Trade-Driven Sectoral Upgrading and the Global Imbalances

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Abstract

This paper analyzes how trade integration may affect international financial flows in a world with heterogeneous financial development. In the presence of financial frictions and sector-specific minimum investment requirements, the static gains from trade triggers cross-sector investment reallocation on **the extensive margin**. If trade-driven sectoral shifts allow the more financially developed country (North) to abandon low-return production activities and upgrade to high-return activities along the value chains, deepening trade integration may amplify the global imbalances (a phenomenon of large financial flows from developing to developed countries observed in the recent decades). This way, **trade-driven sectoral upgrading** in North becomes as a critical mechanism through which the substantial decline in trade and communication costs and the resulting boom in supply-chain trade may contribute to the global imbalances in the recent decades. This finding complements Antras and Caballero (2009, Journal of Political Economy).

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1 Introduction

The recent wave of globalization has two prominent features. First, emerging economies (especially China and other emerging Asian economies) have witnessed large current account surplus, while advanced economies (notably, the United States) have incurred persistent current account deficits in the past two decades. Accordingly, financial flows have been "uphill" from the poor to the rich countries (Kose et al., 2010; Prasad, Rajan, and Subramanian, 2006). Such global imbalances differ significantly from the previous episodes and are in stark contrast to the predictions of neoclassical theories. Second, due to technological progress and the removal of trade barriers, the costs of transportation, communication, and coordination have declined dramatically since 1990s, which accelerates international fragmentation of production (Baldwin, 2013b; Grossman and Rossi-Hansberg, 2006; Timmer et al., 2014). Nowadays, international production, trade and investments are organized within global value chains, which have substantially transformed the landscape of world production networks and influenced trade policies (Amador and Cabral, 2016; Antràs, 2015; Park, Nayyar, and Low, 2013).

Traditionally, trade and capital flows have been analyzed separately in the literature and economists have put little research effort on their interactions. Recent works suggest that such a separation is not always innocuous (Eaton et al., 2016; Ghironi and Melitz, 2005; Jin, 2012; Ju, Shi, and Wei, 2014). In a critical contribution to this literature, Antras and Caballero (2009) show that if the global imbalances are an equilibrium response to heterogeneous degrees of financial development across the world, deepening trade integration raises the return to capital in less financially developed countries (South) and resolves the global imbalances, while trade protectionism may backfire. Their findings have profound implications to the political debates at the peak of global imbalances in 2007-2008. However, the recent global imbalances have emerged and accelerated in parallel to world-wide trade liberalization, which seems at odds with their predictions.

In this paper, I show that how trade integration affects the global imbalances depends critically on how far it reshapes the industrial structure in the more financially developed country (North). If trade-driven sectoral shifts allow North to abandon low-return activities and upgrade to high-return activities, trade liberalization may further raise the interest rate in North, which amplifies the global imbalances. The intuition is as follows.

Consider a two-sector, overlapping-generation model where firms hire physical capital and labor to produce two goods which are combined in the Cobb-Douglas aggregator for consumption and investment. "Physical capital" should be interpreted broadly to include human capital as well as any tangible or intangible capital goods used in production. The two sectors are symmetric, except that individual investment is subject to the minimum investment requirements (hereafter, MIR) in a particular sector. For simplicity, the sector without the MIR is called sector 1, while the one with the MIR is called sector 2. In the absence of financial frictions, domestic savings are allocated efficiently across sectors. As a result, the sectoral output price is equalized and so is the sectoral rate of return.

In the presence of financial frictions and the heterogeneity in individual wealth, the agents with sufficiently high net wealth can meet the MIR and invest in sector 2, and they are called entrepreneurs; other agents have to invest in sector 1 or lend to the credit market, and they are called households. If financial frictions are sufficiently severe, the mass of entrepreneurs is inefficiently low and so are the investment and the output in sector 2. Thus, the rate of return and the output price are higher in sector 2 than in sector 1. The lower the level of financial development, the tighter the borrowing constraints, the smaller the mass of entrepreneurs, the larger the distortions on sectoral investment and output, the larger the sectoral rate-of-return and price differentials, the lower the income per capita. Here, the *extensive margin of investment*[1] is a key channel through which financial frictions and the sector-specific MIR distort the aggregate allocation efficiency.

Consider a small open economy (North) which is more financially developed than the rest of the world (South). At the autarkic steady state, income per capital and sector 1's rate of return are higher in North than in South. As investing in sector 1 and lending are perfect substitutes, the interest rate is coupled with sector 1's rate of return and is also higher in North. If allowed, financial flows are "uphill" from South to North. This way, the global imbalances arise here as an equilibrium responses to heterogeneous financial development across the world. Besides, at the autarkic steady state, the output price in sector 2 (1) is lower (higher) in North than in South, implying that financial development is a determinant of comparative advantage for trade.

Staring from autarkic steady state, trade induces North to specialize towards sector 2 and the static gains raise its national income, which affects the sectoral investment in two ways. First, the higher national income allows each agents to invest more so that the investment scale in each sector tends to rise along **the intensive margin**. The decreasing MRK effect then **dampens** the rise in the national income in the next period. Second, the higher national income allows more agents to overcome the MIR so that domestic investment is reallocated towards sector 2 along **the extensive margin**. It reinforces North's comparative advantage and the enhanced specialization **amplifies** the rise in the national income in the next period. As long as the investment reallocation effect dominates the decreasing MRK effect, North's national income rises over time until the mass of entrepreneurs becomes so large that their total borrowing capacity exceeds the entire household saving. In that case, North abandons sector 1 and specializes fully in sector 2, which turns off the investment reallocation effect. Then, the decreasing MRK effect brings North to a new steady state. This way, the *extensive margin of investment* is the key channel through which trade may induce North to abandon the low-return sector, which fundamentally changes the way the interest rate is determined, as explained below.

When North specializes **partially** towards sector 2, trade reverses cross-country differences in the rate of return in sector 1 through the mechanism described in Antras and Caballero (2009). As sector 1 is still active, the interest rate is coupled with its rate of return, implying that the interest rate is lower in North than in South, a result Antras and Caballero (2009) calls "**the interst rate reversal**". If allowed, financial flows are "downhill" from North to South. In this case, free trade resolves the global imbalances.

When North eventually abandons sector 1 and specializes **fully** in sector 2, sector 1 vanishes so that the interest rate is **decoupled** from sector 1's rate of return and the mechanism of Antras and Caballero (2009) ceases to work. Then, the interest rate in

[1]In each sector, the total investment depends on the investment size of individual investors (the intensive margin) as well as the mass of investors (the extensive margin).

North is coupled with the rate of return in sector 2, which is higher than its autarkic steady-state level, a result I call "**the interst rate re-reversal**". In this case, trade integration amplifies rather than resolves the global imbalances

To sum up, the degree of specialization affects the way the interest rate in North is determined, which then determines whether trade flows reverse or amplify the cross-country interest rate pattern. In other words, whether trade integration resolves or amplifies the global imbalances depends critically on how far it reshapes the industrial composition in the more financially developed country.

This finding allows us to revisit the impacts of the recent boom of value-chain trade on the global imbalances. "Sectors" in my model can be interpreted broadly as production stages or tasks. Since the 1990s, the accelerated international fragmentation of production and supply-chain trade (Baldwin, 2013a; Goos, Manning, and Salomons, 2014; Marcolin, Miroudot, and Squicciarini, 2016; Timmer et al., 2014) has induced advanced economies to offshore low-return fabrication/assembly activities to emerging economies and specialize towards high-return upstream/downstream activities, such as R&D, product design, the manufacturing of key components, marketing, brand building, customer services. As upstream/downstream activities are mainly involved with knowledge-intensive, non-routine tasks, they are subject to relatively high MIR (particularly in terms of human capital) and severe financial frictions. Thanks to their sophisticated financial markets, advanced economies have a comparative advantage in these activities. If supply-chain trade allows them to constantly upgrade along the value chain, the cross-country interest rate differential can be maintained or even amplified rather than reversed.

In this paper, I emphasize **trade-drive sectoral upgrading** as a critical mechanism through which the recent boom in supply-chain trade may contribute to the global imbalances. It fundamentally complements the findings of Antras and Caballero (2009). Undoubtedly, various factors, e.g., globalization, technology progress, industrial policies, and etc., may contribute to the sectoral upgrading of advanced economies along the value chain. No matter what the causes are, as long as they abandon the low-return activities and climb up the value chain, the structural shifts in industrial composition may fundamentally change the way the interest rate is determined and the patterns of international financial flows. The core message of this paper is that one should take into account the underlying shifts of industrial structures when analyzing international capital flows.

This paper is closely related to the recent literature on explaining the global imbalances as an equilibrium response to heterogeneous financial development across countries (Caballero, Farhi, and Gourinchas, 2008; Gourinchas and Rey, 2014; Ju and Wei, 2010; Mendoza, Quadrini, and Rios-Rull, 2009; von Hagen and Zhang, 2014; Zhang, 2017). Building upon this literature, Antras and Caballero (2009) show that deepening trade integration leads to the interest rate reversal, which resolves the global imbalances. They focus on the interest rate response to trade flows in South where both constrained and unconstrained sectors are always active. However, they do not explore explicitly whether and under what conditions trade flows may allow North to abandon the low-return sector.

By introducing sector-specific MIR into the basic setting of Antras and Caballero (2009), I endogenize the extensive margin of investment. It is exactly through this channel that the static gains from trade triggers cross-sector investment reallocation, leading to the possibility of North abandoning the low-return activities and the interest rate re-reversal.

Grossman and Rossi-Hansberg (2006) and Baldwin (2013a,b) show that the recent wave of offshoring and supply-chain trade has fundamentally changed the composition of world trade and transformed the industrial structures in advanced economies and in emerging economies. My paper emphasizes the critical role of supply-chain trade in affecting the industrial structures in advanced economies and the patterns of international financial flows. In subsection 5.4, I use the "smile curve" to illustrate this point explicitly in the context of supply-chain trade.

Kletzer and Bardhan (1987) were the first to show that better access to capital becomes a source of comparative advantage. It was then followed by a strand of theoretical literature on financial development and international trade (Antras and Caballero, 2009; Beck, 2002; Chesnokova, 2007; Ju and Wei, 2005).[2] Matsuyama (2005) introduces sector-specific borrowing constraints in a static model and shows that trade allows the rich (poor) country to fully specialize in the sector with tighter (looser) borrowing constraints. Wynne (2005) argues that a country's wealth can be a determinant of comparative advantage when access to credit differs across sectors, i.e., wealthier nations exhibit a comprehensive advantage towards goods produced in sectors facing more severe financial frictions. Ju and Wei (2011) point out that, in the countries with low-quality institutions, the quality of financial system is an independent source of comparative advantage. Building upon this literature, I analyze the joint determination of trade and capital flows.

Jin (2012) integrates factor-proportions-based trade and financial flows in an OLG model and shows that capital tends to flow to countries that become more specialized in capital-intensive industries. Ju, Shi, and Wei (2014) embed two tradeable sectors with different factor intensity in a small-open-economy setting and show that the current account adjustment to exogenous shocks depends on factor market flexibility. Instead of introducing sector-specific financial frictions or sector-specific factor intensity, I focus on a real friction, i.e., the sector-specific MIR.

In the literature, the MIR is used to capture the investment indivisibility at the individual level, which is a important feature of business ideas, physical and human capital (Aghion and Bolton, 1997; Banerjee and Moll, 2010; Banerjee and Newman, 1993; Galor and Zeira, 1993; Piketty, 1997; Zhang, 2017). Recently, Erosa and Hidalgo-Cabrillana (2008), Barseghyan and DiCecio (2011), Buera, Kaboski, and Shin (2011), Manova (2013), and Midrigan and Xu (2014) introduce the fixed cost or the entry cost at the firm level and show that the individual investment is above a minimum scale in equilibrium. In my

[2]Recently, a booming literature has documented the extensive empirical evidence on the relationship between financial development and trade patterns. Financially developed countries export more in sectors that require more external finance and in sectors with fewer tangible assets (Beck, 2003; Hur, Raj, and Riyanto, 2006; Svaleryd and Vlachos, 2005). Manova (2008) shows that equity market liberalization increases exports disproportionately more in sectors that require more external funds or employ fewer collateralizable assets. Manova (2013) further decompose the trade effect of weak financial markets and show that financially developed countries serve more destination markets and export more products, in more financially vulnerable sectors. Chor and Manova (2012) analyze the collapse of international trade flows during the global financial crisis and show that credit conditions were an important channel through which the financial crisis affected trade volumes.

model, assuming the MIR rather than the fixed cost allows us to characterize analytically the dynamic properties in the entire parameter space.

I also revisit the factor price equalization (hereafter FPE) theorem in the presence of financial and real frictions. Deardorff (2001) shows that free trade leads to the FPE if countries stay in the same cone of diversification. In the dynamic model, the multi-cone equilibrium may arise and the FPE holds within cones, but not between them. In Antras and Caballero (2009), although free trade alone does not lead to the FPE, allowing both free trade and capital flows can do, which may be understood in Deardorff's framework. In my model, if trade has induced North to abandon the lower-return sector, adding capital mobility in the free-trade equilibrium does not achieve the FPE, which is opposite to the finding of Antras and Caballero (2009). Thus, the **timing of economic integration** is critical for the FPE in my model.

The rest of the paper is structured as follows. Section 2 sets up the model and section 3 analyzes the autarkic equilibrium. Section 4 discusses the dynamics of aggregate income and the interest rate when moving from the autarkic steady state to free trade. Section 5 explores the channel for the interest rate re-reversal, discusses the roles of trade costs and the initial conditions in South, and uses the "smile curve" to interpret my key findings in the context of supply-chain trade. Section 6 gives some final remarks. Technical proofs and other related materials are in the appendix.

2 The Model Setting

The world consists of two countries, North and South, which are inherently identical except for the population size and the level of financial development, as specified later. In this section, I mainly describe the economic setting of North and then use the asterisk superscript to denote the variables and the parameters in South.

In North, a continuum of agents are born every period and live for two periods, young and old. In each generation, the population size is constant at one and agents are indexed by $j \in [0,1]$. Agent j is endowed with $l_j = (1-\theta)\epsilon_j$ units of labor when young, where $\epsilon_j \in (1,\infty)$ follows the Pareto distribution, $G(\epsilon_j) = 1 - \epsilon_j^{-\frac{1}{\theta}}$ and $\theta \in (0,1)$.[3] Agents only consume when old and they supply the labor endowment inelastically to the market when young. Every period, the aggregate labor supply is constant at $L = \int_1^\infty l_j dG(\epsilon_j) = 1$.

North has two sectors, indexed by $s \in \{1,2\}$. In sector s, K_t^s units of physical capital and L_t^s units of labor are hired to produce Y_t^s units of goods in period t. Physical capital fully depreciates after use. Sectoral outputs are tradable, while labor and physical capital are not. V_t^1 units of good 1 and V_t^2 units of good 2 are combined to produce Y_t units of final goods which are used for consumption and investment.[4] The markets for goods and

[3]The inverse of θ is the tail index of the Pareto distribution. Pareto distribution is widely used in the literature to feature the income and wealth distribution (Atkinson, Piketty, and Saez, 2011; Gabaix, 2009; Jones, 2015). The top tail of income distribution is very well approximated by a Pareto distribution (Kuznets and Jenks, 1953; Piketty, 2014; Piketty and Saez, 2003).

[4]Under autarky, domestic absorption is equal to domestic output at the sectoral level, $V_t^s = Y_t^s$, while it is not the case with trade flows, $V_t^s \neq Y_t^s$.

productive factors are competitive and final goods serve as the numeraire[5]. There is no uncertainty in the model economy. Let w_t denote the wage rate. Let p_t^s and q_t^s denote respectively the output price and the rental price of capital in sector s.

$$Y_t^s = \left(\frac{K_t^s}{\alpha}\right)^{\alpha}\left(\frac{L_t^s}{1-\alpha}\right)^{1-\alpha}, \quad q_t^s K_t^s = \alpha p_t^s Y_t^s, \quad w_t L_t^s = (1-\alpha) p_t^s Y_t^s, \qquad \alpha \in (0,1), \quad (1)$$

$$Y_t = \left(\frac{V_t^2}{\eta}\right)^{\eta}\left(\frac{V_t^1}{1-\eta}\right)^{1-\eta}, \quad p_t^2 V_t^2 = \eta Y_t, \qquad p_t^1 V_t^1 = (1-\eta) Y_t, \qquad \eta \in (0,1). \quad (2)$$

In order to feature the interest rate response to trade, I exclude international capital flows so that domestic investment is funded by domestic saving and national income is equal to domestic output. Combine equations (1)-(2) to get $w_t = (1-\alpha)\frac{Y_t}{L}$. Thus, the wage rate can serve as a proxy of national income. In the next sections, I use the law of motion for wage to analyze the dynamic and the steady-state properties of the model economy.

