon}_t^{-\frac{1-\theta}{\theta}}}{\psi^*} w_t.$$

$$w_t = \frac{\psi^*}{\underline{\epsilon}_t}\mathbb{F} = (\psi^*)^{\frac{1}{1-\theta}} \left[\frac{\mu^*}{\frac{1}{\eta(1+\varsigma_{t+1}^A)} - (1-\mu^*)}\right]^{\frac{\theta}{1-\theta}} \mathbb{F}$$

Plug $\varsigma_{t+1}^A = \frac{1}{\eta} - 1$ into the above equation to get the threshold value $\hat{w}_T$. Then, ς_{t+1}^A is the piecewise function of w_t,

$$\varsigma_{t+1}^A = \begin{cases} \frac{1}{\eta\left\{1+\mu^*\left[\left(\frac{\hat{w}_T}{w_t}\right)^{\frac{1-\theta}{\theta}} - 1\right]\right\}} - 1 \in (-1, \frac{1}{\eta} - 1), & \text{for } w_t < \hat{w}_T; \\ \frac{1}{\eta} - 1, & \text{for } w_t \geq \hat{w}_T. \end{cases} \tag{50}$$

Combine (48)-(49) with (1)-(2), (46) to get (51) as the law of motion for wage, which depends on whether country i fully specializes in sector A,

$$w_{t+1} = \left(\frac{R}{\rho}w_t\Gamma_t\right)^\alpha,$$

$$\Gamma_t = \begin{cases} \frac{(\mu^*)^\eta}{1-\eta(1-\mu^*)(1+\varsigma_{t+1}^A)} = (\mu^*)^{\eta-1}\left[\mu^* + (1-\mu^*)\left(\frac{w_t}{\hat{w}_T}\right)^{\frac{1-\theta}{\theta}}\right], & \text{for } w_t < \hat{w}_T; \\ (\mu^*)^{\eta-1}, & \text{for } w_t \geq \hat{w}_T. \end{cases} \tag{51}$$

[30] In the current setting, the labor endowment follows the Pareto distribution, $\epsilon_j \in [1, \infty)$, which has no upper bound. There are always some agents who can meet the MIR so that sector A is always active under free trade, $M_t^A > 0$. In the previous version (Zhang, 2014), the labor endowment is distributed with an upper bound. Under free trade, for a sufficiently low aggregate income, the wage rate is so low that even the agent with the highest labor endowment cannot meet the MIR and country i specializes fully in sector B, $M_t^A = 0$. The qualitative results hold in both settings.

Step 2: the shape of the law of motion for wage under trade integration

In the case of $\chi^* < 1$, $\mu^* < 1$. For $w_t > \hat{w}_t$, according to equation (51), the law of motion for wage is concave and has the property as specified by equation (32).

In the following, I focus on the interval of $w_t \in (0, \hat{w}_t)$. Let $\mathbb{P}_t \equiv \frac{1}{\frac{1}{\eta(1+\varsigma^A_{t+1})(1-\mu^*)}-1}$. For $\varsigma^A_{t+1} \in (-1, \frac{1}{\eta} - 1]$, $\mathbb{P}_t \in (0, \frac{1}{\mu^*} - 1]$. Let $\mathbb{N} \equiv \frac{\theta}{\rho(1-\theta)}$. According to equation (51),

$$\mathbb{J}_t \equiv \frac{\partial w_{t+1}}{\partial w_t} = \frac{w_{t+1}}{w_t}\left[\alpha + \frac{\alpha(1-\theta)}{\theta}\frac{\mathbb{P}_t}{1+\mathbb{P}_t}\right] > 0, \tag{52}$$

$$\mathbb{H}_t \equiv \frac{\partial^2 w_{t+1}}{\partial w_t^2} = \frac{\mathbb{J}_t}{w_t}(1-\alpha)\left[\frac{\mathbb{P}_t}{\mathbb{N}(1+\mathbb{P}_t)} - 1 + \frac{\mathbb{P}_t}{\rho\mathbb{N}(1+\mathbb{P}_t)^2}\frac{1}{(\alpha\mathbb{N} + (1-\alpha)\frac{\mathbb{P}_t}{1+\mathbb{P}_t})}\right].$$

Given $\theta \in (0,1)$ and $\mu^* = \mu_A$, for $w_t \to 0$, $\varsigma^A_{t+1} \to -1$, $\mathbb{P}_t \to 0$, $\mathbb{H}_t < 0$; for $w_t \to \hat{w}_T$, $\varsigma^A_{t+1} \to \frac{1}{\eta} - 1$, $\mathbb{P}_t \to \frac{1}{\mu^*} - 1$, $\mathbb{H}_t \to \frac{\mathbb{J}_t}{w_t}\frac{(1-\alpha)\mathbb{P}_t}{(1+\mathbb{P}_t)\mathbb{N}}\left[\frac{1-\mu_A-\mathbb{N}}{1-\mu_A} + \frac{\frac{\mu_A}{\alpha}}{\rho\mathbb{N}+(1-\mu_A)}\right]$.

Let $\bar{\mathbb{N}} \equiv \frac{\sqrt{(\rho-1)^2+\frac{4(\rho+\mu_A)}{1-\mu_A}}-(\rho-1)}{2\rho}(1-\mu_A)$. In the interval of $w_t \in (0, \hat{w}_T)$, the law of motion for wage is concave-convex if $\mathbb{N} \in (0, \bar{\mathbb{N}})$ and concave if $\mathbb{N} \in (\bar{\mathbb{N}}, \infty)$. Intuitively, the smaller the wealth inequality θ, the larger the investment distortion, the larger the sectoral price differential, the stronger the specialization effect, the more likely the law of motion becomes convex.

Step 3: the threshold values for multiple steady states under trade integration

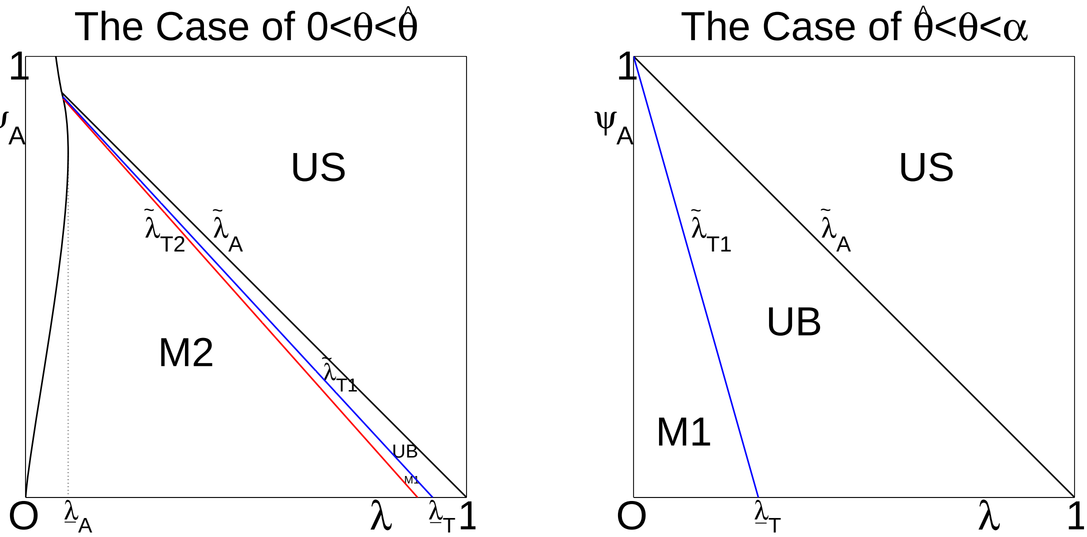

Figure 10: Multiple Steady States under Free Trade: $\theta \in (0, \alpha)$

Given $\theta \in (0, \hat{\theta})$, the left panel of figure 10 shows that multiple steady states arise under trade integration for $\{\lambda, \psi_A\}$ in region M2-M1, while the autarkic steady state is **u**nique and the borrowing constraints are **b**inding (**s**lack) for $\{\lambda, \psi_A\}$ in region UB (US). The right panel is for the case of $\theta \in (\hat{\theta}, \alpha)$. The middle and the right panels of figure 3 is a mapping of figure 10 by using (38)-(39) to convert the borders of four regions from the $\{\lambda, \psi_A\}$ space to the $\{\lambda, Z\}$ space. In the following, I characterize the borders of four regions in the left panel of figure 10.

For $\{\lambda, \psi_A\}$ in region US of figure 9, $\chi_A = \mu_A = 1$. Given $\chi^* = \chi_A = 1$, the law of motion for wage is log-linear with the slope $\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \alpha < 1$. See the bottom-right panel of figure 4. Thus, the autarkic steady state is still the unique steady state under free trade.