As labor is perfectly mobile across sectors and the labor market is frictionless, the marginal revenue of labor (hereafter MRL) is equalized across sectors by the wage rate. As described below, physical capital is sector-specific and sectoral investment is subject to frictions. Thus, the marginal revenue of capital (hereafter, MRK) may differ across sectors, leading to the sectoral rental-price-of-capital wedge.

In period t, young agents invest their entire labor income $w_t L$ in the two sectors, which yields in period $t+1$ K_{t+1}^s units of physical capital in sector s. Let δ_t and ζ_{t+1} denote respectively the fractions of domestic saving ($w_t L$) and labor endowment (L) allocated for the production of good 2 in period $t+1$.

$$K_{t+1}^2 = \delta_t w_t L, \qquad L_{t+1}^2 = \zeta_{t+1} L, \quad (3)$$

$$K_{t+1}^1 = (1-\delta_t) w_t L, \qquad L_{t+1}^1 = (1-\zeta_{t+1}) L. \quad (4)$$

Let $\chi_t \equiv \frac{p_t^1}{p_t^2}$ and $\mu_t \equiv \frac{q_t^1}{q_t^2}$ denote respectively the sectoral output-price ratio and the sectoral rental-price-of-capital ratio. Combine equations (1) and (3)-(4) to get

$$p_t^s = (q_t^s)^{\alpha} w_t^{1-\alpha}, \quad \Rightarrow \quad \chi_t = \mu_t^{\alpha}, \quad (5)$$

$$\zeta_{t+1} = \frac{\delta_t}{(1-\delta_t)\chi_{t+1}^{\frac{1}{\alpha}} + \delta_t} = \frac{\delta_t}{(1-\delta_t)\mu_{t+1} + \delta_t}. \quad (6)$$

Given aggregate investment $w_t L$ and labor input L, equation (6) implicitly describes the production possibility frontier (PPF, hereafter) for North in period $t+1$,[6] reflecting the sectoral output supply. Equation (2) represents the isoquant, featuring the sectoral output demand. In the next sections, I will combine the isoquant and the PPF to show the equilibrium allocations under autarky and under free trade, respectively. In the following, I first derive explicitly the PPF, taking into account sectoral investment distortions.

[5]In Antras and Caballero (2009), capital and labor are hired to produce two final goods which are combined in the Cobb-Douglas aggregator for consumption and investment, while one final good is chosen as the numeraire. In Ju and Wei (2011), the two final goods are combined to produce a composite good in a Cobb-Douglas fashion, while the composite good serves as the numeraire and is used for consumption and investment. The results of my model do not depend on the choice of numeraire.

[6]See the proof of Lemma 1 for derivation.

Agent j born in period t has three options of saving its labor income: investing in the two sectors and lending to the credit market. By investing $k^1_{j,t+1} > 0$ units of final goods in period t, the agent gets in period $t+1$ $k^1_{j,t+1}$ units of physical capital for sector 1 and the rate of return to its investment is q^1_{t+1}. It gets the same one-to-one investment output in sector 2, if its investment size meets the MIR, $k^2_{j,t+1} \geq \mathfrak{m} > 0$; otherwise, its investment in sector 2 yields zero output.

Except for the MIR, the two sectors are symmetric in terms of the investment and production technologies. Let $\mathbb{Y}_{t+1} \equiv \left(\frac{w_t L}{\alpha}\right)^{\alpha} \left(\frac{L}{1-\alpha}\right)^{1-\alpha}$ denotes a sector's maximum possible output if domestic saving and labor are fully allocated in that sector. If credit markets were perfect so that everyone could meet the MIR, investment in the two sectors would be perfect substitutes and the sectoral rates of return would equalize under autarky, $q^1_{t+1} = q^2_{t+1}$ and $\mu_{t+1} = 1$. In this case, for the output of Y^2_{t+1} units of good 2, productive factors would be allocated in the two sectors in equal proportions, $\zeta_{t+1} = \delta_t = \frac{Y^2_{t+1}}{\mathbb{Y}_{t+1}}$, according to equations (6), (3), and (1). Thus, the PPF is linear and has a constant marginal rate of transformation (MRT, hereafter) at unity,

$$Y^2_{t+1} = \delta_t \mathbb{Y}_{t+1}, \quad Y^1_{t+1} = (1-\delta_t)\mathbb{Y}_{t+1}, \quad MRT_{1,2} \equiv -\frac{\partial Y^2_{t+1}}{\partial Y^1_{t+1}} = \mu^{\alpha}_{t+1} = \chi_{t+1} = 1. \tag{7}$$

However, due to limited commitment, agent j can borrow only up to a fraction λ of its investment in sector 2 and has to use its own funds to cover the gap,

$$b_{j,t} \leq \lambda k^2_{j,t+1}, \quad \text{and} \quad k^2_{j,t+1} - b_{j,t} \leq n_{j,t}, \tag{8}$$

where $\lambda \in [0,1)$ measures the level of financial development.[7] If the borrowing constraints are sufficiently tight, the mass of agents who can meet the MIR is inefficiently low and so is the total investment in sector 2. In this case, financial frictions and sector-specific MIR jointly distort sectoral investment, leading to the sectoral return wedge, $q^2_{t+1} > q^1_{t+1}$. According to equation (6), $\mu_{t+1} < 1$ implies that capital and labor are not allocated in the two sectors in equal proportions, $\zeta_{t+1} > \delta_t$. I derive the PPF for this case as follows.

Since all agents can freely invest in sector 1 and lend to the credit market, these two options are perfect substitutes. As good 1 is essential for the final good production, $K^1_{t+1} > 0$ holds under autarky, implying that the interest rate is coupled with the rate of return in sector 1. As shown in section 4, trade may induce North to abandon sector 1 and specialize fully in sector 2. If so, the interest rate is decoupled from (coupled with) the rate of return in sector 1 (sector 2).

$$r_t = \begin{cases} q^1_{t+1}, & \text{if } K^1_{t+1} > 0 \text{ or equivalently } \delta_t < 1; \\ q^2_{t+1}, & \text{if } K^1_{t+1} = 0 \text{ or equivalently } \delta_t = 1. \end{cases} \tag{9}$$

Consider first the case where investment frictions lead to the sectoral return wedge, $q^2_{t+1} > q^1_{t+1}$ and sector 1 is still active $K^1_{t+1} > 0$. In period t, the positive rate-of-return spread ($q^2_{t+1} > r_t = q^1_{t+1}$) induces agent j to invest its entire labor income in

[7] Antras and Caballero (2009) use the same form of borrowing constraints and offer detailed microfoundations. The credit markets would be perfect and the borrowing constraints would be slack if $\lambda = 1$.

sector 2 and borrow to the limit, if it can meet the MIR, $k^2_{j,t+1} = \frac{n_{j,t}}{1-\lambda} \geq \mathfrak{m}$. In period $t+1$, the agent gets the investment return, repays the debt, and consumes the rest. Let $\Omega_{j,t} \equiv \frac{q^2_{t+1}k^2_{j,t+1} - r_t b_{j,t}}{k^2_{j,t+1} - b_{j,t}}$ denote agent j's equity rate, i..e, the rate of return to its own funds. The binding borrowing constraints imply that the equity rate is common for all agents who meet the MIR and the agent's investment size in sector 2 is linear in its net wealth.

$$\Omega_{j,t} = \Omega_t \equiv q^2_{t+1} + (q^2_{t+1} - r_t)(\frac{1}{1-\lambda} - 1) > q^2_{t+1} > r_t, \tag{10}$$

$$k^2_{j,t+1} = \frac{n_{j,t}}{1-\lambda} = \frac{w_t}{1-\lambda}(1-\theta)\epsilon_j \geq \mathfrak{m}, \ \Rightarrow \ \epsilon_j \geq \underline{\epsilon}_t \equiv \frac{1-\lambda}{w_t}\frac{\mathfrak{m}}{1-\theta}, \tag{11}$$

where $\underline{\epsilon}_t$ is a cutoff value. Agents with $\epsilon_j \geq \underline{\epsilon}_t$ can meet the MIR and they are called *entrepreneurs*, with the mass of $\tau_t = \underline{\epsilon}_t^{-\frac{1}{\theta}}$. When young, they invest in sector 2 with their entire labor income $n_{j,t}$ and borrow to the limit $b_{j,t} = \frac{\lambda n_{j,t}}{1-\lambda}$; when old, they consume $c^e_{j,t+1}$,

$$n_{j,t} = w_t l_j \quad \text{and} \quad c^e_{j,t+1} = n_{j,t}\Omega_t. \tag{12}$$

In period t, the fraction of domestic saving allocated in sector 2 is

$$\delta_t = \frac{\int_{\underline{\epsilon}_t}^{\infty} k^2_{j,t+1} dG(\epsilon_j)}{w_t L} = \frac{\underline{\epsilon}_t^{-\frac{1-\theta}{\theta}}}{1-\lambda} = \frac{\tau_t^{1-\theta}}{1-\lambda} = \left(\frac{w_t}{\bar{w}}\right)^{\frac{1-\theta}{\theta}}, \text{ where } \bar{w} \equiv \frac{\mathfrak{m}}{1-\theta}(1-\lambda)^{\frac{1}{1-\theta}}. \tag{13}$$

Agents with $\epsilon_j \in [1, \underline{\epsilon}_t)$ cannot meet the MIR and are called *households*. When young, they invest $k^1_{j,t+1}$ in sector 1 and lend out $n_{j,t} - k^1_{j,t+1}$; when old, they consume $c^h_{j,t+1}$,

$$k^1_{j,t+1} + (n_{j,t} - k^1_{j,t+1}) = n_{j,t} = w_t l_j \quad \text{and} \quad c^h_{j,t+1} = n_{j,t} r_t. \tag{14}$$

The markets for credit, labor, sector-specific capital, and final goods clear every period,

$$\int_{\underline{\epsilon}_t}^{\infty} (k^2_{j,t+1} - n_{j,t}) dG(\epsilon_j) = \int_1^{\underline{\epsilon}_t} (n_{j,t} - k^1_{j,t+1}) dG(\epsilon_j), \quad L^1_t + L^2_t = L, \tag{15}$$

$$K^1_t = (1-\delta_{t-1}) w_{t-1} L, \ K^2_t = \delta_{t-1} w_{t-1} L, \tag{16}$$

$$\int_{\underline{\epsilon}_t}^{\infty} c^e_{j,t} dG(\epsilon_j) + \int_1^{\underline{\epsilon}_t} c^h_{j,t} dG(\epsilon_j) + K^1_{t+1} + K^2_{t+1} = Y_t. \tag{17}$$

Given the binding borrowing constraints in period t, δ_t is predetermined by equation (13) from the perspective of period $t+1$. Thus, the PPF in period $t+1$ can be specified as

$$\left[\frac{Y^1_{t+1}}{(1-\delta_t)^\alpha}\right]^{\frac{1}{1-\alpha}} + \left(\frac{Y^2_{t+1}}{\delta_t^\alpha}\right)^{\frac{1}{1-\alpha}} = \mathbb{Y}_{t+1}^{\frac{1}{1-\alpha}}, \quad MRT_{1,2} \equiv -\frac{\partial Y^2_{t+1}}{\partial Y^1_{t+1}} = \mu^\alpha_{t+1} = \chi_{t+1} < 1. \tag{18}$$

Consider then the case of $q^2_{t+1} = q^1_{t+1}$. Due to the zero spread, the agents who can meet the MIR do not have the incentive to invest their entire labor income in sector 2 or to borrow to the limit. Despite the indeterminacy at the individual level, $\mu_{t+1} = 1$ implies that the PPF in period $t+1$ is specified by equation (7).

Let $\tilde{\delta}_t \equiv \min\{\left(\frac{w_t}{\bar{w}}\right)^{\frac{1-\theta}{\theta}}, 1\}$ denote the maximum possible share of domestic investment in sector 2 when all entrepreneurs borrow and invest to the limit.

Lemma 1. *Let $\tilde{Y}^2_{t+1} \equiv \tilde{\delta}_t \mathbb{Y}_{t+1}$. Given $w_t < \bar{w}$, $\tilde{\delta}_t < 1$ and the PPF in period $t+1$ is piecewise and consists of two parts:*

- *a linear part for $Y^2_{t+1} \leq \tilde{Y}^2_{t+1}$, as specified by equation (7) with $\delta_t = \frac{Y^2_{t+1}}{\mathbb{Y}_{t+1}}$, and*
- *a concave part for $Y^2_{t+1} > \tilde{Y}^2_{t+1}$, as specified by equation (18) with $\delta_t = \left(\frac{w_t}{\bar{w}}\right)^{\frac{1-\theta}{\theta}} < 1$.*

Given $w_t \geq \bar{w}$, $\tilde{\delta}_t = 1$ and the PPF in period $t+1$ is linear, as specified by equation (7).

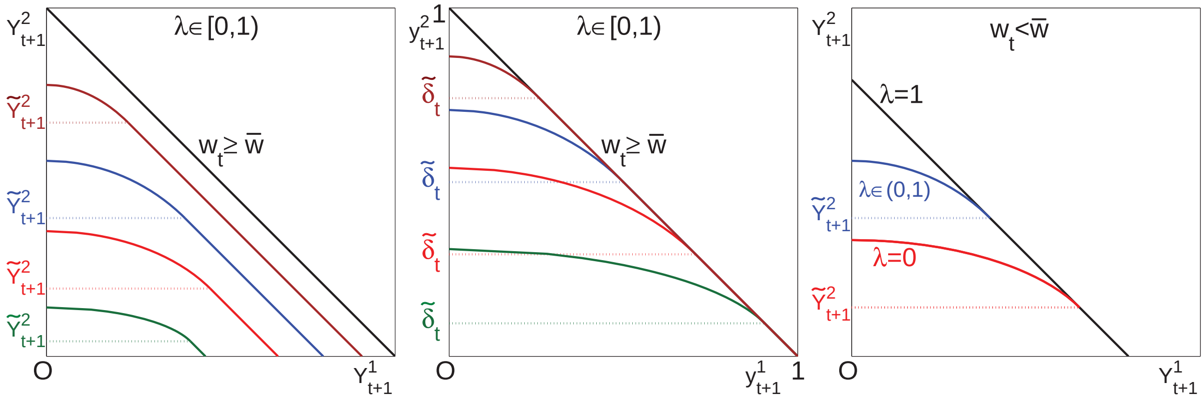

Figure 1: The Production Possibility Frontiers

$\tilde{\delta}_t$ depends positively on the level of financial development (λ) and the level of national income (with w_t as a proxy).

- For the sector-2's output of $Y^2_{t+1} \leq \tilde{Y}^2_{t+1}$, equation (7) specifies the efficient fraction of investment in sector 2, $\hat{\delta}_t = \frac{Y^2_{t+1}}{\mathbb{Y}_{t+1}}$, which is feasible $\hat{\delta}_t \leq \tilde{\delta}_t = \frac{\tilde{Y}^2_{t+1}}{\mathbb{Y}_{t+1}}$. In equilibrium, sectoral investment is efficient $\delta_t = \hat{\delta}_t \leq \tilde{\delta}_t$ and the sectoral rates of return equalize, $\mu_{t+1} = 1$. A marginal rise in sector 2's output requires the reallocation of both labor and investment from sector 1 to 2 in equal proportions, $\zeta_{t+1} = \delta_t$. Sector 1's output declines by the same amount, implying a linear PPF, as specified by equation (7).

- For the sector-2's output of $Y^2_{t+1} > \tilde{Y}^2_{t+1}$, the efficient fraction of investment in sector 2 is not feasible, $\hat{\delta}_t = \frac{Y^2_{t+1}}{\mathbb{Y}_{t+1}} > \tilde{\delta}_t = \frac{\tilde{Y}^2_{t+1}}{\mathbb{Y}_{t+1}}$. In equilibrium, sector 2's share of investment is inefficiently low, $\delta_t = \tilde{\delta}_t < \hat{\delta}_t$, leading to the sectoral return wedge, $q^2_{t+1} > q^1_{t+1}$. Given a predetermined δ_t, a marginal rise in sector 2's output requires the labor reallocation from sector 1 to 2 more than proportionally. Sector 1's output falls by a larger amount, implying a concave PPF, as specified by equation (18). If labor is fully allocated in sector 2, the maximum output is $Y^2_{t+1} = \tilde{\delta}^{\alpha}_t \mathbb{Y}_{t+1} < \mathbb{Y}_{t+1}$.

As shown in the left panel of figure 1, given $w_t < \bar{w}$, a rise in w_t affects the PPF in two ways. First, the higher the w_t, the higher the national income, the higher the domestic saving and investment, the higher the $\mathbb{Y}_{t+1}$. I call it the investment scale effect, which shifts the PPF parallel away from the origin. Second, the higher the w_t, the lower the cutoff value $\underline{\epsilon}_t$, the larger the mass of entrepreneurs τ_t, the higher the $\tilde{\delta}_t$. I call it the investment composition effect, which extends the linear fraction of the PPF.

By normalizing sectoral outputs in equations (7) and (18) by $\mathbb{Y}_{t+1}$, one can explicitly highlight the investment composition effect,

$$y^1_{t+1} + y^2_{t+1} = 1, \quad \text{and} \quad \left[\frac{y^1_{t+1}}{(1-\tilde{\delta}_t)^\alpha}\right]^{\frac{1}{1-\alpha}} + \left(\frac{y^2_{t+1}}{\tilde{\delta}^\alpha_t}\right)^{\frac{1}{1-\alpha}} = 1, \quad \text{where} \quad y^s_{t+1} \equiv \frac{Y^s_{t+1}}{\mathbb{Y}_{t+1}}.$$

As shown in the middle panel of figure 1, given $w_t < \bar{w}$, a rise in w_t raises $\tilde{\delta}_t$ so that the linear part accounts for a larger proportion in the normalized PPF.

For $w_t \geq \bar{w}$, the mass of entrepreneurs is so high that domestic saving can be fully invested in sector 2, $\tilde{\delta}_t = 1$. For sector-2's output of $Y^2_{t+1} \leq \mathbb{Y}_{t+1}$, the efficient fraction of investment in sector 2 is always feasible, $\hat{\delta}_t = \frac{Y^2_{t+1}}{\mathbb{Y}_{t+1}} \leq \tilde{\delta}_t = 1$. Thus, the entire PPF becomes linear, as shown by the diagonal lines in the left and middle panels of figure 1.

The right panel of figure 1 shows that, given $w_t < \bar{w}$, the higher the λ, the lower the cutoff value $\underline{\epsilon}_t$, the larger the mass of entrepreneurs, the higher the $\tilde{\delta}_t$ and $\tilde{Y}^2_{t+1}$, the larger the linear part of the PPF. For $\lambda = 1$, $\tilde{\delta}_t = 1$ and the entire PPF becomes linear.

Under autarky, the sectoral output markets clear domestically,

$$V^s_t = Y^s_t. \tag{19}$$

Definition 1. *Under autarky, a market equilibrium is a set of choices of agents* $\{n_{j,t}, c^e_{j,t}, c^h_{j,t}, k^s_{j,t+1}\}$ *and aggregate variables* $\{Y_t, Y^s_t, K^s_t, L^s_t, V^s_t, \chi_t, \mu_t, w_t, r_t, \Omega_t, \underline{\epsilon}_t\}$*, satisfying equations* (1)-(2)*,* (9)-(12)*,* (14)-(16)*, and* (19)*, where* $s \in \{1, 2\}$.[8]

Free trade aligns the sectoral price ratio to the world level and trade is balanced,

$$\chi_t = \chi^*_t, \quad \chi^*_t(V^1_t - Y^1_t) + (V^2_t - Y^2_t) = 0. \tag{20}$$

Definition 2. *Under free trade, a market equilibrium is a set of choices of agents* $\{n_{j,t}, c^e_{j,t}, c^h_{j,t}, k^s_{j,t+1}\}$ *and aggregate variables* $\{Y_t, Y^s_t, K^s_t, L^s_t, V^s_t, \mu_t, w_t, r_t, \Omega_t, \underline{\epsilon}_t\}$*, satisfying equations* (1)-(2)*,* (9)-(12)*, and* (14)-(16)*, where* $s \in \{1, 2\}$*, while the sectoral price ratio* χ_t *is determined at the world level by equation* (20).

In period t, domestic investment is financed by domestic saving, $\sum_{v=1}^2 K^v_{t+1} = w_t L$; in period $t+1$, the aggregate investment return is $\sum_{v=1}^2 q^v_{t+1} K^v_{t+1} = \frac{\alpha}{1-\alpha} w_{t+1} L$, according to equations (1)-(2). Define the social rate of return as

$$\Upsilon_t \equiv \frac{\sum_{v=1}^2 q^v_{t+1} K^v_{t+1}}{\sum_{v=1}^2 K^v_{t+1}} = (1-\delta_t) q^1_{t+1} + \delta_t q^2_{t+1} = \rho \frac{w_{t+1}}{w_t}, \quad \text{where} \quad \rho \equiv \frac{\alpha}{1-\alpha}. \tag{21}$$

3 The Autarkic Equilibrium

As the labor market is frictionless and labor is perfectly mobile across sectors, the sectoral allocation of labor is efficient and, under autarky, it is equal to the sectoral share in the production of final goods, $\zeta_{t+1} = \eta$. It would also apply to the sectoral allocation of physical capital $\delta_t = \eta$, if sectoral investment were efficient, i.e., $\mu_{t+1} = 1$.

[8] According to the Walras' law, if the markets for labor, credit, sector-specific physical capital, and sectoral outputs clear, the final good market must clear. In this sense, equation (17) is redundant.

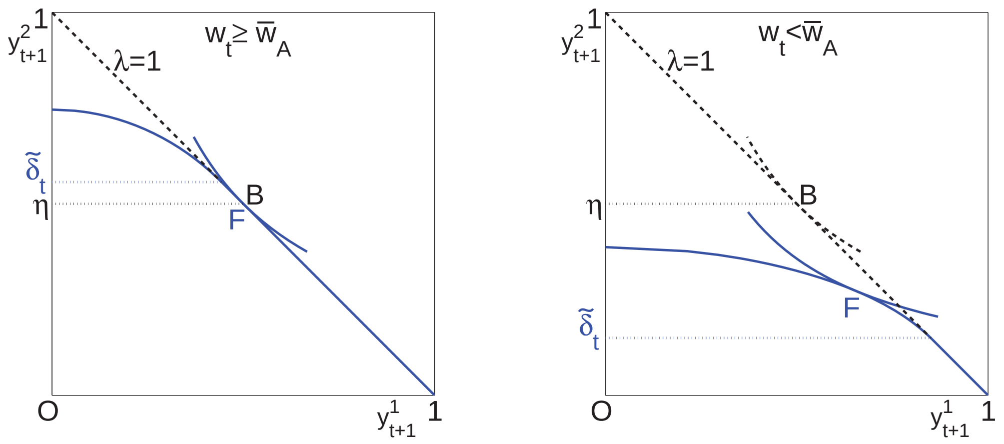

Figure 2: Equilibrium Allocations under Autarky

Start with the benchmark case where the credit markets are perfect, i.e., $\lambda = 1$. In the absence of financial frictions, sectoral investment is efficient and sectoral prices equalize, $\chi_{t+1} = \mu_{t+1} = 1$. The autarkic equilibrium is represented by the tangent point of the isoquant and the PPF. In figure 2, the dashed, diagonal line shows the normalized PPF and point **B** represents the autarkic equilibrium in the **b**enchmark setting. Given the Cobb-Douglas production functions at the sectoral and at the aggregate levels, capital has the decreasing MRK so that the law of motion for wage is concave.

$$w_{t+1} = \left(\frac{w_t}{\rho}\right)^{\alpha}, \Rightarrow \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} < 1, \quad \text{where } \rho \equiv \frac{\alpha}{1-\alpha}. \tag{22}$$

As a convergence force, the decreasing MRK drives North to a unique steady state with the wage rate $w_B = \rho^{-\rho}$. Subscript **B** refers to the ***b***enchmark case.[9] The smaller the α, the stronger the decreasing MRK effect, the faster the convergence.

Consider now the case with financial frictions, $\lambda \in [0, 1)$. Let $\bar{w}_A \equiv \eta^{\frac{\theta}{1-\theta}}\bar{w} < \bar{w}$. If $w_t \geq \bar{w}_A$, the efficient investment allocation is feasible, $\eta \leq \tilde{\delta}_t$. In equilibrium, $\delta_t = \zeta_{t+1} = \eta$ and $\chi_{t+1} = \mu_{t+1} = 1$ hold. In the left panel of figure 2, the solid, concave curve shows the normalized PPF, while point **F** shows the autarkic equilibrium with financial **f**rictions, which coincides with point B. As the equilibrium allocation is efficient in this case, the income dynamics are also characterized by equation (22).

If $w_t < \bar{w}_A$, the efficient investment allocation is infeasible, $\eta > \tilde{\delta}_t$ and the under-investment in sector 2, $\delta_t = \tilde{\delta}_t < \zeta_{t+1} = \eta$, leads to the sectoral return wedge, $\mu_{t+1} < 1$. Given w_t, the income dynamics are featured by $\{\delta_t, \mu_{t+1}, w_{t+1}\}$ satisfying (13), (23)-(24),[10]

$$w_{t+1} = \left(\frac{w_t}{\rho}\Gamma_t\right)^{\alpha}, \quad \text{where } \Gamma_t \equiv \frac{\mu_{t+1}^{\eta}}{1-\eta(1-\mu_{t+1})} < 1, \text{ and } \frac{\partial \Gamma_t}{\partial \mu_{t+1}} > 0, \tag{23}$$

$$\mu_{t+1} = \frac{\frac{1}{\eta}-1}{\frac{1}{\delta_t}-1}, \quad \Rightarrow \quad \frac{\partial \mu_{t+1}}{\partial w_t} = \frac{\partial \mu_{t+1}}{\partial \delta_t}\frac{\partial \delta_t}{\partial w_t} > 0, \tag{24}$$

[9]The dashed curves in figure 4 show the law of motion for wage under autarky and point **B** represents the steady state in the benchmark case.

[10]See the proof of proposition 1 in appendix B for derivation.

where Γ_t measures the aggregate efficiency. Sectoral investment distortion ($\mu_{t+1} < 1$) creates aggregate inefficiency ($\Gamma_t < 1$).[11] In the right panel of figure 2, the solid, concave curve shows the normalized PPF, while the equilibrium allocation represented by point **F** is on an isoquant lower than in the benchmark case. The distance between the two isoquants reflects the efficiency losses in percentage terms, $(1-\Gamma_t^{\alpha})$.

A rise in national income raises the wage rate, which affects the sectoral investment in two ways. First, it allows agents to invest more so that sectoral investment rises on the *intensive margin*. The decreasing MRK effect *dampens* the initial income change. Second, it allows more agents to meet the MIR and become entrepreneurs, which shifts domestic investment towards sector 2 on the *extensive margin*. This investment composition effect improves aggregate efficiency and *amplifies* the initial income change.

$$\begin{aligned}\frac{\partial \ln w_{t+1}}{\partial \ln w_t} &= 1 - \underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} + \underbrace{\frac{\partial \ln w_{t+1}}{\partial \ln \Gamma_t}\frac{\partial \ln \Gamma_t}{\partial \ln \mu_{t+1}}\frac{\partial \ln \mu_{t+1}}{\partial \ln w_t}}_{\text{investment composition effect}} \\ &= \alpha + \alpha\eta(1-\mu_{t+1})\frac{1-\theta}{\theta}.\end{aligned} \tag{25}$$

Let X_A denote the steady-state value of variable X_t under **a**utarky. If there is a steady state with $w_A \in (0, \bar{w}_A)$, the slope of the law of motion for wage at this steady state is $\frac{\partial w_{t+1}}{\partial w_t}|_{w_A} = \alpha + \alpha\eta(1-\mu_A)\frac{1-\theta}{\theta}$. Since $\mu_A < 1$, the investment composition effect is positive. The higher the θ, the more dispersed the wealth distribution, the less sensitive the mass of entrepreneurs and sectoral investment to income changes, the weaker the investment composition effect; the lower the α, the stronger the decreasing MRK effect and the weaker the investment composition effect.

Proposition 1. *Let* $\underline{\theta} \equiv \frac{\alpha}{\alpha + \frac{1-\alpha}{\eta}} < \alpha$, $Z \equiv \frac{\frac{1}{\rho}}{\eta^{\frac{\theta}{\rho(1-\theta)}}\left(\frac{m}{1-\theta}\right)^{\frac{1}{\rho}}}$, *and* $\tilde{\lambda}_A \equiv 1 - Z^{\rho(1-\theta)}$.
For $\theta \in [\underline{\theta}, 1)$ *and* $\lambda \in [0, \tilde{\lambda}_A)$, *there is a unique, stable steady state under autarky where the borrowing constraints are binding and sectoral investment is inefficient.*

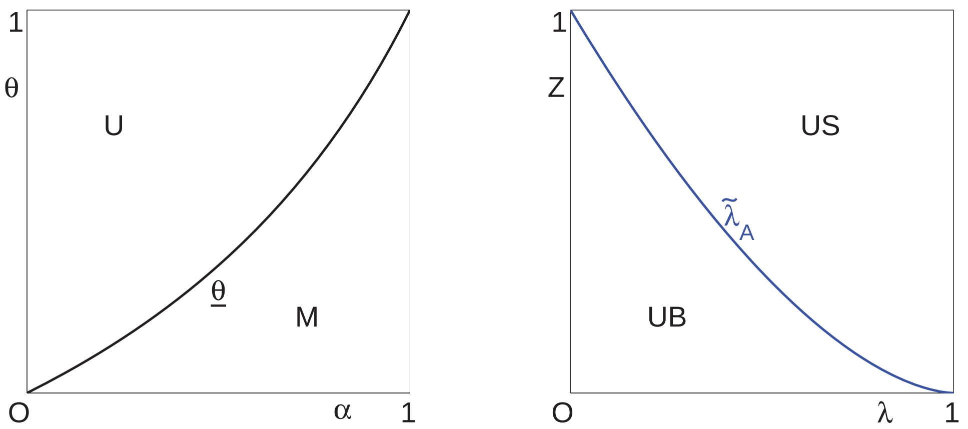

Figure 3: Threshold Values for the Autarkic Equilibrium

Figure 3 shows the two threshold values, i.e., $\underline{\theta}$ and $\tilde{\lambda}_A$, in the $\{\alpha, \theta\}$ space and in the $\{\lambda, Z\}$ space, respectively. For $\{\alpha, \theta\}$ in region **U** of the left panel and $\{\lambda, Z\}$ in region

[11] In the benchmark case, $\mu_{t+1} = 1$ implies that the aggregate efficiency index is constant at $\Gamma_t = 1$.

UB (**US**) of the right panel of figure 3, there exists a **u**nique steady state under autarky where $w_A < \bar{w}_A$ ($w_A > \bar{w}_A$) and the borrowing constraints are **b**inding (**s**lack).[12]

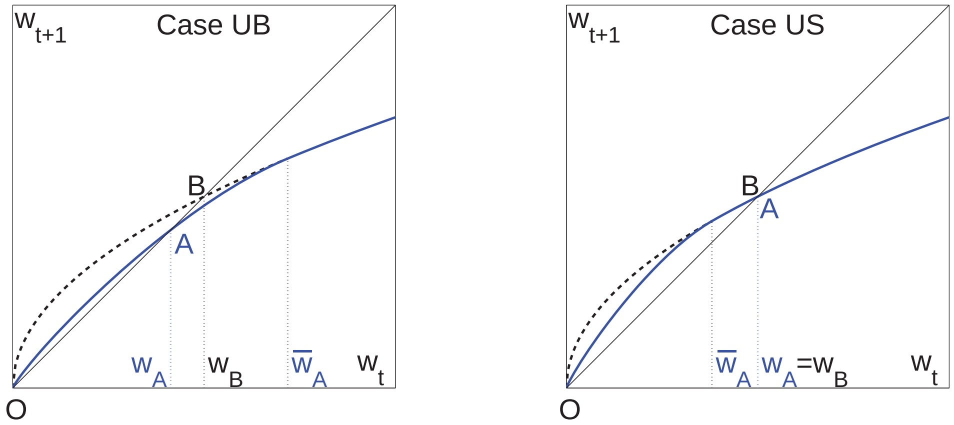

Figure 4: Laws of Motion for Wage under Autarky: $\theta \geq \underline{\theta}$

In figure 4, the solid curves show respectively the laws of motion for wage in these two cases, while the dashed curves show those in the case of $\lambda = 1$. For $w_t \in (0, \bar{w}_A)$, the solid curve lies below the dashed curve and the gap reflects the efficiency losses, $(1 - \Gamma_t^\alpha)\left(\frac{w_t}{\rho}\right)^\alpha$.

3.1 The World Economy under Autarkic Equilibrium

South and North are inherently identical, except that North is more financially developed and its population share in the world economy is negligible.