For $\{\lambda, \psi_A\}$ in region UB of figure 9, $\chi_A = \mu_A^\alpha < 1$. According to the upper panels of figure 4, multiple steady states arise in two cases, given $\chi^* = \chi_A < 1$.

- Case 1: Free trade destabilizes the autarkic steady state, $\mathbb{J}_t \mid_{w_A} > 1$. Use equation (52) to get (24). In the boundary case, $\mathbb{J}_t \mid_{w_A} = 1$ gives $1 - \mu_A = \frac{\mathbb{N}}{\eta}$. Combine it with $\mu_A = \frac{\lambda}{1-\psi_A}$ to get a threshold value

$$\tilde{\lambda}_{T2} = (1 - \psi_A)(1 - \frac{\theta}{\rho\eta(1-\theta)}) \tag{53}$$

which defines the border between region M2 and M1. Combine it with (38) to get,

$$\tilde{\lambda}_{T2} \equiv \frac{1}{1 + \frac{\theta(1-\hat{\theta})}{\hat{\theta}-\theta}} - \frac{Z^{\rho(1-\theta)}\eta^{\rho(1-\theta)-\theta}\left(1 + \frac{\theta(1-\hat{\theta})}{\hat{\theta}-\theta}\right)^{(1-\eta)\rho(1-\theta)-1}}{\left[1 + \frac{\theta(1-\hat{\theta})}{\hat{\theta}-\theta}(1-\eta)\right]^{\rho(1-\theta)-\theta}}. \tag{54}$$

- Case 2: The kink point of the law of motion for wage at $w_t = \hat{w}_T$ lies at the 45° line.

$$\hat{w}_T = \left(\frac{R}{\rho}\hat{w}_T\mu_A^{\eta-1}\right)^\alpha, \;\Rightarrow\; \psi_A^{\frac{1}{1-\theta}}\mathbb{F} = \left(\frac{R}{\rho}\mu_A^{\eta-1}\right)^\rho \tag{55}$$

Combine (55) and (42) with $\psi_H = \psi_A$ to get the boundary condition $\mu_A = \frac{\frac{1}{\eta}-1}{\eta^{-\frac{1}{1-\mathbb{N}}}-1}$. Combine it with $\mu_A = \frac{\lambda}{1-\psi_A}$ to get a threshold value

$$\tilde{\lambda}_{T1} = (1 - \psi_A)\frac{\frac{1}{\eta} - 1}{\eta^{-\frac{1}{1-\mathbb{N}}} - 1}, \tag{56}$$

which defines the border between region M1 and UB. Combine it with (38) to get

$$\tilde{\lambda}_{T1} \equiv \frac{1}{1 + \frac{\eta^{-\frac{(1-\alpha)\theta}{\alpha-\theta}}-1}{1-\eta}} - Z^{\rho(1-\theta)}\eta^{\rho(1-\theta)}\left(1 + \frac{\eta^{-\frac{(1-\alpha)\theta}{\alpha-\theta}} - 1}{1-\eta}\right)^{(1-\eta)\rho(1-\theta)-1}. \tag{57}$$

Given the model parameters, one can prove that $\tilde{\lambda}_{T2} < \tilde{\lambda}_{T1} < \tilde{\lambda}_A$.

Let $\hat{\theta} \equiv \frac{1}{1+\frac{1}{\rho\eta}} < \alpha$. Region M2 exists iff the horizontal intercept of $\tilde{\lambda}_{T2}$ is in region $(0,1)$, which requires $\theta < \hat{\theta}$, while region M1 exists iff the horizontal intercept of $\tilde{\lambda}_{T1}$ is in region $(0,1)$, which requires $\mathbb{N} < 1$ or equivalently $\theta < \alpha$. For $\theta \in (0, \hat{\theta})$, multiple steady states arise in two cases, as shown by region M2-M1 in the left panel of figure 10; for $\theta \in (\hat{\theta}, \alpha)$, multiple steady states arise in the second case, as shown by region M1 in the right panel of figure 10. □

Proof of Lemma 3

Proof. According to the proof of proposition 2, if multiple steady states arise under trade integration, the law of motion for wage has the concave-convex shape in the interval of $w_t \in (0, \hat{w}_T)$. Given $\chi^* = \chi_A$, the law of motion for wage must intersect with the 45° line at the autarkic steady state. If $\frac{\partial w_{t+1}}{\partial w_t} \mid_{w_A} > 1$, there must exist a steady state L with $w_L < w_A$ and $\frac{\partial w_{t+1}}{\partial w_t} \mid_{w_L} < 1$. Then, the law of motion for wage is convex for $w_t \in (w_A, \hat{w}_T)$ and there cannot exist a steady state in this interval. For $w_t > \hat{w}_T$, the law of motion for wage is concave and there must exist one and only one steady state in this interval.

If $\frac{\partial w_{t+1}}{\partial w_t} |_{w_A} < 1$, the autarkic steady state must be the steady state with the lowest steady-state income level. As shown in the proof of proposition 2, multiple steady states may arise if the kink point at $w_t = \hat{w}_T$ is above the 45° line. Thus, there exists one and only one steady state in the interval of $w_t > \hat{w}_T$. □

Proof of Lemma 4

Proof. Assumption 1 ensures the uniqueness of the autarkic steady state where the borrowing constraints are binding in each countries. Use equation (20) and (11) to get

$$\ln Z = \ln(1 - \frac{\lambda^i}{\mu_A^i}) + \left[1 - \frac{\theta}{\rho(1-\theta)}\right] \ln\left(1 + \frac{1-\eta}{\eta\mu_A^i}\right) + (1-\eta)\ln \mu_A^i \tag{58}$$

$$\frac{\partial \ln \mu_A^i}{\partial \ln \lambda^i} > 0 \quad \Leftrightarrow \quad \frac{\frac{\lambda^i}{\mu_A^i}}{1 - \frac{\lambda^i}{\mu_A^i}} + (1-\eta) > \left[1 - \frac{\theta}{\rho(1-\theta)}\right] \frac{\frac{1-\eta}{\eta\mu_A^i}}{1 + \frac{1-\eta}{\eta\mu_A^i}} \tag{59}$$

One can prove that condition (59) holds under assumption 1.

Figure 11 shows that μ_A is an increasing function of λ as long as $\{\lambda, \psi_A\}$ is to the right of the dashed curve in region M. Assumption 1 certainly satisfies this condition.

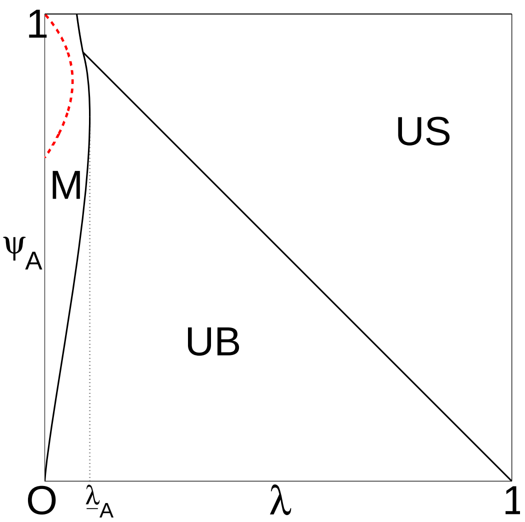

Figure 11: Financial Development and Steady-State Aggregate Output: $\theta \in (0, \hat{\theta})$

Under autarky, the markets for final goods clear domestically, $V_t^f = Y_t^f$. As both sectors are active, $r_t = Rq_{t+1}^B$. According to equations (1)-(2), $w_{t+1}^{1-\alpha}(q_{t+1}^A)^{\alpha\eta}(q_{t+1}^B)^{\alpha(1-\eta)} = 1$. Combine them to get $w_{t+1} = \left(\frac{R}{r_t}\mu_{t+1}^\eta\right)^\rho$. Combine it with equation (20) to get equation (25). In the autarkic steady state, $\Upsilon_A = \rho$ and the interest rate is $r_A = \rho[1 - \eta(1-\mu_A)]$. Following the proof of proposition 1, one can prove that, for $\{\lambda, Z\}$ in region US of the right panel of figure 1, $\frac{\partial \mu_A}{\partial \lambda} > 0$ and hence, $\frac{\partial \chi_A}{\partial \lambda} > 0$ and $\frac{\partial r_A}{\partial \lambda} > 0$. □

Proof of Lemma 5

Proof. Let $\psi^* \equiv 1 - \frac{\lambda}{\mu^*}$ and $\bar{\psi} \equiv 1 - \lambda$. In the case of full specialization in sector A, combine $\varsigma_{t+1}^A = \frac{1}{\eta} - 1$ with equation (48), (11), and the investment demand and supply in sector A to get

$$w_t = M_t^{i,A} = \int_{\underline{\epsilon}_t}^{\infty} \frac{n_{j,t}}{\psi_t} dG(\epsilon_j) = \frac{\underline{\epsilon}_t^{-\frac{1-\theta}{\theta}}}{\psi_t} w_t, \quad \psi_t = \left(\frac{w_t}{\mathbb{F}}\right)^{1-\theta}. \tag{60}$$

According to equation (50), if $w_t < \hat{w}_T$, the country does not fully specialize in sector A and the positive investment in sector B ensures $r_t = q^B_{t+1}R$. Given $\mu_{t+1} = \mu^*$, $\psi_t = \psi^* \equiv 1 - \frac{\lambda}{\mu^*}$.