Assumption 1. *$0 < \lambda^* < \lambda < \tilde{\lambda}_A$ and $\frac{L}{L+L^*} \to 0$, given $\theta > \underline{\theta}$.*[13]

Given assumption 1, the parameter configurations of both countries are in region U of the left panel and in region UB of the right panel of figure 3. In each country, there has a **u**nique, autarkic steady state where the borrowing constraints are **b**inding.

Under autarky, the interest rate is coupled with the rate of return in sector 1,

$$r_t = q_{t+1}^1 = \Upsilon_t[1 - \eta(1 - \mu_{t+1})] < \Upsilon_t. \tag{26}$$

Lemma 2. *$w_A^* < w_A < w_B$, $\chi_A^* < \chi_A < 1$, $\mu_A^* < \mu_A < 1$, and $r_A^* < r_A < \rho$.*

A higher level of financial development not only allows each entrepreneur to borrow and invest more, but also allows more agents to meet the MIR and invest in sector 2. The improvement of investment composition $\frac{\partial \mu_{t+1}}{\partial \lambda} > 0$ enhances the aggregate efficiency, $\frac{\partial \Gamma_t}{\partial \lambda} > 0$. In period $t+1$, higher aggregate output implies higher aggregate saving, which further improves investment composition and production efficiency. In the long run, national income is higher $\frac{\partial w_A}{\partial \lambda} > 0$ and so is the sectoral price ratio, $\frac{\partial \chi_A}{\partial \lambda} > 0$.

The dashed curve in figure 5 shows the steady-state sectoral outputs as the functions of $\lambda \in [0, \tilde{\lambda}_A)$. At the autarkic steady state, the (absolute) gradient of the PPF reflects

[12]The proof of proposition 1 analyzes the case of $\{\alpha, \theta\}$ in region M where multiple steady states arise.

[13]By definition, the composite parameter Z is independent of the level of financial development and the population size. Thus, Z takes the same value for both countries.

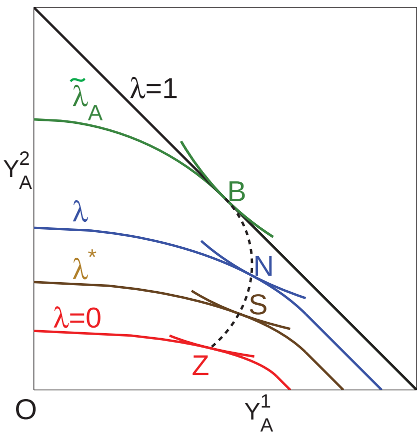

Figure 5: The Steady-State Pattern of Sectoral Outputs under Autarky: $\lambda \in [0, 1]$

the sectoral output-price ratio, which rises in λ. This way, **the level of financial development is a determinant of comparative advantage** in this model, i.e., good 2 (1) is cheaper in North (South) at the autarkic steady state.

Financial development affects the interest rate in three ways. First, it allows each entrepreneur to borrow more as well as allows more agents to become entrepreneurs $\frac{\partial \epsilon_t}{\partial \lambda} < 0$. The higher (lower) aggregate credit demand (supply) on the ***i***ntensive and ***e***xtensive ***m***argins leads to a higher interest rate. We call it the **IEM** effect. Second, by reallocating domestic investment towards the sector with the higher return, financial development improves aggregate efficiency and raises the social rate of return, leading to a higher interest rate in period t. Third, higher aggregate output in period $t+1$ implies higher domestic saving and investment. Due to the decreasing MRK effect, the social rate of return declines in the long run and so does the interest rate.

At the autarkic steady state, the decreasing MRK effect and the aggregate efficiency effect cancel out.[14] Due to the positive **IEM** effect, the interest rate is higher in North than in South. Starting from the autarkic steady state, if agents are allowed to borrow and lend abroad[15], financial flows are "uphill" from South to North, which widens the cross-country income gap. Thus, "global imbalances" arise in this model as an equilibrium response to heterogeneous financial development across countries.

4 Can Free Trade Resolve the Global Imbalances?

The world economy is initially at the autarkic steady state. In period 0, the two countries announce that goods 1 and 2 will be freely traded from period 1 on.[16] Due to its negligible

[14] Due to inelastic aggregate saving $w_t L$ and the Cobb-Douglas production functions, the social rate of return at the autarkic steady state is constant at ρ, independent of λ. See equation (21).
In von Hagen and Zhang (2014), aggregate saving is elastic and the extensive margin is mute. At the autarkic steady state, the decreasing MRK effect dominates the aggregate efficiency effect so that the social rate of return declines in the level of financial development. Nevertheless, the intensive-margin effect always dominates so that the interest rate is still higher in the more financially developed country.

[15] In this case, the final good is tradable and serves as the vehicle for international borrowing/lending.

[16] If free trade is announced and implemented in period 0, χ_0 is aligned immediately with χ_A^*, which unexpectedly affects the investment return of those born in period $t = -1$. In my two-period OLG

world population share, North is a small open economy. Thus, trade integration does not affect the equilibrium allocation in South. In the absence of trade costs, the sectoral price ratio in North is aligned to the world level from period $t = 1$ on, $\chi_t = \chi^* = \chi_A^*$ and so is the sectoral return ratio, $\mu_t = \chi_t^{\frac{1}{\alpha}} = (\chi^*)^{\frac{1}{\alpha}} = \mu^*$. Without international factor mobility, trade is balanced, as specified by equation (20), and the social rate of return is still characterized by equation (21).

4.1 Specialization as An Amplification Mechanism

Lemma 3. *Given $\mu^* < \mu_A < 1$, free trade induces North to specialize partially (fully) in sector 2 if $w_t < \bar{w}$ ($w_t \geq \bar{w}$).*

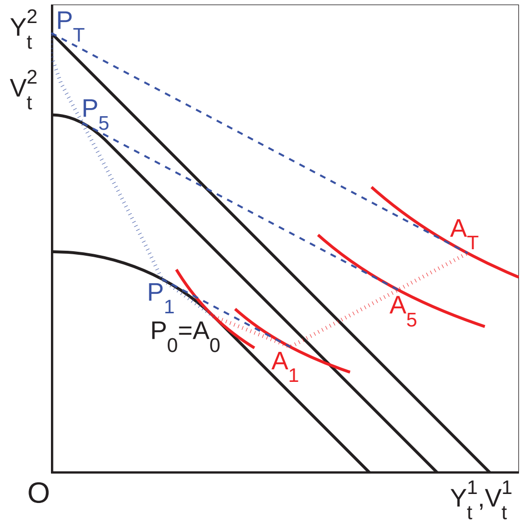

Figure 6: The Equilibrium Dynamics in North under Free Trade

Figure 6 shows the sectoral dynamics in North, where $P_t \equiv (Y_t^1, Y_t^2)$ denotes domestic **p**roduction and $A_t \equiv (V_t^1, V_t^2)$ denotes domestic **a**bsorption of the two sectors. In period 0, North is at the autarkic steady state with $A_0 = P_0$ and $MRS_{1,2} = MRT_{1,2} = \chi_A$. The announcement of free trade does not affect the sectoral investment in period 0, $\delta_0 = \left(\frac{w_0}{\bar{w}}\right)^{\frac{1-\theta}{\theta}} = \delta_A$, so that the PPF in period 1 is the same as before. By aligning the sectoral price ratio to the world level $\chi_1 = \chi^* < \chi_A$, free trade induces North to import (export) good 1 (2) in period 1, while the decline (rise) in the MRL in sector 1 (2) reallocates labor from sector 1 to 2, $\zeta_1 = \frac{1}{1+(\frac{1}{\delta_A}-1)(\chi^*)^{\frac{1}{\alpha}}} > \zeta_A$. Thus, domestic production moves from point P_0 to P_1. Trade decouples domestic absorption from domestic production and the national income line (the dashed line linking P_1 and A_1) is flatter than in period 0, $\chi_1 = \chi^* < \chi_0 = \chi_A$. The static gains from trade raise national income, as shown by the fact that domestic absorption A_1 is on the isoquant higher than A_0. It allows more agents to overcome the MIR and invest in sector 1, $\delta_1 = \left(\frac{w_1}{\bar{w}}\right)^{\frac{1-\theta}{\theta}} > \delta_0$, which enhances North's comparative advantage. Thus, North specializes further towards sector 2 along the labor margin in period 2, $\zeta_2 = \frac{1}{1+(\frac{1}{\delta_1}-1)(\chi_A^*)^{\frac{1}{\alpha}}} > \zeta_1$.

setting, announcing the policy one period in advance avoids such an uncertainty.

This way, the static gains from trade in period 1 trigger a dynamic, virtuous cycle, i.e., the rise in national income improves North's comparative advantage along the extensive margin of sectoral investment and the enhanced specialization in the next period raises national income further. I call it the trade-driven investment reallocation effect. As long as it dominates the decreasing MRK effect, such an amplification process propagates over time until the mass of entrepreneurs becomes so large that their borrowing capacity exceeds the entire household labor income and sector 1 vanishes in North, i.e., $w_t \geq \bar{w}$ and $\delta_t = 1$. From then on, the decreasing MRK effect brings North to a new steady state.

As explained in section 2, a higher national income affects the PPF in the next period through the scale effect and the composition effect. The former moves the PPF parallel away from the origin, while the latter extends the linear part of the PPF by raising $\tilde{\delta}_t$, given $w_t \leq \bar{w}$. Let X_T denote the steady-state value of variable X_t under **t**rade. In figure 6, the dotted line linking P_1 and P_T shows the path of domestic production, which eventually aligns with the vertical axis. It implies that North abandons sector 1 at some point in time. The dotted line linking A_1 and A_T features the path of domestic absorption, showing the rise in national income over time.

The dynamic impacts of trade on national income can be illustrated by the law of motion for wage, which is piecewise and characterized by equation (27),

$$w_{t+1} = \left(\frac{1}{\rho} w_t \Gamma_t\right)^{\alpha}, \text{where } \Gamma_t = (\mu^*)^{\eta}[1 + (\frac{1}{\mu^*} - 1)\delta_t],\ \delta_t = \begin{cases} \left(\frac{w_t}{\bar{w}}\right)^{\frac{1-\theta}{\theta}}, & \text{if } w_t < \bar{w}; \\ 1 & \text{if } w_t \geq \bar{w}. \end{cases} \tag{27}$$

$$\frac{\partial \ln w_1}{\partial \ln \chi^*} = \frac{\partial \ln \Gamma_0}{\partial \ln \mu^*} = \frac{\mu^* - \mu_A}{\frac{\mu^*}{\eta} + \frac{\mu_A}{1-\eta}} \tag{28}$$

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} + \underbrace{\alpha \frac{\partial \ln \Gamma_t}{\partial \ln \delta_t} \frac{\partial \ln \delta_t}{\partial \ln w_t}}_{\text{investment reallocation effect}} = \alpha + \alpha \frac{\frac{\partial \ln \delta_t}{\partial \ln w_t}}{1 + \frac{1}{(\frac{1}{\mu^*} - 1)\delta_t}}, \tag{29}$$

$$\text{where } \frac{\partial \ln \delta_t}{\partial \ln w_t} = \begin{cases} \frac{1-\theta}{\theta}, & \text{if } w_t < \bar{w}; \\ 0 & \text{if } w_t \geq \bar{w}. \end{cases} \tag{30}$$

Given $\mu^* = \mu_A^* < \mu_A$, $\frac{\partial \ln w_1}{\partial \ln \chi^*} < 0$ so that the static gains raise national income in period 1, $w_1 > w_0$. From period 1 on, the dynamics of national income are driven by the competition of the investment reallocation effect and the decreasing MRK effect for $w_t < \bar{w}$ and by the decreasing MRK effect alone for $w_t \geq \bar{w}$.

Proposition 2. *Free trade may induce North to converge from the autarkic steady state to a* ***u****nique steady state where it abandons sector 1 and* ***f****ully specializes in sector* ***2****.*

Let us call the case described in proposition 2 as case **UF2**. It arises if the static gains from trade in period 1 and the investment reallocation effect are sufficiently large.

- The larger the cross-country differences in financial development $\lambda - \lambda^*$, the larger the cross-country differences in the sectoral price ratio $\chi_A - \chi_A^*$, the larger the trade flows and the static gains in period 1. See equation (28) .

- The lower the λ^*, the lower the χ^* and the μ^*, the larger the sectoral price differential and the sectoral rate-of-return differential on the world market, the larger the gains for North by specializing towards sector 2, the stronger the investment reallocation effect. See equation (29).

- Given $\theta \geq \underline{\theta}$, the lower the θ, the less dispersed the wealth distribution, the more sensitive the mass of entrepreneurs and sectoral investment to the static gains and income changes, the stronger the investment reallocation effect. See equation (30).

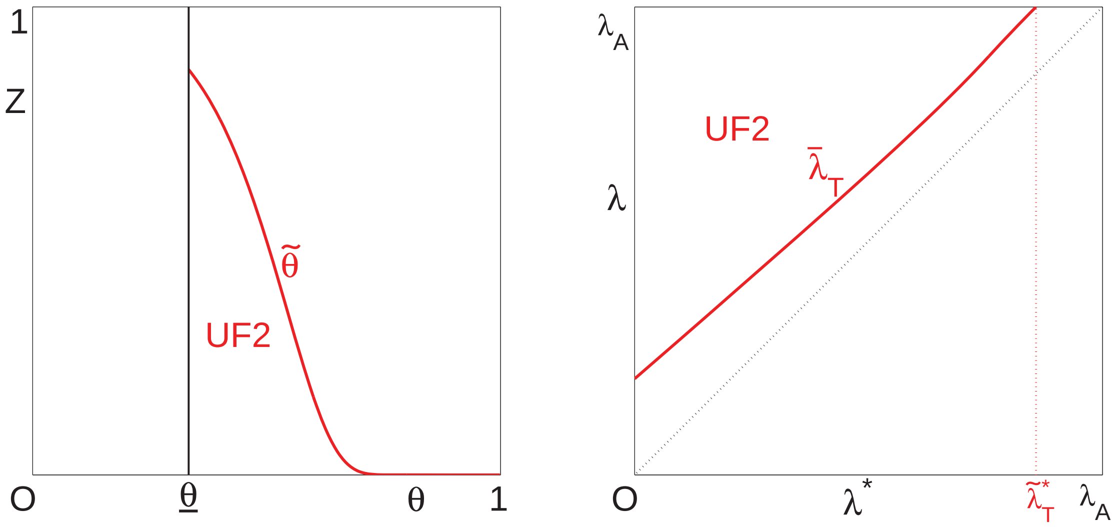

Figure 7: Threshold Values for the Free-Trade Equilibrium

To be specific, case UF2 arises for $\{\theta, Z, \lambda^*, \lambda\}$ in region UF2 of figure 7, i.e., $\theta \in [\underline{\theta}, \tilde{\theta})$, $\lambda^* < \tilde{\lambda}^*_T$, and $\lambda > \bar{\lambda}_T$ where the threshold value $\bar{\lambda}_T > \lambda^*$ and it increases in λ^*.[17] In the following, I focus on case UF2 and analyze the dynamic responses of national income and the interest rate with respect to trade integration.

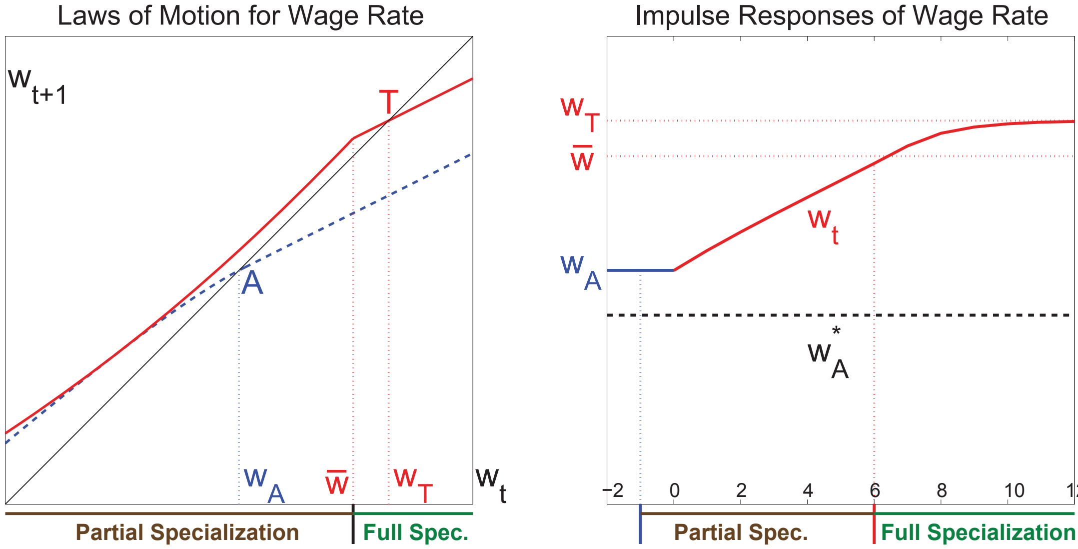

Figure 8: Dynamics of National Income: From Autarkic Steady State to Free Trade

In figure 8, the solid (dashed) curve in the left panel shows the law of motion for wage under trade (autarky), while the solid curve in the right panel shows the impulse

[17] Appendix A gives a complete analysis of the dynamic and steady-state properties under free trade.

responses of wage under trade.[18] For period $t \leq 6$, $w_t < \bar{w}$ and hence, $\delta_t < 1$. As the investment reallocation effect dominates the decreasing MRK effect, national income rises over time. For period $t \geq 7$, $w_t > \bar{w}$ and North abandons sector 1, $\delta_t = 1$. Then, the decreasing MRK effect brings North to steady state T where the wage rate is w_T.

4.2 Interest Rate Reversal and Re-reversal under Free Trade

Lemma 4. *Under free trade, the interest rate in North is a piecewise function of national income, depending on whether North abandons the low-return sector.*

1.) For $w_t \geq \bar{w}$, the mass of entrepreneurs is so large that their total debt capacity exceeds the entire household labor income and the aggregate credit demand pushes the interest rate equal to the rate of return in sector 2,

$$r_t = q_{t+1}^2 = \Upsilon_t = \frac{\rho w_{t+1}}{w_t}, \qquad \frac{\partial \ln r_t}{\partial \ln w_t} = - \underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} < 0. \tag{31}$$

The higher the national income, the higher the domestic saving and investment, the lower the MRK in sector 2, the lower the sector-2's return and the interest rate.

2.) For $w_t \in (0, \bar{w})$, the mass of entrepreneurs is so small that their total debt capacity is below the entire household labor income. Besides lending to entrepreneurs, households invest the rest of their labor income in sector 1. According to the no-arbitrage condition,

$$r_t = q_{t+1}^1 = \frac{\Upsilon_t}{1 + \frac{1-\mu^*}{\mu^*}\delta_t}, \qquad \frac{\partial \ln r_t}{\partial \ln w_t} = - \underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} - \underbrace{\frac{(1-\alpha)\left(\frac{1}{\theta}-1\right)}{1+\frac{1}{(\frac{1}{\mu^*}-1)\delta_t}}}_{\text{reallocation effect}} < 0. \tag{32}$$

In period 1, trade only triggers the labor reallocation, i.e., $\zeta_1 > \zeta_A$ and $\delta_0 = \delta_A$, leading to a higher capital-labor ratio in sector 1,

$$\frac{K_1^1}{L_1^1} = \frac{1-\delta_0}{1-\zeta_1}\frac{w_0 L}{L} = \left[1 + \delta_A(\frac{1}{\mu^*} - 1)\right] w_0 > \frac{K_0^1}{L_0^1} = \left[1 + \delta_A(\frac{1}{\mu_A} - 1)\right] w_A.$$

Meanwhile, the price of good 1 falls in period 1, $p_1^1 = \chi_1^\eta = (\chi^*)^\eta < p_0^1 = \chi_A^\eta$. Both effects lead to a fall in the sector 1's rental price of capital,

$$q_1^1 = p_1^1 \frac{\alpha Y_1^1}{K_1^1} = p_1^1 \left(\frac{K_1^1}{\rho L_1^1}\right)^{\alpha-1} < q_0^1 = p_0^1 \left(\frac{K_0^1}{\rho L_0^1}\right)^{\alpha-1}.$$

The interest rate falls in period 0, too. I call it the reallocation effect. It dominates the **IEM** effect so that the interest rate in North is even lower than the world level, $r_0 < r_A^* < r_A$, a result Antras and Caballero (2009) calls **the interest rate reversal**.

From period $t = 1$ on, trade triggers sectoral reallocation of labor and investment. Labor reallocation is frictionless, while it is not the case for investment. Besides the scale

[18] For illustration clarity, the axes in the left panel are scaled in logarithm and so is the vertical axis in the right panel. This scaling approach also applies to figure 9.

effect, the rise in national income also raises the capital-labor share in sector 1 through the disproportional sectoral reallocation of labor and investment. Use equation (6) to get

$$\frac{K_{t+1}^1}{L_{t+1}^1} = \frac{1-\delta_t}{1-\zeta_{t+1}}\frac{w_t L}{L} = \underbrace{\left[1+\delta_t\left(\frac{1}{\mu^*}-1\right)\right]}_{\text{reallocation effect}}\underbrace{w_t}_{\text{scale effect}}.$$

Thus, the reallocation effect and the decreasing MRK effect jointly reduce the MRK in sector 1 over time and the interest rate falls further.

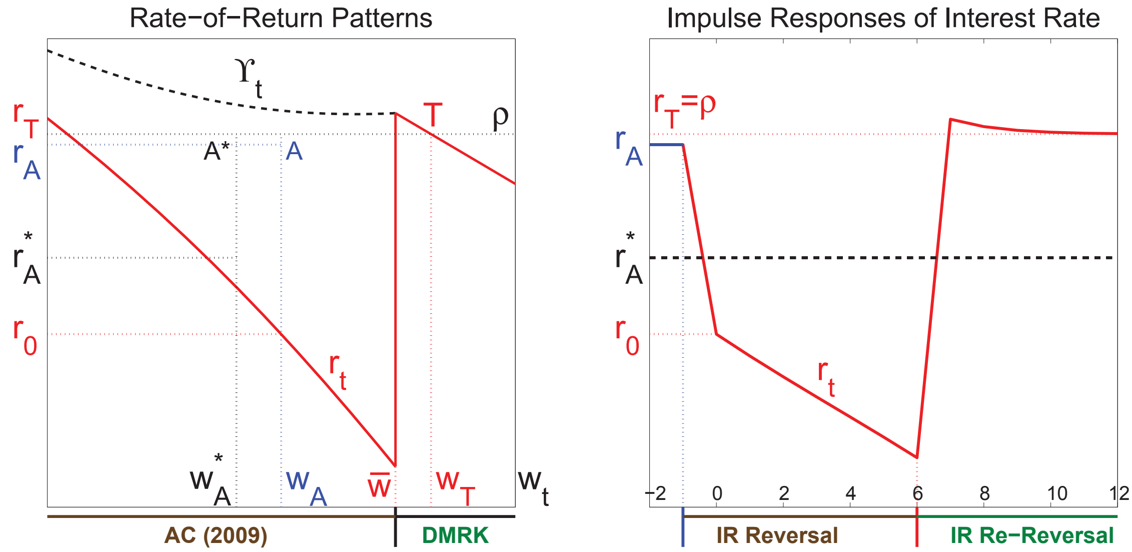

Figure 9: The Interest Rate Dynamics: From Autarkic Steady State to Free Trade

The left panel of figure 9 shows the interest rate as a piecewise function of national income, while the right panel shows the interest rate responses to free trade. In period 0, the announcement of free trade leads to the interest rate reversal, $r_0 < r_A^* < r_A$. From period 1 on, the reallocation effect and the decreasing MRK effect reduce the interest rate further. This process continues as long as sector 1 is active. In period 7, North abandons sector 1 and the interest rate is decoupled from (coupled with) the rate of return to sector 1 (2), featured by a jump of the interest rate in the right panel. I call it **the interest rate re-reversal**. Then, the decreasing MRK effect reduces the interest rate over time until North reaches the new steady state with $r_T = \Upsilon_T = \rho > r_A > r_A^*$.

Proposition 3. *In case UF2, North may witness a non-monotonic interest rate pattern during its transition towards the new steady state under free trade. The interest rate in the new steady state is higher than in the autarkic one,* $r_T > r_A > r_A^*$.

To sum up, whether trade lead to the interest rate re-reversal depends on how far it reshapes the sectoral composition in North. For the parameter configurations outside region UF2 of figure 7, the static gains and the investment reallocation effect are too weak to ensure that North abandons sector 1, when starting from the autarkic steady state. In this case, trade leads to the interest rate reversal.

4.3 FPE and The Timing of Economic Integration

In Antras and Caballero (2009), financial frictions distort domestic allocation along two dimensions. Along the intratemporal dimension, the deviation of the sectoral price ratio from its efficient level $(1-\chi_t)$ reflects the distortions on cross-sector investment allocation; along the intertemporal dimension, the deviation of the interest rate from the social rate of return $(1-\frac{r_t}{\Upsilon_t})$ reflects the distortion on aggregate credit demand. Trade alone only equalizes across countries the intertemporal distortion, while capital mobility alone only equalizes across countries the intratemporal distortion. In either case, factor prices (i.e., the wage rate, the interest rate, etc) do not equalize across countries. If trade and capital flows are allowed simultaneously, the distortions are equalized along both dimensions, which equalizes across countries the factor prices as well as income per capita.

Their results can be shown in my model. According to the right panels of figure 8 and 9, trade induces North to specialize *partially* towards sector 2 until period 6, with the interest rate reversal. If agents can borrow/lend abroad before or in period 6, financial flows are "downhill" from North to South and the interest rate in North is aligned to the world level, $r_t = r_t^*$. Due to the interest rate coupling, the rate of return in sector 1 is also equalized, $q_{t+1}^1 = r_t = r_t^* = q_t^{1,*}$. Meanwhile, trade equalizes the sectoral price ratio $\chi_t = \chi_t^*$, which implicitly equalizes the sectoral rental-price-of-capital ratio, $\mu_t = \chi_t^{\frac{1}{\alpha}} = (\chi_t^*)^{\frac{1}{\alpha}} = \mu_t^*$. This way, trade and capital mobility jointly equalize the rate of return in sector 2, $q_t^2 = \frac{q_t^1}{\mu_t} = \frac{q_t^{1,*}}{\mu_t^*} = q_t^{2,*}$. Despite international labor immobility, the wage rate is equalized across countries and so is income per capita,[19]

$$\frac{Y_t}{2L} = \frac{w_t}{2(1-\alpha)} = \frac{[(q_t^1)^{1-\eta}(q_t^2)^{\eta}]^{-\rho}}{2(1-\alpha)} = \frac{[(q_t^{1,*})^{1-\eta}(q_t^{2,*})^{\eta}]^{-\rho}}{2(1-\alpha)} = \frac{w_t^*}{2(1-\alpha)} = \frac{Y_t^*}{2L^*}.$$

Here, the factor price equalization (FPE, hereafter) and cross-country income convergence depend critically on the fact that trade only leads to partial specialization and hence, the interest rate is coupled with the sector 1's rate of return before and in period 6.

Suppose that agents are allowed to borrow/lend abroad only from period 7 on. Since North has abandoned sector 1, the interest rate is decoupled from the rate of return there. Thus, the interest rate equalization does not imply the cross-country equalization of sector 1's rate of return, which invalidates the aforementioned mechanism. In fact, the interest rate is re-reversed from period $t \geq 7$ on. If allowed, financial flows are "uphill" from South to North, which further widens the cross-country gap of income per capita. This is in stark contrast to the findings of Antras and Caballero (2009).

To sum up, whether trade and capital mobility jointly lead to the FPE depends critically on the **timing of integration**, i.e., whether the more financially developed country has abandoned the low-return sector upon financial integration.

[19] In period t, each country is populated with the young and the old of the same mass. Thus, the size of total population is $2L$ in North and $2L^*$ in South, respectively.

5 Discussions and Extensions

The endogenous responses of δ_t to the static gains from trade are critical for the possibility of North abandoning sector 1 and the interest rate re-reversal. In this section, I compare two channels through which the static gains affect δ_t and discuss some model extensions.

5.1 Endogenous Responses of δ_t through Two Channels

Let W_t^e and W_t denote respectively entrepreneurial wealth and national wealth. In the case of the binding borrowing constraints, the fraction of domestic investment in sector 2, $\delta_t = \frac{\frac{W_t^e}{1-\lambda}}{W_t} = \frac{1}{1-\lambda}\frac{W_t^e}{W_t}$, depends on the leverage multiplier $\frac{1}{1-\lambda}$, which is constant by assumption, and the entrepreneurial wealth share $\frac{W_t^e}{W_t}$, which is higher if the mass of entrepreneurs is higher (the **extensive** margin) and/or if the average wealth of entrepreneurs rises relative to the national average (the **intensive** margin). As shown below, δ_t may respond to the static gains from trade through these two margins.

In my model, agents live for **two** periods; each agent invests its entire labor income when young and consumes when old. Due to the absence of wealth accumulation at the individual level, agent j's wealth relevant for investment is just its labor income, $n_{j,t} = w_t(1-\theta)\epsilon_j$. One can decompose along two margins the period-1 response of the entrepreneurial wealth share in North to the static gains from trade,

$$\frac{W_1^e}{W_1} - \frac{W_A^e}{W_A} = \underbrace{\frac{\int_{\underline{\epsilon}_A}^{\infty} n_{j,1} dG(\epsilon_j)}{w_1 L} - \frac{\int_{\underline{\epsilon}_A}^{\infty} n_{j,A} dG(\epsilon_j)}{w_A L}}_{\text{the intensive margin effect}} + \underbrace{\frac{\int_{\underline{\epsilon}_1}^{\underline{\epsilon}_A} n_{j,1} dG(\epsilon_j)}{w_1 L}}_{\text{the extensive margin effect}} \tag{33}$$

$$= \underbrace{\frac{w_1 L \tau_A^{1-\theta}}{w_1 L} - \frac{w_A L \tau_A^{1-\theta}}{w_A L}}_{\text{the intensive margin effect (0)}} + \underbrace{\frac{w_1 L(\tau_1^{1-\theta} - \tau_A^{1-\theta})}{w_1 L}}_{\text{the extensive margin effect (+)}} . \tag{34}$$

Consider the agents who are born in period 1 and would meet the MIR at the autarkic steady state, i.e., $\epsilon_j \geq \underline{\epsilon}_A$. The static gains affect in equal proportions the wealth of these "existing" entrepreneurs,[20] $n_{j,t} = w_t(1-\theta)\epsilon_j$, and the national wealth, $\frac{w_t L}{L} = w_t$, through the wage rate. Thus, the national wealth share of "existing" entrepreneurs does not respond to the static gains so that the intensive margin is mute. Due to financial frictions and sector-specific MIR, the cutoff value $\underline{\epsilon}_t$ and the mass of entrepreneurs $\tau_t = \underline{\epsilon}_t^{-\frac{1}{\theta}}$ are endogenous. Thus, the static gains raise $\frac{W_t^e}{W_t}$ purely through the extensive margin, which then raises $\delta_t = \frac{1}{1-\lambda}\frac{W_t^e}{W_t}$ and creates the possibility of North abandoning sector 1.

In the static model of Antras and Caballero (2009), the extensive margin of entrepreneurial wealth is mute, due to two assumptions: **(1)** there is no MIR; **(2)** only a fixed mass τ of agents have the technology to invest in the constrained sector and are called entrepreneurs. Consider the case of the homogeneous labor endowment, $l_j = 1$. If this static model is embedded into my two-period OLG setting where wealth accumulation is absent at the individual level, the wealth share of entrepreneurs is constant at

[20] Referring these agents as "existing" entrepreneurs is purely for quantitative exposition. It may be misleading, because they are born in period 1 and did not "exist" in period 0.

$\frac{W_t^e}{W_t} = \frac{\tau w_t L}{w_t L} = \tau$. Under assumption 1 of Antras and Caballero (2009), the borrowing constraints are so tight that the investment share of the constrained sector is inefficiently low, $\delta = \frac{\tau}{1-\lambda} < \eta$. Since δ is constant and does not respond to the static gains from trade, the unconstrained sector is always active, $\frac{K_{t+1}^1}{w_t L} = 1 - \delta > 1 - \eta > 0$ and the interest rate re-reversal never happens. This way, the endogenous extensive margin of entrepreneurial wealth is key to the interest rate re-reversal in my two-period OLG model.

Antras and Caballero (2009) embed their static model into a dynamic, continuous-time setting with two key assumptions: **(1)** agents are born at a constant rate per unit of time and die at the same rate, **(2)** agents save all their (labor and investment) income and consume only when they die. Due to the law of large numbers, the mass of entrepreneurs is constant and hence, the extensive margin of entrepreneurial wealth is mute. Different from the two-period OLG setting, agents accumulate wealth over their lifetime in this model. Due to the privilege of investing in the constrained sector at a higher rate of return, entrepreneurs accumulate wealth at a faster speed than others under autarky and hence, their wealth share $\frac{W_t^e}{W_t}$ becomes endogenous on the intensive margin. By aligning the sectoral price ratio at the world level, trade raises (reduces) the sectoral rate-of-return differential in North (South), which induces entrepreneurs to accumulate wealth at a faster (slower) speed than in the autarkic steady state. This way, trade affects the entrepreneurial wealth share along **the intensive margin** and δ_t responds accordingly.