If $w_t > \hat{w}_T$, the country fully specializes in sector A and the zero investment in sector B ensures $r_t > q^B_{t+1}R$. In this case, $\psi_t > \psi^*$ and, according to equation (27), the interest rate rises in aggregate income iff $\psi^i_t > \tilde{\psi}_T \equiv \frac{1}{1+\frac{1-\theta}{1-\alpha}}$. For $w_t > \bar{w}_T$ or equivalently $\psi_t > \bar{\psi}$, the borrowing constraints are slack $\mu_{t+1} = 1$ so that the interest rate declines in aggregate income.

$\hat{w}_T$, $\tilde{w}_T$, and $\bar{w}_T$ are converted respectively from ψ^*, $\tilde{\psi}_T$, and $\bar{\psi}$ by using equation (60).

□

Proof of Proposition 3

Proof. Let $\gamma \equiv \frac{\lambda}{\lambda^*} > 1$ denote the relative level of financial development in the two countries. Given assumption 1, use equations (20)-(21) to get

$$w^*_A = \left(\frac{R}{\rho}\frac{(\mu^*_A)^\eta}{1-\eta+\eta\mu^*_A}\right)^\rho = (\psi^*_A)^{\frac{1}{1-\theta}}\left(\frac{\eta\mu^*_A}{1-\eta+\eta\mu^*_A}\right)^{\frac{\theta}{1-\theta}}\mathbb{F}. \tag{61}$$

The solid (dashed) curve in the left panel of figure 12 shows the law of motion for wage in country N under trade (autarky), while the solid (dashed) curve in the right panel shows the pattern of the interest rate (the social rate of return) with respect to the wage rate under trade.

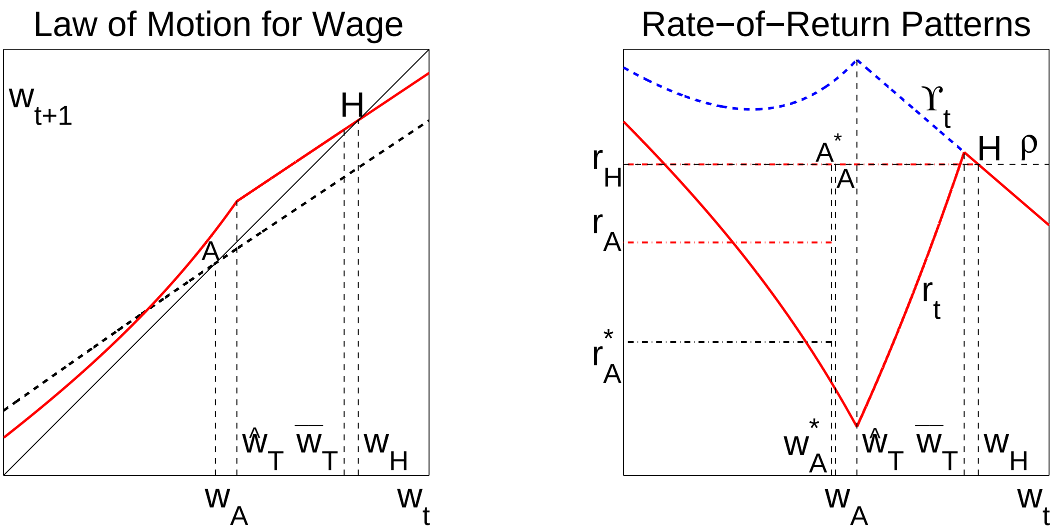

Figure 12: Law of Motion for Wage and Interest Rate Pattern under Free Trade

Starting from the autarkic steady state A, country N ends up in steady state H under free trade with $w_H = \left[\frac{R}{\rho}(\mu^*_A)^{\eta-1}\right]^\rho$. As described in subsection 4.1, the interest rate is a piecewise function of w_t. According to figure 12, $w_H > \bar{w}_T$ and hence, the borrowing constraints are slack at steady state H with $r_H = \rho > r_A > r^*_A$. In this case, free trade strictly widens the cross-country interest rate differential, independent of the value of γ. Combine equation (61) with $w_H = \bar{w}_T$ and $\gamma = 1$ to get the sufficient condition for this case,

$$\frac{1-\lambda^*}{\psi^*_A} = \eta^{\rho(1-\theta)}\left(1 + \frac{1-\eta}{\eta}\frac{1-\psi^*_A}{\lambda^*}\right)^{\rho(1-\theta)-\theta}, \tag{62}$$

which defines the threshold value $\tilde{\lambda}_C$ as the function of ψ^*_A and other parameters. $\tilde{\lambda}_C$ is shown as the border between region C and D in figure 13.

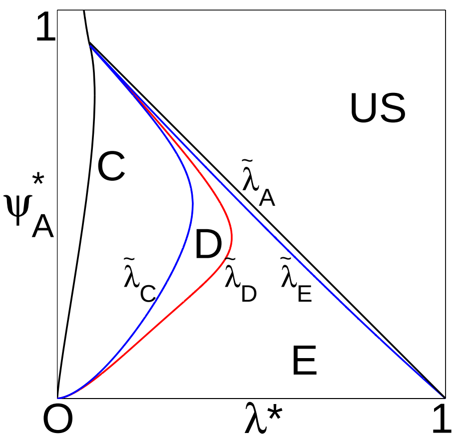

Figure 13: Parameter Configuration for Interest Rate Pattern under Free Trade: $\theta \in (0, \hat{\theta})$

Figure 12 shows one possible scenario where free trade does not lead to the interest rate reversal. Alternatively, according to the left panels of figure 6, $w_H < \bar{w}_T$ and hence, the borrowing constraints are binding in steady state H, $r_H = \frac{\rho\gamma\lambda^*}{1-\psi_H} < \rho$. The sufficient condition under which free trade does not reverse the cross-country interest rate differential is

$$r_H = \frac{\rho\gamma\lambda^*}{1-\psi_H} > r_A^* = \rho[1-\eta(1-\mu_A^*)], \tag{63}$$

where $\psi_H \in (1 - \frac{\lambda}{\mu_A^*}, 1 - \rho\gamma\lambda^*)$ is endogenous. Let us solve for ψ_H. In steady state H, country N specializes fully in sector A, implying that domestic saving is fully used to finance domestic investment in sector A. According to equation (30), $M_t^A = \frac{\epsilon_t^{-\frac{1-\theta}{\theta}}}{\psi_t} w_t = w_t$, or equivalently, $\psi_t = \epsilon_t^{-\frac{1-\theta}{\theta}}$. Combine it with equation (11) to get $w_H = \left(\frac{R}{\rho}(\mu_A^*)^{\eta-1}\right)^{\rho} = \psi_H^{\frac{1}{1-\theta}}\mathbb{F}$. Combine it with (61) to get $\psi_H = \psi_A^* \eta^{\rho(1-\theta)}(1 + \frac{1-\eta}{\eta}\frac{1-\psi_A^*}{\lambda^*})^{\rho(1-\theta)-\theta}$. Then, condition (63) is rewritten as

$$\gamma\lambda^* > \left[1 - \eta\left(1 - \frac{\lambda^*}{1-\psi_A^*}\right)\right]\left[1 - \psi_A^*\eta^{\rho(1-\theta)}\left(1 + \frac{1-\eta}{\eta}\frac{1-\psi_A^*}{\lambda^*}\right)^{\rho(1-\theta)-\theta}\right] \tag{64}$$

There are two possible cases.