Antras and Caballero (2009) focus on the interest rate response to trade flows for the **less** financially developed country. As both sectors are always active there, the interest rate reversal holds strictly. They do not analyze explicitly whether and under what conditions trade can induce the **more** financially developed country to abandon the low-return sector.[21] In contrast, I highlight the **extensive margin** as a key channel through which the entrepreneurial wealth share responds to the static gains and show that trade may lead to the interest rate re-reversal if it substantially changes the sectoral composition in the **more** financially developed country,[22] which complements their findings.

5.2 The Implications of Trade Costs and Initial Conditions

So far, I have made two simplifying assumptions: **(1)** there is no trade cost, and **(2)** the world economy is initially at the autarkic steady state. Given $\{\theta, Z\}$ in region UF2 of the left panel of figure 7, the left panel of figure 10 shows that case UF2 arises if $\chi^* < \tilde{\chi}_T^*$ and $\chi_A > \bar{\chi}_T$. Let $\underline{\chi}_A$ denote the sectoral price ratio at the autarkic steady state in the case of $\lambda = 0$.[23] It holds that $\chi_A, \chi_A^* \geq \underline{\chi}_A$.

Under the two assumptions, the world sectoral price ratio that North faces is $\chi^* = \chi_A^*$. Then, one can convert the threshold values ($\tilde{\chi}_T^*$ and $\bar{\chi}_T$) into the $\{\lambda^*, \lambda\}$ space, as shown by the right panel of figure 7. In the following, I discuss how trade costs and South's initial condition affect the likelihood of case UF2 for North.

Let us first consider the trade costs. Suppose that the world economy is initially at

[21] It is beyond the scope of my paper to explore this possibility in their dynamic model.

[22] Allowing wealth accumulation at the individual level in my model would strengthen my results through the additional, positive intensive margin effect.

[23] It is the (absolute) gradient of the PPF at point Z in figure 5.

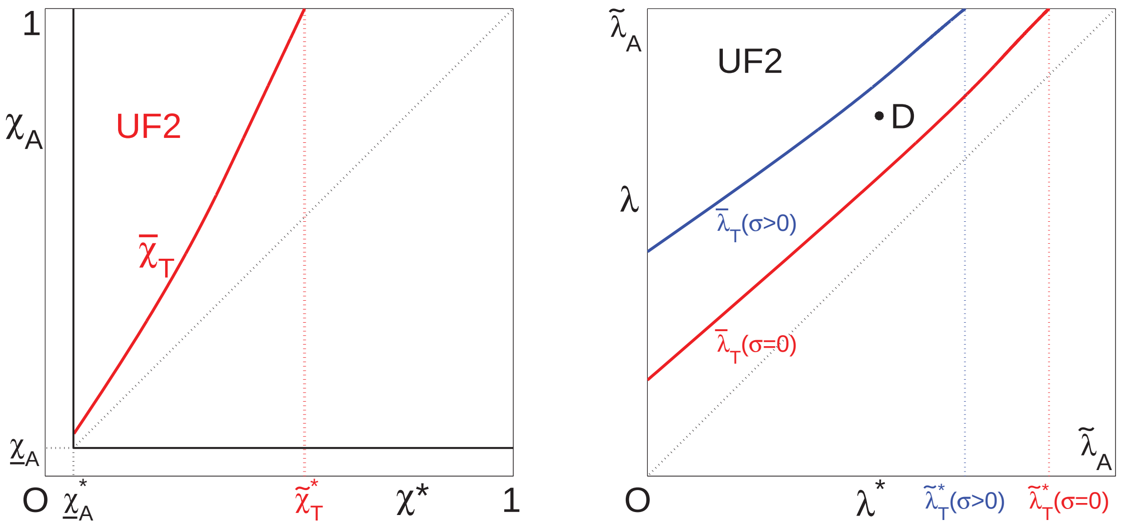

Figure 10: Threshold Values for the Trade Equilibrium with Trade Costs $\sigma \in (0, \bar{\sigma})$

the autarkic steady state as before. Let $\sigma \in (0, 1)$ denote the proportional trade costs. Although trade costs do not affect χ_A and χ_A^*, they raise the world sectoral price ratio that North faces $\chi^* = \chi_A^* \frac{1+\sigma}{1-\sigma}$, which narrows the international sectoral price differential, $\chi_A - \chi^* = \chi_A - \chi_A^* \frac{1+\sigma}{1-\sigma}$, and reduces the trade flows in period 1. As a result, trade costs weaken the static gains from trade and the investment reallocation effect. For the existence of case UF2, χ_A^* needs to be lower than in the absence of trade costs ($\sigma = 0$) and so does λ^*. In the right panel of figure 10, trade costs reduce the threshold value $\tilde{\lambda}_T^*$ and raise the threshold value $\bar{\lambda}_T$. The larger the trade costs, the smaller the region UF2 in the right panel of figure 10, the less likely case UF2 arises.

Lemma 5. *Let* $\bar{\sigma} \equiv \frac{\eta^{\frac{(1-\alpha)\theta}{(1-\eta)(1-\theta)}} - \underline{\chi}_A}{\eta^{\frac{(1-\alpha)\theta}{(1-\eta)(1-\theta)}} + \underline{\chi}_A}$. *For* $\sigma \in [\bar{\sigma}, 1]$, *case UF2 does not arise. For* $\sigma \in (0, \bar{\sigma})$, *the interest rate in North may have a non-monotonic pattern with regard to trade costs.*

Take as an example the parameter configuration represented by point D in the right panel of figure 10. For the sufficiently large trade costs, point D is outside of region UF2 and trade does not induce North to abandon sector 1. By reducing $\chi^* = \chi_A^* \frac{1+\sigma}{1-\sigma}$, a marginal decline in trade costs widens the international sectoral price differential, $\chi_A - \chi^* = \chi_A - \chi_A^* \frac{1+\sigma}{1-\sigma}$, which raises the trade flows and allows North to specialize further towards sector 2. Meanwhile, the decline in trade costs reduces the threshold value $\bar{\lambda}_T$ and shifts rightwards the border of region UF2, making case UF2 more likely. If the border is far above point D, a marginal decline in trade costs reduces χ^* and the interest rate in North falls, as mentioned in subsection 4.2. However, if the decline in trade costs is so large that the border of region UF2 is below point D, North eventually abandons sector 1, leading to the interest rate re-reversal. This way, deepening trade integration may have the non-monotonic effect on the interest rate in North.

Let us consider next the initial condition of South. Suppose that there is no trade cost ($\sigma = 0$) and North is initially at the autarkic steady state ($w_0 = w_A$) as before, while South is below its autarkic steady state ($w_0^* < w_A^*$). Take as an example the parameter configuration represented by point T in the left panel of figure 11. As it is outside of

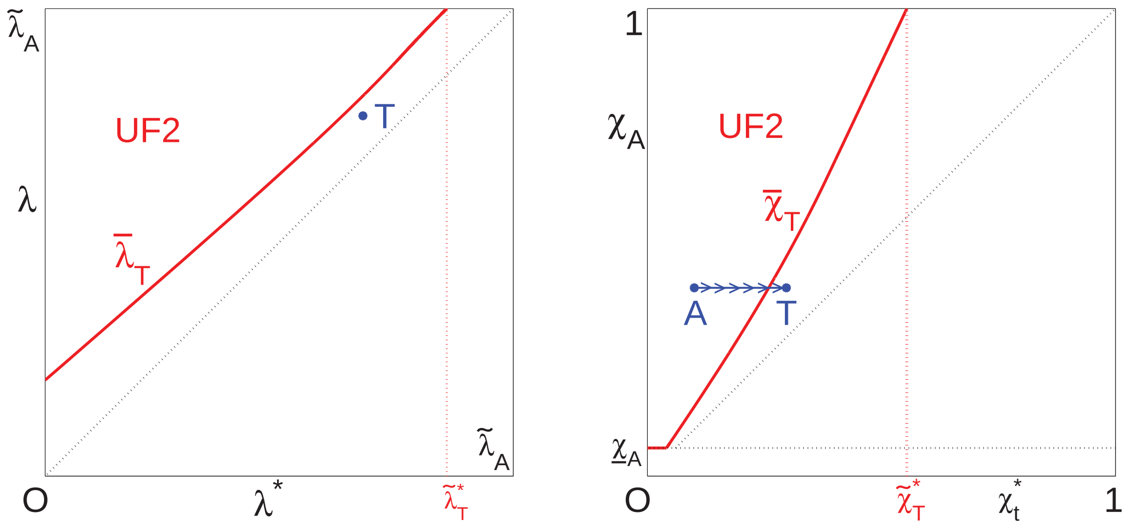

Figure 11: Threshold Values for the Trade Equilibrium with $w_0^* < w_A^*$

region UF2, case UF2 does arise in the long run when South eventually converges to its steady state with $w_t^* = w_A^*$ and $\chi_t^* = \chi_A^*$. According to equations (13) and (24), $w_0^* < w_A^*$ implies $\mu_1^* < \mu_A^*$ and $\chi_1^* < \chi_A^*$ so that case UF2 may arise for North in the short run. The right panel of figure 11 shows the threshold values in the $\{\chi_t^*, \chi_A\}$ space, with point A and T denoting the sectoral price ratio in South in period 1 and in the long run. Starting with point A, trade integration may induce North to gradually abandon sector 1 in the medium run if the convergence process of South towards its steady state is sufficiently slow, with χ_t^* stays in region UF2. However, when South is sufficiently close to its steady state, χ_t^* is so high that trade flows become small. As a result, sector 1 becomes active again in North. In this sense, the economic development in South may affect the sectoral composition in North, which may have significant implications to the global imbalances.

5.3 From the Two-Sector to the Multi-Sector Setting

Section 4 shows that the interest rate reversal and re-reversal may occur when trade induces North to gradually abandon the low-return sector. The two-sector setting allows me to explore the core mechanism intuitively and analytically. However, in this setting, North eventually specializes fully in the high-return sector and the borrowing constraints become slack, which seems implausible.

One may generalize my model into a S-sector setting where sector 1 is not subject to the MIR as in the present model, $\mathfrak{m}_1 = 0$, while the other S-1 sectors are ranked in ascending order with regard to the MIR, i.e., $\mathfrak{m}_{s-1} < \mathfrak{m}_s$ for $s \in \{2, ..., S\}$. If the MIR differs substantially across sectors, the borrowing constraints are slack in sector 1 and binding in all other sectors at the autarkic steady state. Then, the investment rate of return also ranks in ascending order across sectors. Given that North is more financially developed than South, North (South) has the comparative advantage in the high-rank (low-rank) sectors.

By the same logic as mentioned in section 4, trade may allow North to gradually upgrade its sectoral structure by abandoning the low-rank sectors sequentially over time.

Once the lowest-rank sector among all active ones is abandoned, the interest rate is decoupled from the rate of return in that sector and coupled with the rate of return in the next lowest-rank sector. As a result, the interest rate pattern in the left panel of figure 9 will become a triangle wave. In other words, North may witness the interest rate reversal and re-reversal recurrently along its convergence path under free trade. During this process, the borrowing constraints are binding in the high-rank sectors in North as long as no less than two sectors are active in North.

5.4 Supply-Chain Trade and the Global Imbalances

In my model, "sectors" can be interpreted broadly as production stages or tasks. Due to technological progress and world-wide economic liberalization, the costs of transportation, communication, and coordination have declined substantially. Since the 1990s, production process has become more and more fragmented globally in the sense that vertically and horizontally linked production stages/tasks have been increasingly conducted in different countries (Baldwin, 2013a; Baldwin and Lopez-Gonzalez, 2015; Gereffi, 1999; Koopman, Wang, and Wei, 2014). In OECD countries, fabrication and assembly activities account for an increasingly lower value-added share, compared with upstream and downstream activities[24] (Baldwin, 2013a; OECD, 2013). Furthermore, developed economies increasingly specialize towards high-skilled, non-routine activities, while emerging economies specialize towards capital-intensive, routine activities (Goos, Manning, and Salomons, 2014; Marcolin, Miroudot, and Squicciarini, 2016; Timmer et al., 2014).

In my model, investment can be tangible or intangible.[25] Fabrication and assembly are involved intensively with routine tasks, which require mainly the input of tangible investment; upstream and downstream activities are involved intensively with non-routine tasks, which require heavily the input of intangible investment. Compared with tangible investment, intangible investment is subject to more severe financial frictions because it has a riskier return and is not commonly accepted as collateral for loans. In the following, I apply the core mechanism of my model to supply-chain trade and explain intuitively its impacts on the interest rate patterns between developed and emerging economies.

Suppose that North is more developed than South in financing intangible investment (Corrado et al., 2013). Thus, North has a comparative advantage in upstream and downstream activities, while South has a comparative advantage in fabrication and assembly. Under autarky, the interest rate is coupled with the MRK in the active, lowest-return production stages/tasks, $r^i = \min\{MRK_s^i \| K_s^i > 0\}$, where superscript $i \in \{N, S\}$ is the country index and subscript s is the index of production stage. According to lemma 2,

[24] In the manufacturing industry, upstream activities (such as R&D, product design, or the manufacturing of key parts and components) and downstream activities (such as marketing, branding, and customer service) constitute a large share of value-added, while the intermediate production stages (such as component fabrication and final assembly) account for a small value-added share (Kimura, 2003). The smile curve was introduced by Stan Shih, the founder of Acer, to feature such an "U-shaped" value-added pattern along the production chain (Baldwin, Ito, and Sato, 2014; Ye, Meng, and Wei, 2015).

[25] "physical capital" in my model should be interpreted broadly to include human capital as well as any tangible or intangible capital goods used in production.

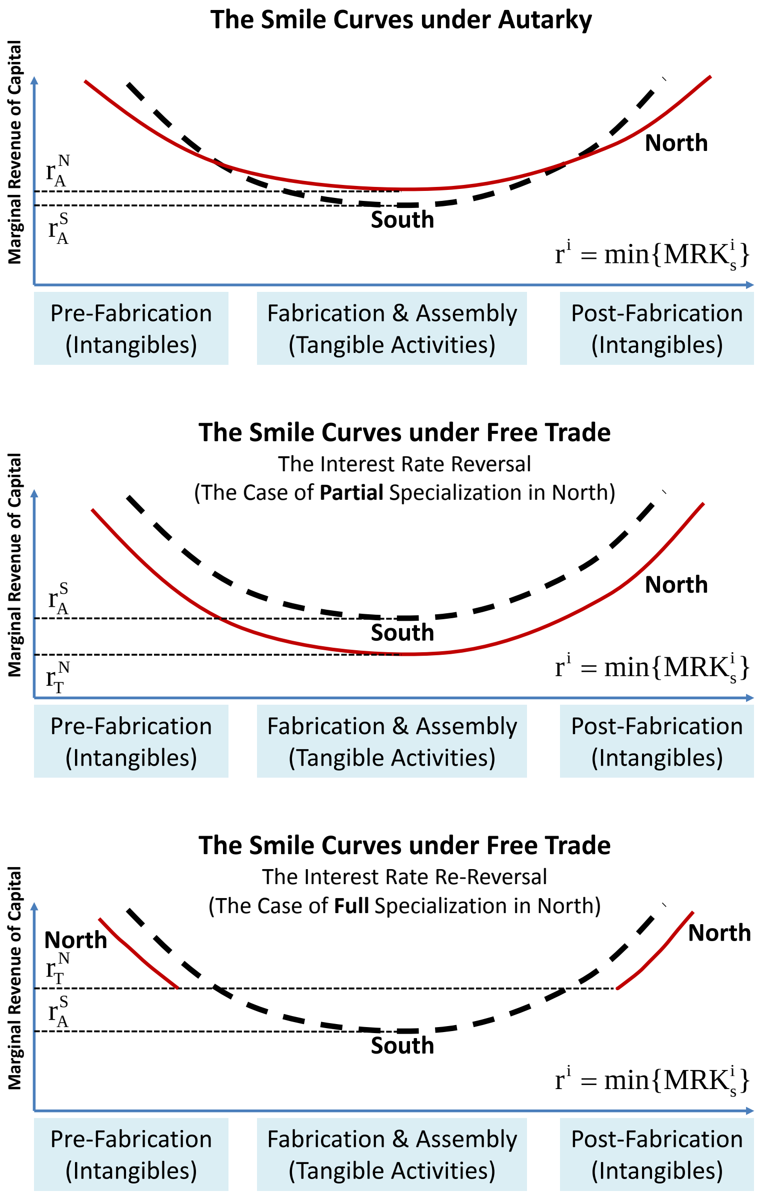

Figure 12: The "Falling vs. Missing Jaw" of the Smile Curve

the smile curve in terms of the MRK pattern[26] is flatter and the interest rate is higher in North than in South at autarkic steady state, $r_A^S < r_A^N$. See the upper panel of figure 12.