Case 1: condition (64) holds, independent of γ. Plug $\gamma = 1$ into (64) to get $\tilde{\lambda}_D$ as the function of ψ_A^* and other parameters, which is shown as the border between region D and E in figure 13.
Case 2: condition (64) holds, if $\gamma > \underline{\gamma} > 1$. Consider the extreme case of $\lambda = \gamma\lambda^*$ and $\psi_A = 1 - \gamma\lambda^*$ such that the borrowing constraints are weakly binding at the autarkic state state in country N, $\mu_A = 1$. Given assumption 1, two countries only differ in the level of financial development. Use equations (20)-(21) to get the respective steady-state wage rates in two countries under autarky. Then, combine them to get

$$\gamma\lambda^* = 1 - \psi_A^*\left[\eta + \frac{1-\eta}{\mu_A^*}\right]^{\rho(1-\theta)-\theta}(\mu_A^*)^{(1-\eta)\rho(1-\theta)} \tag{65}$$

Combine equations (64)-(65) to get $\tilde{\lambda}_E$ as the function of ψ_A^* and other parameters, which is shown as the right border of region E in figure 13 shows $\tilde{\lambda}_E$.

Figure 5 is a mapping of figure 13 by using (38) to convert the borders of four regions from the $\{\lambda, \psi_A\}$ space to the $\{\lambda, Z\}$ space. □

B The Choice of Numeraire

Introducing final goods in my model is purely for narrative simplicity and the choice of numeraire does not matter for my results. In this section, I prove the analytical equivalence of the equilibrium allocations in setting C where the final good is introduced and chosen as the numeraire versus in setting A where good A is chosen as the numeraire. The choice of numeraire only affects the prices, with no impacts on the quantities, the relative prices, or the ratios in the model economy. Let X_t and $\hat{X}_t$ denote the prices in setting C and A, respectively. The analysis in my paper is conducted in setting C. In the following, I derive the autarkic allocation in setting A and the country index is suppressed.

Agents have the identical Cobb-Douglas preference over good A and B when old. To be specific, agent j born in period $t-1$ spends its entire net wealth in period t to maximize

$$U_{j,t-1} = \left(\frac{c^A_{j,t}}{\eta}\right)^{\eta}\left(\frac{c^B_{j,t}}{1-\eta}\right)^{1-\eta}.$$

A fraction η of its spending is devoted to good A, which is taken as the numeraire. Let $\hat{p}^f_t$ and $\hat{P}_t$ denote the price of good $f \in \{A, B\}$ and the aggregate price index, respectively. Thus, $\hat{p}^A_t = 1$ holds by definition and the aggregate price index is

$$\hat{P}_t = (\hat{p}^A_t)^{\eta}(\hat{p}^B_t)^{1-\eta} = (\hat{p}^B_t)^{1-\eta}. \tag{66}$$

Following the assumption of Antras and Caballero (2009), an investment good that combines good A and B according to the utility aggregator is converted into sector-specific physical capital, $K^f_{t+1} = RM^f_t$. The cost minimization implies that the price of the investment good is $\hat{P}_t$.

Since good A and B are produced from capital and labor in the Cobb-Douglas fashion and they enter into consumption and investment with the Cobb-Douglas aggregator, I get

$$\hat{Y}_t = \frac{Y^A_t}{\eta} = \frac{\hat{p}^B_t Y^B_t}{1-\eta} = \frac{\hat{w}_t L}{1-\alpha} = \frac{\hat{w}_t L^A_t}{(1-\alpha)\eta} = \frac{\hat{w}_t L^B_t}{(1-\alpha)(1-\eta)} = \frac{\hat{q}^A_t RM^A_{t-1}}{\alpha\eta} = \frac{\hat{q}^B_t RM^B_{t-1}}{\alpha(1-\eta)}, \tag{67}$$

$$\hat{w}_t^{1-\alpha}(\hat{q}^A_t)^{\alpha\eta}(\hat{q}^B_t)^{\alpha(1-\eta)} = \hat{P}_t, \tag{68}$$

where $\hat{q}^f_t$ denotes the MRK in sector $f \in \{A, B\}$ and $\mu_t \equiv \frac{\hat{q}^B_t}{\hat{q}^A_t}$ denotes the relative sectoral MRK. Due to the frictionless labor market and perfect cross-sector labor mobility, the sectoral labor allocation is efficient, $L^A_t = \eta L$ and $L^B_t = (1-\eta)L$, according to the sectoral share in the Cobb-Douglas aggregator. As the investment in sector A is subject to the MIR and financial frictions, the sectoral investment may not be efficient. Under autarky, domestic investment is financed by domestic saving. Combine it with (67),

$$M^A_t + M^A_t = \frac{\hat{w}_t L}{\hat{P}_t}, \;\Rightarrow\; M^A_t = \frac{\mu_{t+1}\frac{\hat{w}_t}{\hat{P}_t}}{1-\eta(1-\mu_{t+1})}\eta L \text{ and } M^B_t = \frac{\frac{\hat{w}_t}{\hat{P}_t}}{1-\eta(1-\mu_{t+1})}(1-\eta)L, \tag{69}$$

Use equations (1) and (67)-(69) to get

$$\hat{p}^B_{t+1} = \frac{1-\eta}{\eta}\frac{Y^A_{t+1}}{Y^B_{t+1}} = \frac{1-\eta}{\eta}\frac{\frac{L^A_{t+1}}{1-\alpha}\left(\frac{M^A_t}{L^A_{t+1}}\frac{R}{\rho}\right)^{\alpha}}{\frac{L^B_{t+1}}{1-\alpha}\left(\frac{M^B_t}{L^B_{t+1}}\frac{R}{\rho}\right)^{\alpha}} = \mu^{\alpha}_{t+1}. \tag{70}$$

The law of motion for wage is

$$\hat{w}_{t+1}\frac{L^A_{t+1}}{1-\alpha}=Y^A_{t+1}=\frac{L^A_{t+1}}{1-\alpha}\left(\frac{RM^A_t}{\rho L^A_{t+1}}\right)^{\alpha},\ \Rightarrow\ \hat{w}_{t+1}=\left(\frac{R}{\rho}\frac{\mu_{t+1}}{1-\eta(1-\mu_{t+1})}\frac{\hat{w}_t}{\hat{P}_t}\right)^{\alpha}. \tag{71}$$

Households have two options to save the labor income over time. They can lend one unit of funds in period t and get the gross interest payment r_t in period $t+1$. Alternatively, they can spend one unit of funds to form $\frac{R}{\hat{P}_t}$ unit of physical capital in sector B and get the investment revenue $\frac{R\hat{q}^B_{t+1}}{\hat{P}_t}$ in period $t+1$. The no-arbitrage condition gives

$$\hat{r}_t=\frac{R\hat{q}^B_{t+1}}{\hat{P}_t}. \tag{72}$$

Entrepreneurs can spend one units of funds in period t to form $\frac{R}{\hat{P}_t}$ units of capital in sector A and get the investment revenue $\frac{\hat{q}^A_{t+1}R}{\hat{P}_t}$ in period $t+1$. In period t, they can borrow against a fraction of the present value of investment revenue, $\lambda\frac{\hat{q}^A_{t+1}R}{\hat{r}_t\hat{P}_t}$, and use own funds to cover the gap. Let ψ_t denote the leverage ratio. If $\hat{q}^B_{t+1}<\hat{q}^A_{t+1}$, the borrowing constraints are binding and

$$\psi_t=1-\lambda\frac{\hat{q}^A_{t+1}R}{\hat{r}_t\hat{P}_t}=1-\frac{\lambda}{\mu_{t+1}}. \tag{73}$$

The individual's investment size in sector A must be no less than $\mathfrak{m}$ units of investment goods. The cutoff value $\underline{\epsilon}_t$ is associated with the agents who meet the MIR at the margin,

$$\frac{\hat{w}_t(1-\theta)\underline{\epsilon}_t}{\psi_t}=\mathfrak{m}\hat{P}_t \tag{74}$$

In terms of aggregate investment in sector A, the supply is equal to the demand,

$$\frac{\hat{w}_tL}{\psi_t}\underline{\epsilon}_t^{-\frac{1-\theta}{\theta}}=\hat{P}_tM^A_t=\frac{\eta\mu_{t+1}}{1-\eta(1-\mu_{t+1})}\hat{w}_tL,\ \ \Rightarrow\ \ \underline{\epsilon}_t^{-\frac{1-\theta}{\theta}}=\frac{\psi_t\eta\mu_{t+1}}{1-\eta(1-\mu_{t+1})} \tag{75}$$

The social rate of return is by definition

$$\hat{\Upsilon}_t=\frac{\hat{q}^A_{t+1}K^A_{t+1}+\hat{q}^B_{t+1}K^B_{t+1}}{\hat{P}_t(M^A_t+M^B_t)}=\rho\frac{\hat{w}_{t+1}L}{\hat{w}_tL} \tag{76}$$

Since the sectoral production function is Cobb-Douglas and goods enter into consumption and investment in the Cobb-Douglas fashion,

Combine equations (66), (68), and (70)-(72) to get

$$\hat{r}_t=\frac{R\hat{q}^B_{t+1}}{\hat{P}_t}=\frac{R\mu_{t+1}\hat{w}_{t+1}}{\frac{R}{\rho}\frac{\mu_{t+1}}{1-\eta(1-\mu_{t+1})}\hat{w}_t}=\rho\frac{\hat{w}_{t+1}}{\hat{w}_t}[1-\eta(1-\mu_{t+1})]. \tag{77}$$

Under autarky, the aggregate dynamics are characterized by $\{\hat{w}_t,\hat{p}^B_t,\hat{P}_t,\hat{\Upsilon}_t,\hat{r}_t,\psi_t,\underline{\epsilon}_t,\mu_t\}$ satisfying equations (66), (70), (71), (73)-(76), and (77).

Let us compare the equilibrium conditions in the two settings. As the choice of numeraire does not affect the ratios, equations (73) and (75) in setting A are identical as (29) and (30) in setting C. Since the price of good B in setting A is by definition equal to the relative sectoral price in setting C, $\hat{p}^B_t=\chi_t$, equation (70) is identical as (25). Convert the wage rate from setting A to setting C $w_t = \frac{\hat{w}_t}{\hat{P}_t}$ and combine it with (66)-(71) to get the law of motion for wage in setting C,

$$\hat{w}_{t+1} = w_{t+1}\hat{P}_{t+1} = \left[\frac{R}{\rho L}\frac{\mu_{t+1}}{1-\eta(1-\mu_{t+1})}\frac{\hat{w}_t}{\hat{P}_t}\right]^\alpha, \quad \Rightarrow \quad w_{t+1} = \left[\frac{R}{\rho L}\frac{\mu_{t+1}^\eta}{1-\eta(1-\mu_{t+1})}w_t\right]^\alpha,$$

which is identical as (20). Similarly, equations (76) and (77) can be converted into setting C as

$$\Upsilon_t = \rho\frac{\frac{\hat{w}_{t+1}}{\hat{P}_{t+1}}L}{\frac{\hat{w}_t}{\hat{P}_t}L} = \rho\frac{w_{t+1}}{w_t}, \text{ and } r_t = \frac{R\mu_{t+1}\frac{\hat{w}_{t+1}}{\hat{P}_{t+1}}}{\frac{R}{\rho}\frac{\mu_{t+1}}{1-\eta(1-\mu_{t+1})}\frac{\hat{w}_t}{\hat{P}_t}} = \rho\frac{w_{t+1}}{w_t}[1-\eta(1-\mu_{t+1})],$$

which is identical as (18) and (25), respectively.

In the two settings, since the equilibrium conditions are equivalent, the dynamic and stability properties of the model characterized in the $\{\lambda, Z\}$ space are also the same. one can use the law of motion for wage to analyze the model properties, as shown in the proof of proposition 1.

In setting C, the law of motion for wage is characterized by (20), where μ_{t+1} is an increasing function of w_t implicitly defined as below,

$$w_t = \frac{\psi_t^{\frac{1}{1-\theta}}}{\left[\frac{1-\eta}{\eta\mu_{t+1}}+1\right]^{\frac{\theta}{1-\theta}}}\mathbb{F} = \frac{\left(1-\frac{\lambda}{\mu_{t+1}}\right)^{\frac{1}{1-\theta}}}{\left(\frac{1-\eta}{\eta\mu_{t+1}}+1\right)^{\frac{\theta}{1-\theta}}}\mathbb{F}, \text{ and } \frac{\partial\mu_{t+1}}{\partial w_t} > 0. \tag{78}$$

In setting A, combine equations (71), (73)-(75) to get

$$\hat{w}_t^{\frac{1}{\alpha}} = \left(\frac{1-\frac{\lambda}{\mu_t}}{\frac{1-\eta}{\eta\mu_t}+1}\right)^{\frac{1}{1-\theta}}\frac{R}{\rho L\eta}\mathbb{F} \quad \text{and} \quad \hat{w}_t = \frac{\left(1-\frac{\lambda}{\mu_{t+1}}\right)^{\frac{1}{1-\theta}}}{\left(\frac{1-\eta}{\eta\mu_{t+1}}+1\right)^{\frac{\theta}{1-\theta}}}\mu_t^{\alpha(1-\eta)}\mathbb{F}, \tag{79}$$

where $\frac{\partial\mu_t}{\partial\hat{w}_t} > 0$ and $\frac{\partial\mu_{t+1}}{\partial\hat{w}_t} > 0$. Then, the law of motion for wage can be reformulated as

$$\hat{w}_{t+1} = \left(\frac{R}{\rho}\frac{\mu_{t+1}}{1-\eta(1-\mu_{t+1})}\frac{\hat{w}_t}{\mu_t^{\alpha(1-\eta)}}\right)^\alpha, \tag{80}$$

where μ_t and μ_{t+1} are the increasing functions of $\hat{w}_t$ implicitly defined by equations (79).

When analyzing the law of motion for wage, one only needs to keep track of μ_{t+1} as the functions of w_t in setting C, while one has to keep track of both μ_t and μ_{t+1} as the functions of w_t in setting A. Thus, the analysis is simpler in setting C than in setting A and hence, the analysis in my paper is conducted in setting C.

Due to the endogenous extensive margin of sectoral investment, the relative sectoral MRK is time-varying in my model, which essentially makes the analysis in setting A more complicated than in setting C. In Antras and Caballero (2009), as the extensive margin of sectoral investment is mute, the relative sectoral price is constant under autarky $\chi_t = \chi_A$ and so is the relative sectoral MRK $\mu_t = \mu_A$. Thus, the choice of numeraire does not matter for the analysis at all.

C Symmetry Breaking under Trade Equilibrium

Under autarky, the relative sectoral price is determined domestically in each country and, for the parameter configuration in region UB-US of the right panel of figure 1, the world economy has a unique, symmetric steady state. Trade integration decouples the relative sectoral price from domestic conditions and aligns it with the world level. In the following, I identify the conditions under which trade integration *inevitably* "breaks" the symmetric, autarkic steady state and lead to asymmetric steady states where inherently identical countries end up with different income levels.

C.1 The Symmetric Steady State

For the parameter configuration in region US-UB-M1 of the middle and the right panels of figure 3, trade integration does not destabilize the autarkic steady state for the small open economy so that the symmetric autarkic steady state in the world economy is still stable; for the parameter configuration in region M2, trade integration destabilizes the autarkic steady state for a small open economy so that the world economy does not have the stable, symmetric steady state.

C.2 The Asymmetric Steady States

According to the upper-left panel of figure 4 and equation (51), given χ^*, if free trade destabilizes the autarkic steady state, country i may end up either in steady state H where it specializes fully in sector A with aggregate income $Y_H = \frac{\left(\frac{R}{\rho}(\mu^*)^{\eta-1}\right)^\rho}{1-\alpha}$ and exports $\frac{(1-\eta)Y_H}{p^{*,A}}$ units of good A or in steady state L where it specializes partially towards sector B with $Y_L = \frac{\left(\frac{R}{\rho}\frac{(\mu^*)^\eta}{1-\eta(1-\mu^*)(1+\varsigma_L^A)}\right)^\rho}{1-\alpha}$ and imports $-\frac{\varsigma_L^A \eta Y_L}{p^{*,A}}$ units of good A, where $\varsigma_L^A < 0$ is a function of μ^*.

Suppose that the world economy is in a stable, asymmetric steady state where the fraction δ of countries end up in steady state H with aggregate income Y_H and the rest in steady state L with Y_L. The market of good A clears at the world level,[31]

$$\delta\frac{(1-\eta)Y_H}{p^{A,*}} = -(1-\delta)\varsigma_L^A\frac{\eta Y_L}{p^{A,*}}, \Rightarrow \delta = \frac{-\varsigma_L^A\left(\frac{\mu^*}{1-\eta(1-\mu^*)(1+\varsigma_L^A)}\right)^\rho}{\frac{1}{\eta} - 1 - \varsigma_L^A\left(\frac{\mu^*}{1-\eta(1-\mu^*)(1+\varsigma_L^A)}\right)^\rho}, \tag{81}$$

There exists a δ that supports the world relative sectoral price $\chi^* = (\mu^*)^\alpha$.

Proposition 5. *Given $\theta \in (0, \alpha)$, there exists a threshold value $\tilde{\lambda}_{TD}$ as a function of Z such that, for $\lambda < \tilde{\lambda}_{TD}$, the world economy has a continuum of stable, asymmetric steady states under trade integration where a fraction $\delta \in (\delta^-, \delta^+) \subset (0, 1)$ of the countries have the income $Y_H > Y_A$ and the rest have the income $Y_L < Y_A$.*

Proof. Suppose that the world economy is at an asymmetric steady state under trade integration where $\chi^* < 1$ and $\mu^* \in (\lambda, 1)$ are determined endogenously.[32] Since χ^* is not necessarily equal to χ_A, the autarkic steady state is not necessarily a steady state under free trade, either. Given χ^*, the law of motion for wage in an individual country is a piecewise function over two intervals, according to equation (51); it crosses the 45° line once at point H with $w_H > \hat{w}_T$ and twice at

[31]Given the balanced trade at the country level, if the market for one good clears at the world level, the market for the other good must also clear, according to the Walras' law.