As a small open economy, North faces the world relative price $\chi_A^S \frac{1+\sigma}{1-\sigma}$, where σ denotes the trade cost. Given the moderate trade costs, the cross-country relative price differential $\chi_A^N - \chi_A^S \frac{1+\sigma}{1-\sigma}$ is positive but small. Trade integration only allows North to offshore **part** of fabrication and assembly activities to South. Thus, the interest rate in North is still coupled with the MRK in these activities and, as mentioned in subsection 4.2, the interest rate reversal happens, $r_T^N < r_A^S < r_A^N$, as shown in the middle panel of figure 12.

As discussed in subsection 5.2, a sufficiently large decline in σ may allow North to

[26]Originally, the smile curve shows the value-added pattern along the production chain. Since the production stages/tasks that account for the high value-added shares usually have the high MRK, I use the smile curve to show the MRK pattern along the value chain.

abandon all fabrication/assembly activities through offshoring and specialize fully in upstream/downstream activities. In that case, the middle part of the smile curve is missing and the interest rate in North is then coupled with the MRK in upstream and downstream activities. As shown the lower panel of figure 12, free trade leads to the interest rate re-reversal, which amplifies the initial interest rate differential, $r_T^N > r_A^N > r_A^S$. This way, the boom of supply-chain trade may contribute to the global imbalances.

6 Final Remarks

This paper highlights the extensive margin of sectoral investment as a key channel through which trade integration affects domestic sectoral composition and the incentives for international financial flows in a world with heterogeneous financial development. Whether trade flows amplify or dampen the global imbalances depends on how far trade-driven sectoral shifts reshape the industrial structure in the more financially developed country, which fundamentally complements the findings of Antras and Caballero (2009). The logic of my model results can be applied to the case of supply-chain trade. In particular, supply-chain trade may amplify the global imbalances, if it allows the developed countries to abandon the low-return production activities and upgrade to the high-return activities along the value chains.

For simplicity, I take the level of financial development as exogenously given and analyze the impacts of economic integration. In fact, trade and capital account liberalization may affect the incentives for financial intermediaries (Alessandria and Qian, 2005; Svaleryd and Vlachos, 2002; Tressel and Verdier, 2011) and the structures of financial markets from the political economy perspective (Braun and Raddatz, 2008; Rajan and Zingales, 2003, 2004). Besides, market-reform policies that improve domestic financial sectors and institutional structures are more fundamental to the CEECs' income convergence than simply reducing the barriers to trade or financial flows.[27] Taking into account these factors, developing countries may use trade and capital account liberalization as a triggering device and combine them with other market-reform policies to promote financial development and productivity growth. These topics are kept for future research.

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[27]Since the 2004 EU enlargement, the adoption of EU laws and directives has improved financial sector quality in the CEECs by upgrading their legal, regulatory, and supervisory framework to the same standard as in the Western Europe, while the dominance of foreign banks in the CEECs' financial markets has also improved the quality of domestic banking sectors (Herrmann and Winkler, 2009a,b). As a result, the CEECs have witnessed rapid and massive capital inflows (von Hagen and Zhang, 2014).

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Appendix

A Equilibrium Allocation under Free Trade

For $\theta \in (\underline{\theta}, 1)$ and $\lambda, \lambda^* \in [0, \tilde{\lambda}_A)$, four cases may arise under free trade, given $\chi^* = \chi_A^*$:

- Case **M**: **m**ultiple steady states arise, i.e., there are two stable steady states denoted by H and L, and one unstable steady state denoted by M. The steady states are ranked in terms of income level, $w_H > \bar{w} > w_M > w_L > w_A$.
- Case **UF2**: North converges to a **u**nique steady state where it **f**ully specializes in sector **2**;
- Case **UP2**: North converges to a **u**nique steady state where it **p**artially specializes in sector **2**;
- Case **UP1**: North converges to a **u**nique steady state where it **p**artially specializes in sector **1**;

Define some threshold values as follows,

$$\hat{Z} \equiv (1-\mathfrak{s})^{\mathfrak{s}-\eta}(1-\eta\mathfrak{s})^{1-\mathfrak{s}} \text{ and } \hat{\lambda}_T^* \equiv 1 - \left(\frac{Z}{\hat{Z}}\right)^{\rho(1-\theta)}, \text{ where } \mathfrak{s} \equiv \frac{\theta}{\rho(1-\theta)} \tag{35}$$

$$\tilde{Z} \equiv \eta^{\frac{\mathfrak{s}(\mathfrak{s}-\eta)}{1-\eta}}(1-\eta+\eta^{\frac{\mathfrak{s}}{1-\eta}+1})^{1-\mathfrak{s}} \text{ and } \tilde{\lambda}_T^* \equiv 1 - \left(\frac{Z}{\tilde{Z}}\right)^{\rho(1-\theta)} \tag{36}$$

In the following, I show graphically the threshold conditions in the $\{\theta, Z, \lambda^*, \lambda\}$ space that define the four cases[28] as well as show the law of motion for wage in each case.

Under free trade, multiple steady states arise if four conditions are satisfied simultaneously,

[28]The proof of proposition 2 provides the technical derivation for these threshold values.

- $\theta \in (\underline{\theta}, \alpha)$, i.e., $\{\alpha, \theta\}$ in region M of the left panel of figure 13, and
- $Z < \hat{Z}$, i.e., $\{\theta, Z\}$ in region M of the right panel of figure 13, and
- $\lambda^* < \hat{\lambda}_T^*$ and $\lambda \in (\tilde{\lambda}_T, \hat{\lambda}_T)$, i.e., $\{\lambda^*, \lambda\}$ in region M of the left panel of figure 14.

In the upper-left panel of figure 15, the solid (dashed) curve shows the law of motion for wage under free trade (autarky) for case M.

In section 4, I focus on case UF2 which arises if four conditions are satisfied simultaneously, i.e., $\theta \in (\underline{\theta}, 1)$ and $Z < \tilde{Z}$, i.e., $\{\theta, Z\}$ in regions M and UF of the right panel of figure 13,

- given $\{\theta, Z\}$ in region UF of the right panel of figure 13, it is required that $\{\lambda^*, \lambda\}$ in region UF2 of the middle panel of figure 14, i.e., $\lambda^* < \tilde{\lambda}_T^*$ and $\lambda \in (\tilde{\lambda}_T, \tilde{\lambda}_A)$;
- given $\{\theta, Z\}$ in region M of the right panel of figure 13, it is required that $\{\lambda^*, \lambda\}$ in region UF2 of the middle panel of figure 14, i.e, $\lambda^* \in (0, \hat{\lambda}_T^*)$ and $\lambda \in (\hat{\lambda}_T, \tilde{\lambda}_A)$, or alternatively, $\lambda^* \in (\hat{\lambda}_T^*, \tilde{\lambda}_T^*)$ and $\lambda \in (\tilde{\lambda}_T, \tilde{\lambda}_A)$.

In the upper-right panel of figure 15, the solid (dashed) curve shows the law of motion for wage under free trade (autarky) for case UF2.

Given $\theta \in (\underline{\theta}, 1)$, free trade induces North to move from the autarkic steady state to a **u**nique steady state where it specializes **p**artially in sector **2** (**1**) if parameters are in region region UP2 (UP1) of figure 14. The three panels in figure 14 correspond to $\{\theta, Z\}$ in the three regions of the right panel of figure 13.

In the two bottom panels of figure 15, the solid (dashed) curve shows the law of motion for wage under free trade (autarky) in case UP2 and UP1, respectively.

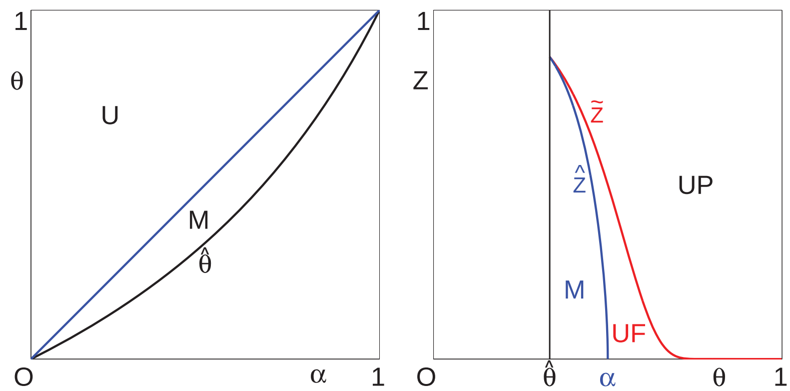

Figure 13: Threshold Values under Free Trade

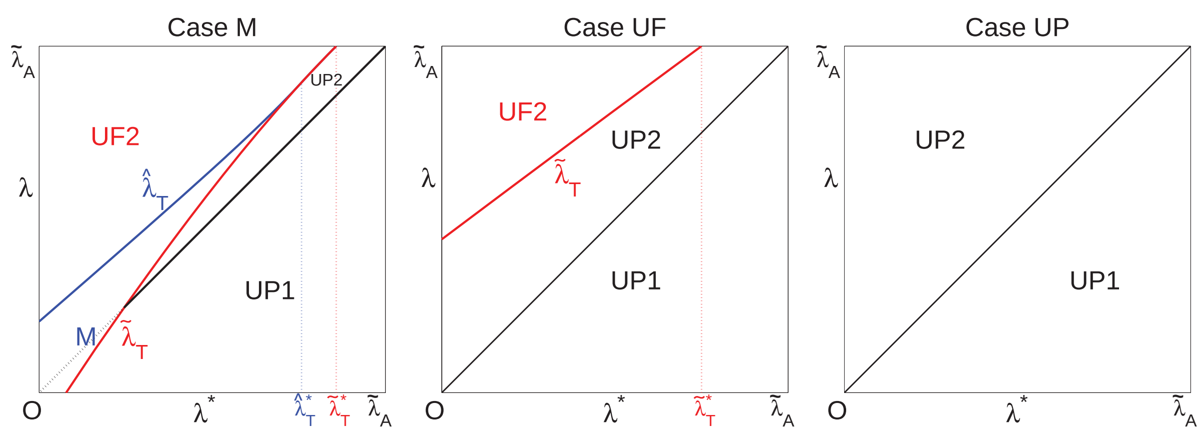

Figure 14: Threshold Values under Free Trade

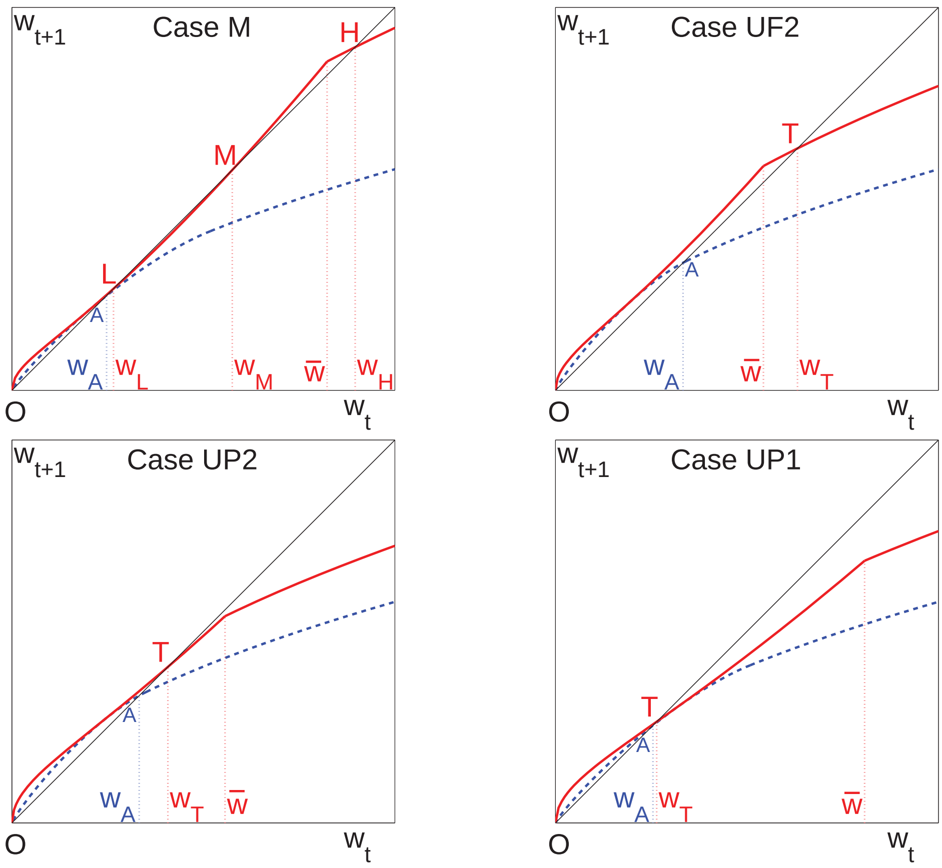

Figure 15: Laws of Motion for Wage under Free Trade: $\theta \in (\underline{\theta}, \alpha)$

B Proofs

Proof of Lemma 1

Proof. First, combine equations (1) and (3)-(4) to get equations (5)-(6) as follows,

$$\frac{q^2_{t+1}\delta_t w_t L}{\alpha} = p^2_{t+1}Y^2_{t+1} = \frac{w_{t+1}\zeta_{t+1}L}{1-\alpha}, \quad \frac{q^1_{t+1}(1-\delta_t)w_t L}{\alpha} = p^1_{t+1}Y^1_{t+1} = \frac{w_{t+1}(1-\zeta_{t+1})L}{1-\alpha} \tag{37}$$

$$\frac{w_{t+1}}{q^2_{t+1}} = \frac{\delta_t w_t}{\rho\zeta_{t+1}}, \quad \frac{w_{t+1}}{q^1_{t+1}} = \frac{(1-\delta_t)w_t}{\rho(1-\zeta_{t+1})}, \quad \mu_{t+1} \equiv \frac{q^1_{t+1}}{q^2_{t+1}} = \frac{\frac{\delta_t}{1-\delta_t}}{\frac{\zeta_{t+1}}{1-\zeta_{t+1}}}, \quad \zeta_{t+1} = \frac{\delta_t}{(1-\delta_t)\mu_{t+1}+\delta_t} \tag{38}$$

$$Y^2_{t+1} = \left(\frac{\delta_t w_t L}{\alpha}\right)^{\alpha}\left(\frac{\zeta_{t+1}L}{1-\alpha}\right)^{1-\alpha} = \left(\frac{p^2_{t+1}Y^2_{t+1}}{q^2_{t+1}}\right)^{\alpha}\left(\frac{p^2_{t+1}Y^2_{t+1}}{w_{t+1}}\right)^{1-\alpha}, \tag{39}$$

$$Y^1_{t+1} = \left[\frac{(1-\delta_t)w_t L}{\alpha}\right]^{\alpha}\left[\frac{(1-\zeta_{t+1})L}{1-\alpha}\right]^{1-\alpha} = \left(\frac{p^1_{t+1}Y^1_{t+1}}{q^1_{t+1}}\right)^{\alpha}\left(\frac{p^1_{t+1}Y^1_{t+1}}{w_{t+1}}\right)^{1-\alpha}, \tag{40}$$

$$p^2_{t+1} = (q^2_{t+1})^{\alpha}w^{1-\alpha}_{t+1}, \quad p^1_{t+1} = (q^1_{t+1})^{\alpha}w^{1-\alpha}_{t+1}, \quad \Rightarrow \quad \frac{p^1_{t+1}}{p^2_{t+1}} = \left(\frac{q^1_{t+1}}{q^2_{t+1}}\right)^{\alpha}, \quad \chi_{t+1} = \mu^{\alpha}_{t+1}. \tag{41}$$

Second, derive the PPF in two cases.

- In the case of efficient sectoral investment, $\mu_{t+1} = 1$ and, according to equation (38), the sectoral shares of labor and capital equalize, $\zeta_{t+1} = \delta_t$. Use equations (39)-(40) to get

$$Y^1_{t+1} = (1-\zeta_{t+1})\mathbb{Y}_{t+1}, \quad Y^2_{t+1} = \zeta_{t+1}\mathbb{Y}_{t+1}, \quad \text{where} \quad \mathbb{Y}_{t+1} \equiv \left[\frac{w_t L}{\alpha}\right]^{\alpha}\left[\frac{L}{1-\alpha}\right]^{1-\alpha}. \tag{42}$$

  In this case, the PPF is specified by equation (7).