[32]If $\chi^* = \mu^* = 1$, the law of motion for wage is concave and specified as equation (32) so that the world economy must have the symmetric steady state.

point L and M with $w_L < w_M < \hat{w}_T$. Let w_M denotes the wage at the unstable steady state. Compare the autarkic steady state versus the unstable steady state under trade integration,

$$\mathbb{P}_A \equiv \frac{\eta(1-\mu_A)}{1-\eta(1-\mu_A)}, \qquad w_A = \left[\frac{R}{\rho}(\mu_A)^\eta(1+\mathbb{P}_A)\right]^\rho = \psi_A^{\frac{1}{1-\theta}}\left(\frac{\mathbb{P}_A\mu_A}{1-\mu_A}\right)^{\frac{\theta}{1-\theta}}\mathbb{F},$$

$$\mathbb{P}_M \equiv \frac{\eta(1-\mu^*)(1+\varsigma_M^A)}{1-\eta(1-\mu^*)(1+\varsigma_M^A)}, \quad w_M = \left[\frac{R}{\rho}(\mu^*)^\eta(1+\mathbb{P}_M)\right]^\rho = (\psi^*)^{\frac{1}{1-\theta}}\left(\frac{\mathbb{P}_M\mu^*}{1-\mu^*}\right)^{\frac{\theta}{1-\theta}}\mathbb{F},$$

$$\frac{1+\mathbb{P}_M}{\mathbb{P}_M^{\mathbb{N}}} = \left(\frac{\psi^*}{\psi_A}\right)^{\frac{1}{\rho(1-\theta)}}\left(\frac{1-\psi_A}{1-\psi^*}\right)^{1-\eta}\frac{1}{\eta}\frac{1}{(1-\mu^*)^{\mathbb{N}}(\mu^*)^{1-\mathbb{N}}\mathbb{S}_A^{1-\mathbb{N}}}, \tag{82}$$

where $\mathbb{N} \equiv \frac{\theta}{\rho(1-\theta)}$ and $\mathbb{S}_A \equiv 1 + \frac{1-\eta}{\eta\mu_A}$. According to (52), multiple steady states arise if $\mathbb{P}_M > \frac{\mathbb{N}}{1-\mathbb{N}}$. Meanwhile, $\varsigma_M^A < \frac{1}{\eta} - 1$ or equivalently $\mathbb{P}_M < \frac{1}{\mu^*} - 1$. Overall, it gives $\mu^* < \bar{\mu}^* \equiv 1 - \mathbb{N}$.

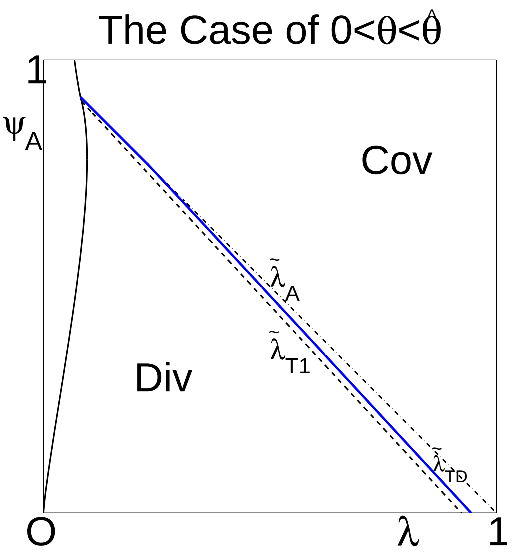

Figure 14: Trade Globalization and Symmetry Breaking

Given $\theta \in (0, \hat{\theta})$, the left panel of figure 14 shows that the world economy has asymmetric steady states under trade integration for $\{\lambda, \psi_A\}$ in region Div, while it has the unique, symmetric steady state for $\{\lambda, \psi_A\}$ in region Cov. One can use (38) to convert the border of the two regions from the $\{\lambda, \psi_A\}$ space in the left panel into the $\{\lambda, Z\}$ space in the right panel. In the following, I characterize the border of the two regions in figure 14.

Consider the boundary case where the law of motion for wage is tangent with 45° line at point M, i.e., $\mathbb{P}_M = \frac{\mathbb{N}}{1-\mathbb{N}}$. Then, equation (82) becomes

$$\left(\frac{1-\frac{\lambda}{\mu^*}}{\psi_A}\right)^{\frac{1}{\rho(1-\theta)}}\left[(1-\psi_A)\frac{\mu^*}{\lambda}\right]^{1-\eta}\frac{1}{\eta} = \left(\frac{1-\mu^*}{\mathbb{N}}\right)^{\mathbb{N}}\left[\frac{\mu^*}{1-\mathbb{N}}\left(1+\frac{1-\eta}{\eta}\frac{1-\psi_A}{\lambda}\right)\right]^{1-\mathbb{N}} \tag{83}$$

The procedure to solve the threshold value λ is as follows.

- given ψ_A, input $\mu^* \in (0, \bar{\mu}^*)$ to get $\lambda < \mu^*$ satisfying the above equation.
- find the maximum value of λ for each $\psi_A \in (0, 1)$.

□

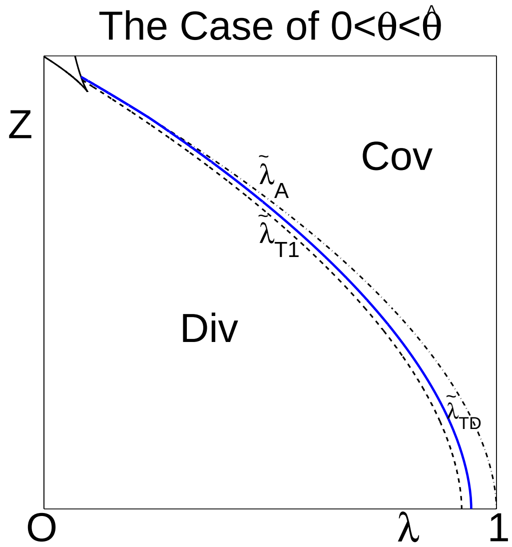

Figure 15: Symmetry Breaking in a World Economy under Trade Integration

Given $\theta \in (0, \hat{\theta})$, the solid curve in the right panel of figure 14 represents $\tilde{\lambda}_{TD}$ and splits the parameter space into region Div and Cov. For $\{\lambda, Z\}$ in region Div, $\lambda < \tilde{\lambda}_{TD}$ and trade integration may lead to asymmetric steady states in the world economy. In the small-open-economy setting, free trade leads to multiple steady states under the condition of $\chi^* = \chi_A$, given $\theta \in (0, \hat{\theta})$ and $\{\lambda, Z\}$ in region M2-M1 of the middle panel of figure 3. In the world-economy setting, I relax this condition so that trade integration leads to asymmetric steady states even if $\{\lambda, Z\}$ is in part of region UB of the middle panel of figure 3.[33]

If the asymmetric steady state is stable, trade integration may generate income divergence rather than convergence among inherently identical countries. In this case, the world economy is polarized into two groups of countries with different income levels. This way, I offer a theoretical support for the view that international trade is a mechanism through which rich countries become richer at the expense of poor countries.

According to the right panel of figure 14, whether trade integration leads to convergence depends on financial development and on wealth inequality. Thus, my model helps reconcile the mixed empirical evidence from the institutional perspective.

D A Simplified Model of Antras and Caballero (2009)

For comparison purposes, I first analyze a simplified version of Antras and Caballero (2009). The model setting differs from ours in the following aspects. First, there is no sector-specific MIR, i.e., $\mathfrak{m} = 0$. Second, agents differ in the technology endowment. Some agents can invest in sector A and they are called entrepreneurs, while others can only invest in sector B and they are called households. The mass of entrepreneurs is **exogenous** at τ. Besides, each agent is endowed with one unit of labor when young. Third, entrepreneurs can borrow up to $(\lambda^i - 1)$ times of their net wealth, where $\lambda^i \geq 1$ reflects the level of financial development in country i. If the borrowing constraints are binding, the leverage ratio is **exogenous** at $\frac{1}{\lambda^i}$. Fourth, the

[33]The dash-dotted curve shows $\tilde{\lambda}_A$ and and the dashed curve shows $\tilde{\lambda}_{T1}$. The region between the two curves is region UB of the middle panel of figure 3.

world economy consists of two countries, N (North) and S (South), which only differ in the level of financial development.

Assumption 3. $0 < \tau < \eta < 1$.

Under autarky, equations (4)-(5) describe the sectoral demand for investment and labor. In the case of efficient allocation, the sectoral rate of return equalizes, $\mu^i_{t+1} = 1$, and the sectoral investment is proportional to the sector share in the aggregate production function, $M^{i,A}_t = \eta w^i_t$ and $M^{i,B}_t = (1-\eta)w^i_t$. Under assumption 3, the efficient investment size in sector A exceeds the total net wealth of entrepreneurs, $M^{i,A}_t > w^i_t \tau$, implying that entrepreneurs need to borrow the amount of $(\eta - \tau)w^i_t$.

Let $\bar{\lambda} \equiv \frac{\eta}{\tau}$. If $\lambda^i \geq \bar{\lambda}$, the total debt capacity of entrepreneurs exceeds the amount of loans required for the efficient investment in sector A, $(\lambda^i - 1)w^i_t\tau > (\eta - \tau)w^i_t$. Thus, the sectoral investment is efficient, $\mu^i_{t+1} = 1$, and the borrowing constraints are slack.

For $\lambda^i \in [1, \bar{\lambda})$, the total debt capacity of entrepreneurs is below the efficient amount of loans. Thus, the sectoral investment is inefficient, $\mu^i_{t+1} < 1$ and the borrowing constraints are binding. Due to the exogeneity in the mass of entrepreneurs τ and in the leverage ratio $\frac{1}{\lambda^i}$, the sectoral supply of investment is a constant fraction of aggregate saving,

$$\frac{M^{i,A}_t}{w^i_t} = \lambda^i\tau < \eta \quad \text{and} \quad \frac{M^{i,B}_t}{w^i_t} = 1 - \lambda^i\tau > 1 - \eta. \tag{84}$$

Then, the changes in aggregate income only affect the sectoral investment on the intensive margin. As *the extensive margin is mute*, the sectoral investment ratio is constant and so is the relative sectoral rate of return. Combine (4) and (84) to get

$$\frac{M^{i,A}_t}{M^{i,B}_t} = \frac{\lambda^i\tau}{1-\lambda^i\tau} \quad \text{and} \quad \mu^i_{t+1} = \mu^i_A = \frac{\frac{1}{\eta}-1}{\frac{1}{\lambda^i\tau}-1} \in (\frac{\frac{1}{\eta}-1}{\frac{1}{\tau}-1}, 1) \quad \text{and} \quad \frac{\partial \mu^i_A}{\partial \lambda^i} > 0. \tag{85}$$

Combine them with equations (20)-(25) to get the law of motion for wage and the solution to the interest rate. Note that the aggregate efficiency indicator is constant and so is the ratio of the interest rate over the social rate of return,

$$\Gamma^i_t = \Gamma^i_A = \frac{(\mu^i_A)^\eta}{1-\eta+\eta\mu^i_A} \quad \text{and} \quad \frac{r^i_t}{\Upsilon^i_t} = 1 - \eta + \eta\mu^i_A.$$

Assumption 4. $1 \leq \lambda^S < \lambda^N \leq \bar{\lambda}$.

Lemma 6. *Under autarky, if assumptions 3-4 hold, the borrowing constraints are binding and there exists a unique, stable steady state in each country where* $Y^N_A > Y^S_A$, $\chi^N_A > \chi^S_A$, $\mu^N_A > \mu^S_A$, *and* $r^N_A > r^S_A$.

Proof. According to equation (85), for $\lambda^i \in [1, \bar{\lambda})$, $\mu^i_A < 1$, implying that the cross-sector investment is inefficient and the rate of return is higher in sector A than in sector B. Thus, entrepreneurs borrow to the limit.

According to equation (20), given $\mu^i_{t+1} = \mu^i_A$, the law of motion for wage is log-linear with the slope $\frac{\partial \ln w^i_{t+1}}{\partial \ln w^i_t} = \alpha < 1$ so that there exists a unique steady state with $w^i_A = \left(\frac{R}{\rho}\Gamma^i_A\right)^\rho$.

According to equations (20)-(25), the wage rate is $w^i_A = \left(\frac{R}{\rho}\Gamma^i_A\right)^\alpha$ with $\frac{\partial w^i_A}{\partial \mu^i_A} > 0$ and the interest rate is $r^i_A = \rho[1-\eta+\eta\mu^i_A]$ with $\frac{\partial r^i_A}{\partial \mu^i_A} > 0$ in the autarkic steady state. Combine them with (85) to get $\mu^N_A > \mu^S_A$, $\chi^N_A = (\mu^N_A)^\eta > (\mu^S_A)^\eta = \chi^S_A$, $r^N_A > r^S_A$, and $w^N_A > w^S_A$. □

Lemma 6 is identical as lemma 4. Suppose that the world economy is in the autarkic steady state before period 0. If agents are allowed to borrow and lend abroad from period 0 on, financial capital flows are "uphill" from country S to N. Here, the global imbalances are an equilibrium response to cross-country differences in financial development (Caballero, Farhi, and Gourinchas, 2008; Mendoza, Quadrini, and Rios-Rull, 2009; von Hagen and Zhang, 2014).

D.1 Interest Rate Reversal under Free Trade

Suppose that the world economy moves from autarky to free trade in the way specified in section 3. Equations (48) and (49) specify the sectoral demand for investment in period t and for labor in period $t+1$, respectively. Since $\mu^*_{t+1} < \mu^N_A < 1$, the borrowing constraints are binding so that equations (84) specify the sectoral supply of investment. $M^{i,B}_t > 0$ implies the *coupling* of the interest rate with the rate of return in sector B. Combine (48) and (84) to get

$$\varsigma^{i,A}_{t+1} = \frac{1}{\eta\left[1 + \mu^*_{t+1}(\frac{1}{\lambda^i \tau} - 1)\right]} - 1 \in (-1, \frac{1}{\eta} - 1), \tag{86}$$

which reflects the degree of specialization.[34]

Given the world relative sectoral rate of return μ^*_{t+1}, the law of motion for wage and the interest rate in country i are specified as follows,

$$w^i_{t+1} = \left(\frac{R}{\rho} w^i_t \Gamma^i_t\right)^{\alpha}, \quad \text{where } \Gamma^i_t = (1-\tau\lambda^i)(\mu^*_{t+1})^{\eta} + \tau\lambda^i(\mu^*_{t+1})^{\eta-1}, \tag{87}$$

$$r^i_t = \Upsilon^i_t[1 - \eta(1-\mu^*_{t+1})(1+\varsigma^{i,A}_{t+1})] = \frac{\Upsilon^i_t}{1 + \tau\lambda^i(\frac{1}{\mu^*_{t+1}} - 1)} < \Upsilon^i_t \equiv \rho \frac{w^i_{t+1}}{w^i_t}. \tag{88}$$

The world market clearing condition for good A determines the world relative sectoral price χ^*_{t+1} as well as the world relative sectoral rate of return μ^*_{t+1},

$$V^{N,A}_{t+1}\varsigma^{N,A}_{t+1} + V^{S,A}_{t+1}\varsigma^{S,A}_{t+1} = 0, \quad \Rightarrow \quad w^N_{t+1}\varsigma^{N,A}_{t+1} + w^S_{t+1}\varsigma^{S,A}_{t+1} = 0. \tag{89}$$

Proposition 6. *Starting from the autarkic steady state, free trade induces country N (S) to specialize* ***partially*** *to sector A (B). It raises the level and maintains the rank of aggregate income in the two countries. It reverses the cross-country interest rate pattern.*

Proof. According to equation (89), $\varsigma^{N,A}_{t+1}$ and $\varsigma^{S,A}_{t+1}$ must have the opposite sign. Combine it with equation (86) to get $\varsigma^{N,A}_{t+1} > 0 > \varsigma^{S,A}_{t+1}$ and $\mu^*_{t+1} \in (\mu^S_A, \mu^N_A)$, implying that country N (S) specializes toward sector A (B) and exports good A (B). According to equation (86), one can prove that $\varsigma^{N,A}_{t+1} \in (0, \bar{\varsigma})$ and $\varsigma^{S,A}_{t+1} \in (\underline{\varsigma}, 0)$, where $\bar{\varsigma} \equiv \frac{1}{\eta + (1-\eta)\frac{\frac{1}{\lambda^N} - \tau}{\frac{1}{\lambda^S} - \tau}} - 1 < \frac{1}{\eta} - 1$ and $\underline{\varsigma} \equiv \frac{1}{\eta + (1-\eta)\frac{\frac{1}{\lambda^S} - \tau}{\frac{1}{\lambda^N} - \tau}} - 1 > -1$, implying the partial specialization in both countries.

According to equation (87),

$$\frac{\partial \Gamma^i_t}{\partial \mu^*_{t+1}} = (\mu^*_{t+1} - \mu^i_A)\frac{(1-\tau\lambda^i)\eta}{(\mu^*_{t+1})^{2-\eta}}, \quad \Rightarrow \quad \text{sgn}\left(\frac{\partial \Gamma^i_t}{\partial \mu^*_{t+1}}\right) = \text{sgn}(\mu^*_{t+1} - \mu^i_A). \tag{90}$$

[34] If country i fully specializes in sector A, $\varsigma^{i,A}_t = \frac{1}{\eta} - 1$; if it fully specializes in sector B, $\varsigma^{i,A}_t = -1$. If it partially specializes in sector A, $\varsigma^{i,A}_t \in (0, \frac{1}{\eta} - 1)$; if it partially specializes in sector B, $\varsigma^{i,A}_t \in (-1, 0)$.

For country N, $\mu^*_{t+1} < \mu^N_A$ implies $\frac{\partial \Gamma^N_t}{\partial \mu^*_{t+1}} < 0$. Then, the fall in μ^N_{t+1} from μ^N_A to μ^*_{t+1} implies that $\Gamma^N_t > \Gamma^N_A$ and $Y^N_{t+1} > Y^N_A$ under free trade. For country S, $\mu^*_{t+1} > \mu^S_A$ implies $\frac{\partial \Gamma^S_t}{\partial \mu^*_{t+1}} > 0$. Then, the rise in μ^S_{t+1} from μ^S_A to μ^*_{t+1} implies that $\Gamma^S_t > \Gamma^S_A$ and $Y^S_{t+1} > Y^S_A$.

$$\frac{\partial \Gamma^i_t}{\partial \lambda^i} = \tau(\mu^*_{t+1})^\eta(\frac{1}{\mu^*_{t+1}} - 1) > 0 \tag{91}$$

Thus, $\lambda^N > \lambda^S$ gives $\Gamma^N_t > \Gamma^S_t$ and hence $Y^N_T > Y^S_T$.

According to lemma 6, $\lambda^N > \lambda^S$ gives $r^N_A > r^S_A$ in the autarkic steady state. According to equation (88), given μ^*_{t+1}, the social rate of return $\Upsilon^i_T = \rho$ and $\varsigma^{N,A}_T > 0 > \varsigma^{S,A}_T$ in the steady state under free trade jointly imply $r^N_T < r^S_T$, . □

Let us first consider country S. Given $\chi^S_A < \chi^N_A$, country S has a comparative advantage in good B. By the same logic as mentioned in section 3, free trade raises (reduces) the domestic demand for labor and investment in sector B (A). Due to the exogeneity in the mass of entrepreneurs and the leverage ratio, the sectoral investment does not respond to the demand change. Since free trade only triggers the cross-sector reallocation of labor, the capital-labor ratio declines (rises) in sector B (A) in period 1. Thus, the MRK rises (declines) in sector B (A) in period 1 and so does the sectoral rate of return in period 0. *Coupled with the rate of return in sector B*, the interest rate also rises in period 0, $r^S_0 = q^{S,B}_1 R > q^{S,B}_A R = r^S_A$. By the same logic, free trade triggers the labor reallocation in country N towards sector A and the rise in the capital-labor ratio in sector B leads to the declines in the rate of return in sector B and the interest rate.

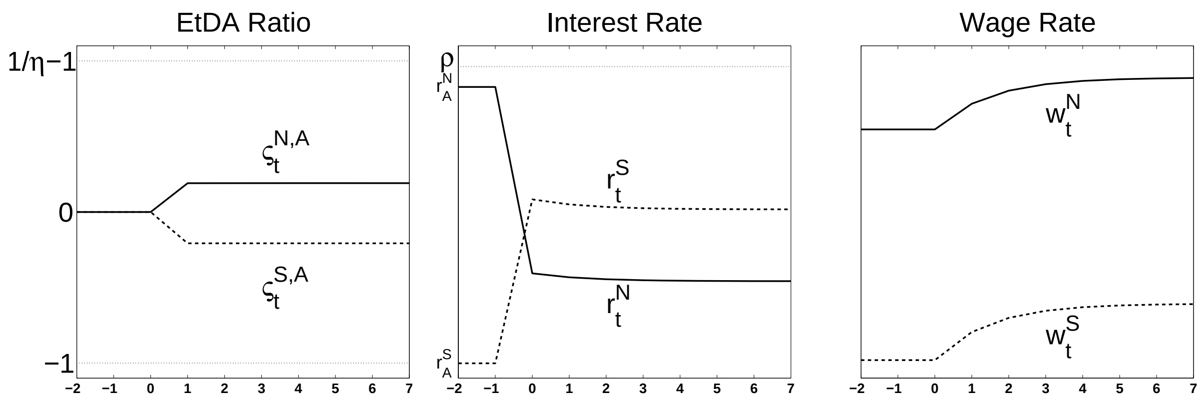

Figure 16: Model Dynamics under Free Trade with the Exogenous Extensive Margin

Figure 16 shows the impulse responses of three endogenous variables upon free trade in period $t = 1$. The left panel shows that free trade induces country N (S) to specializes *partially* towards sector A (B). The middle panel shows that moving from autarky to free trade not only raises (reduces) the interest rate in country S (N) but also reverses the cross-country interest rate patterns. Thus, Antras and Caballero (2009) argue that free trade can reverse the direction of financial capital flows.

D.2 Can Free Trade and Capital Flows Lead to FPE?

Free trade affects aggregate income in each country through the terms-of-trade effect and the specialization effect. As free trade only triggers the cross-sector reallocation of labor, the specialization effect is small and dominated by the terms-of-trade effect so that aggregate income rises in country S. Due to the positive terms-of-trade and specialization effects, aggregate income rises in country N. Overall, free trade does not change the rank of aggregate income in the two countries. See the right panel of figure 16.

Intuitively, the sector-specific financial frictions distort aggregate allocation in two dimensions. On the intra-temporal dimension, the cross-sector investment composition is distorted so that the MRK is not equalized in the two sectors, $\mu_{t+1}^i < 1$; on the inter-temporal dimension, the aggregate credit demand and supply are distorted so that the private and the social rates of return are not equalized, $\frac{r_t^i}{\Upsilon_t^i} = 1 - \eta(1 - \mu_{t+1}^i) < 1$. Free trade alone equalizes globally the ratio rather than the level of sectoral MRK, while financial capital mobility alone equalizes globally the interest rate rather than the social rate of return. In either case, the factor prices are not equalized.

Let us start from the steady state under free trade. The relative sectoral price is equalized $\chi_t^S = \chi_t^N$ and so is the relative sectoral rate of return, $\mu_{t+1}^S = (\chi_{t+1}^S)^{\frac{1}{\alpha}} = (\chi_{t+1}^N)^{\frac{1}{\alpha}} = \mu_{t+1}^N$. If agents are also allowed to borrow and lend abroad, the interest rate is equalized, $r_t^S = r_t^N$. According to equation (84), sector B is always active so that the interest rate is coupled with the rate of return in sector B. Thus, the rate of return in sector B is also equalized, $q_{t+1}^{S,B}R = r_t^S = r_t^N = q_{t+1}^{N,B}R$.

Overall, allowing free trade and capital flows implicitly equalizes the rate of return in sector A, $q_{t+1}^{S,A}R = \frac{q_{t+1}^{S,B}R}{\mu_{t+1}^S} = \frac{q_{t+1}^{N,B}R}{\mu_{t+1}^N} = q_{t+1}^{N,A}R$. Despite international immobility of labor, the wage rate is equalized and so is aggregate income,

$$Y_{t+1}^S = \frac{w_{t+1}^S}{1-\alpha} = \frac{[(q_{t+1}^{S,A})^\eta (q_{t+1}^{S,B})^{1-\eta}]^{-\rho}}{1-\alpha} = \frac{[(q_{t+1}^{N,A})^\eta (q_{t+1}^{N,B})^{1-\eta}]^{-\rho}}{1-\alpha} = \frac{w_{t+1}^N}{1-\alpha} = Y_{t+1}^N.$$

In this model and in the model of Antras and Caballero (2009), allowing both free trade and capital flows leads to FPE and income convergence. Thus, global imbalances vanish.

The results of the interest rate reversal and FPE depend critically on the coupling of the interest rate with the rate of return in sector B. This feature exists as long as the unconstrained sector B is active $M_t^{i,B} > 0$ in each country, which is then ensured by *the exogeneity in the mass of entrepreneurs and the leverage ratio* in this model.