- In the case of inefficient sectoral investment, $\mu_{t+1} < 1$ and the borrowing constraints are binding. According to equation (13), sector 2's share of physical capital in period $t+1$, i.e., δ_t, is predetermined and depends on Y_t. Rewrite equations (39)-(40) as

$$\left[\frac{Y^1_{t+1}}{(1-\delta_t)^\alpha}\right]^{\frac{1}{1-\alpha}} = (1-\zeta_{t+1})\mathbb{Y}_{t+1}^{\frac{1}{1-\alpha}}, \quad \left(\frac{Y^2_{t+1}}{\delta_t^\alpha}\right)^{\frac{1}{1-\alpha}} = \zeta_{t+1}\mathbb{Y}_{t+1}^{\frac{1}{1-\alpha}}. \tag{43}$$

  In this case, the PPF is specified by equation (18).

Third, derive the boundary condition for the two cases. The level of national income in period t determines sector 2's maximum share of physical capital in period $t+1$, according to equation (13). Maintaining the efficient sectoral investment $\mu_{t+1} = 1$, sector 2's maximum possible share of domestic labor is $\zeta_{t+1} = \tilde{\delta}_t$, corresponding to the threshold output $\tilde{Y}^2_{t+1}$.

Fourth, derive the MRT from the gradient of the PPF. Combine equations (1) and (3)-(4),

$$\ln Y^2_{t+1} = \alpha \ln \delta_t + (1-\alpha)\ln \zeta_{t+1} + \ln \mathbb{Y}_{t+1}, \; \ln Y^1_{t+1} = \alpha \ln(1-\delta_t) + (1-\alpha)\ln(1-\zeta_{t+1}) + \ln \mathbb{Y}_{t+1}$$

$$\frac{\partial \ln Y^2_{t+1}}{\partial \ln \zeta_{t+1}} = \alpha \frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1-\alpha), \quad \frac{\partial \ln Y^1_{t+1}}{\partial \ln \zeta_{t+1}} = \alpha \frac{-\delta_t}{1-\delta_t}\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1-\alpha)\frac{-\zeta_{t+1}}{1-\zeta_{t+1}}$$

$$MRT_{1,2} \equiv -\frac{\partial Y^2_{t+1}}{\partial Y^1_{t+1}} = -\frac{\partial \ln Y^2_{t+1}}{\partial \ln Y^1_{t+1}}\frac{Y^2_{t+1}}{Y^1_{t+1}} = \frac{\alpha \frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1-\alpha)}{\alpha \frac{\delta_t}{1-\delta_t}\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1-\alpha)\frac{\zeta_{t+1}}{1-\zeta_{t+1}}}\frac{Y^2_{t+1}}{Y^1_{t+1}}.$$

In the case of efficient sectoral investment, $\mu_{t+1} = 1$ and combine it with equation (38) to get $\zeta_{t+1} = \delta_t$ and $\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} = 1$. In the case of inefficient sectoral investment, $\mu_{t+1} < 1$ and, given the predetermined δ_t, $\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} = 0$. In either case, the (absolute) gradient of the PPF is

$$MRT_{1,2} = \frac{1-\zeta_{t+1}}{\zeta_{t+1}}\frac{Y^2_{t+1}}{Y^1_{t+1}}. \tag{44}$$

□

Proof of Proposition 1

Proof. First, derive the autarkic equilibrium. Use equation (2) to get the gradient of isoquant,

$$\ln Y_{t+1} = (1-\eta)\ln V^1_{t+1} + \eta \ln V^2_{t+1} - \ln \eta^\eta (1-\eta)^{1-\eta}, \; MRS_{1,2} \equiv -\frac{\partial V^2_{t+1}}{\partial V^1_{t+1}} = \frac{1-\eta}{\eta}\frac{V^2_{t+1}}{V^1_{t+1}}. \tag{45}$$

In the autarkic equilibrium, the PPF and the isoquant are tangent and the markets for sectoral outputs clear domestically. Combine equations (19), (44), and (45) to get

$$MRS_{1,2} = MRT_{1,2}, \; \Rightarrow \; \frac{1-\eta}{\eta} = \frac{1-\zeta_{t+1}}{\zeta_{t+1}}, \; \Rightarrow \; \zeta_{t+1} = \eta. \tag{46}$$

Thus, the share of domestic labor allocated in a sector under autarky is equal to the sector's share in the production of final goods. Whether it also applies to the investment share depends on how far financial frictions distort the maximum share of domestic investment in sector 2.

$$\delta_t = \begin{cases} \tilde{\delta}_t < \zeta_{t+1} = \eta, & \text{if } \tilde{\delta}_t < \eta; \\ \zeta_{t+1} = \eta, & \text{if } \tilde{\delta}_t \geq \eta. \end{cases} \tag{47}$$

Second, combine equations (46)-(47) with (1)-(4) and (6) to get (23) as the law of motion for wage. In the case of efficient sectoral investment, $\mu_{t+1} = 1$ and the law of motion for wage degenerates into equation (22). In the case of inefficient sectoral investment, equation (13) specifies the actual fraction of domestic investment in sector 2 (δ_t) as a function of w_t. Combine equation (6) and (46) to get (24) specifying the sectoral return wedge μ_{t+1} as a function of δ_t.

Third, derive the condition for the unique steady state, i.e., the steady state if exists is stable. For $w_t \geq \bar{w}_A$, the law of motion for wage in logarithm is linear with the slope less than unity, as shown by equation (22). If there exists a steady state with $w_A \geq \bar{w}_A$, it is stable.

For $w_t < \bar{w}_A$, sectoral investment is inefficient. Use equations (13), (23), (24) to get

$$\ln \delta_t = \frac{1-\theta}{\theta}(\ln w_t - \ln \bar{w}), \qquad \frac{\partial \ln \delta_t}{\partial \ln w_t} = \frac{1-\theta}{\theta}, \tag{48}$$

$$\ln \mu_{t+1} = \ln \delta_t - \ln(1-\delta_t) + \ln \frac{1-\eta}{\eta}, \qquad \frac{\partial \ln \mu_{t+1}}{\partial \ln \delta_t} = \frac{1}{1-\delta_t}, \tag{49}$$

$$\ln \Gamma_t = \eta \ln \mu_{t+1} - \ln(1-\eta+\eta\mu_{t+1}), \qquad \frac{\partial \ln \Gamma_t}{\partial \ln \mu_{t+1}} = \eta(1-\mu_{t+1})(1-\delta_t). \tag{50}$$

Combine them with equation (23) to get (25). The condition for the stable steady state is.

$$\frac{\partial w_{t+1}}{\partial w_t}|_{w_{t+1}=w_t} < 1, \;\Rightarrow\; \mu_A > 1 - \frac{\$}{\eta}, \text{ where } \$ \equiv \frac{\frac{1}{\alpha}-1}{\frac{1}{\theta}-1}. \tag{51}$$

Given $\mu_{t+1} \in (0,1]$, a sufficient condition for inequality (51) to hold is

$$1 - \frac{\$}{\eta} \leq 0, \;\Rightarrow\; \theta \geq \underline{\theta} \equiv \frac{\eta\alpha}{\eta\alpha + (1-\alpha)}. \tag{52}$$

Consider first the case of $\theta \geq \underline{\theta}$, i.e., $\{\alpha, \theta\}$ in region U of the left panel of figure 3. Derive the condition under which the borrowing constraints are binding at the autarkic steady state, i.e., $\mu_A < 1$ or equivalently $\tilde{\delta}_A < \eta$. Combine $w_{t+1} = w_t = w_A$ with equations (13) and (23)-(24) to get $\mu_A < 1$ as a function of λ,

$$w_A = \left(\frac{1}{\rho}\frac{\mu_A^\eta}{1-\eta+\eta\mu_A}\right)^\rho = \delta_A^{\frac{\theta}{1-\theta}}\bar{w} = \left[\frac{\eta\mu_A}{1-\eta+\eta\mu_A}\right]^{\frac{\theta}{1-\theta}} \frac{\mathfrak{m}}{1-\theta}(1-\lambda)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \quad Z = \frac{\frac{1}{\rho}}{(\frac{\mathfrak{m}}{1-\theta}\eta^{\frac{\theta}{1-\theta}})^{\frac{1}{\rho}}} = (1-\lambda)^{\frac{1}{\rho(1-\theta)}} \frac{(\mu_A)^{\$-\eta}}{(1-\eta+\eta\mu_A)^{\$-1}}, \tag{53}$$

$$\frac{\partial \ln \mu_A}{\partial \ln \lambda} = \frac{\frac{1}{\theta}\frac{\lambda}{1-\lambda}}{(1-\delta_A)[1-\frac{(1-\mu_A)\eta}{\$}]}. \tag{54}$$

Given $\theta > \underline{\theta}$, $\$ > \eta$ and hence, $\frac{(1-\mu_A)\eta}{\$} < 1$ and hence, $\frac{\partial \ln \mu_A}{\partial \ln \lambda} > 0$. Let $\tilde{\lambda}_A$ denote the level of financial development that ensures $\tilde{\delta}_A = \eta$ and then $\mu_A = 1$. Combine them with equation (53),

$$\tilde{\lambda}_A \equiv 1 - Z^{\rho(1-\theta)}. \tag{55}$$

Finally, in the case of $\theta < \underline{\theta}$, i.e., $\{\alpha, \theta\}$ in region M of the left panel of figure 3, $\$ < \eta$. Derive the threshold condition for multiple steady states, i.e., the existence of unstable steady state M. In the case of $\lambda = \hat{\lambda}_A$, the law of motion for wage is tangent with the 45° line at $w_M \in (0, \bar{w}_A)$,

$$\frac{\partial w_{t+1}}{\partial w_t}|_{w_M} = 1, \;\Rightarrow\; \mu_M = 1 - \frac{\$}{\eta}. \tag{56}$$

Combine equations (56) and (53) to get the threshold condition,

$$\hat{\lambda}_A = 1 - \left[Z\frac{(\frac{\eta-\$}{\eta})^{\eta-\$}}{(1-\$)^{1-\$}}\right]^{\rho(1-\theta)}, \tag{57}$$

In the case of $\theta \in (0, \underline{\theta})$, figure 16 shows two threshold values, $\hat{\lambda}_A$ and $\tilde{\lambda}_A$, in the $\{\lambda, Z\}$ space. The solid curves in the three panels of figure 17 show the law of motion for wage, given $\{\lambda, Z\}$ in the three regions of figure 16, respectively, while the dashed curves show the laws of motion for wage in the benchmark case of $\lambda = 1$. The steady state properties in the three cases are summarized as follows.

- For $\{\lambda, Z\}$ in region N, the economy converges to the steady state with the income level at zero, $w_A = 0$. In this case, there does **n**ot exist a non-trivial steady state.
- For $\{\lambda, Z\}$ in region MB, there is an unstable steady state M and a stable steady state A with $0 < w_M < w_A < w_B$, and the borrowing constraints are **b**inding in steady state A.

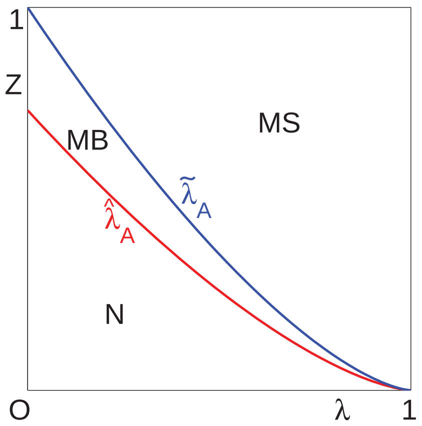

Figure 16: Steady-State Properties under Autarky

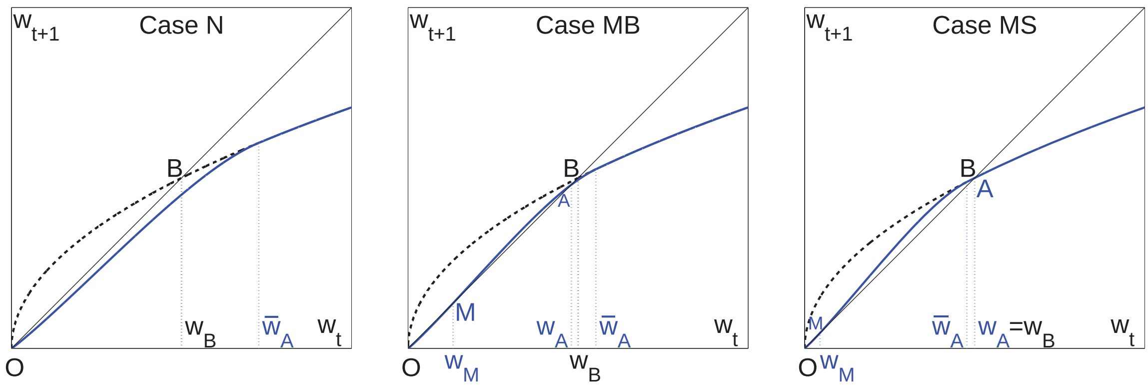

Figure 17: Laws of Motion for Wage under Autarky: $\theta < \underline{\theta}$

- For $\{\lambda, Z\}$ in region MS, there is an unstable steady state M and a stable steady state A with $0 < w_M < w_A = w_B$, and the borrowing constraints are slack in steady state A.

□

Proof of Lemma 2

Proof. Under autarky, the markets for sectoral outputs clear domestically, $V_t^s = Y_t^s$, implying $K_{t+1}^1 > 0$ and $r_t = q_{t+1}^1$. According to equations (1)-(2), $w_{t+1}^{1-\alpha}(q_{t+1}^1)^{\alpha(1-\eta)}(q_{t+1}^2)^{\alpha\eta} = 1$. Combine them to get $w_{t+1} = \left(\frac{1}{r_t}\mu_{t+1}^{\eta}\right)^{\rho}$. Combine it with equation (23) to get (26).

For $\theta > \underline{\theta}$, there exists a unique steady state under autarky with $w_A = \left(\frac{\Gamma_A}{\rho}\right)^{\rho}$ where

$$\Gamma_A = \frac{\mu_A^{\eta}}{1 - \eta + \eta\mu_A} \quad \text{and} \quad \frac{\partial \ln \Gamma_A}{\partial \ln \mu_A} = \eta(1 - \delta_A) > 0, \quad \text{given} \ \ \lambda \in [0, \tilde{\lambda}_A). \tag{58}$$

In the autarkic steady state, $w_{t+1} = w_t$ implies that $\Upsilon_A = \rho$ and $r_A = \rho[1 - \eta + \eta\mu_A]$. According to equation (54), $\frac{\partial \mu_A}{\partial \lambda} > 0$ for $\theta > \underline{\theta}$. Thus, $\frac{\partial r_A}{\partial \lambda} > 0$, $\frac{\partial w_A}{\partial \lambda} > 0$, $\frac{\partial \ln \chi_A}{\partial \ln \lambda} > 0$. Given $\lambda^* < \lambda < \tilde{\lambda}_A$, it holds that $Y_A^* < Y_A$, $\chi_A^* < \chi_A < 1$, and $r_A^* < r_A < \rho$. □

Proof of Lemma 3

Proof. For $w_t < \bar{w}$, the maximum possible share of investment in sector 2 is less than unity. In this case, sector 1 is active $K_{t+1}^1 > 0$ and the interest rate coupling gives $r_t = q_{t+1}^1$. Thus, the borrowing constraints are binding in North, $\mu_{t+1} = \frac{q_{t+1}^1}{q_{t+1}^2} = \frac{r_t}{q_{t+1}^2} = \mu_A^* < 1$ and $\delta_t = \tilde{\delta}_t < 1$ and $\zeta_{t+1} = \frac{1}{(\frac{1}{\delta_t} - 1)\mu^* + 1} \in (\delta_t, 1)$.

For $w_t \geq \bar{w}$, North abandons sector 1, $K^1_{t+1} = 0$, implying the interest rate decoupling from the rate of return in sector 1. The credit market competition implies the interest rate coupling with the rate of return in sector 2. Combine them with equations (20), (1) and (3)-(6) to get equation (27). $\square$

Proof of Lemma 4

Proof. Following the approach described in the proof of lemma 2 and taking into account equation (27), one can derive the interest rate as the piecewise function of national income characterized by equations (31)-(32). $\square$

Proof of Proposition 2

Proof. For $\theta \in (\underline{\theta}, 1)$, there exists a unique, autarkic steady state. As the static gains from trade raise North's income, the law of motion for wage under free trade lies strictly above that under autarky. According to equations (29) and (30), the law of motion for wage in logarithm has a slope less than unity for $w_t > \bar{w}$. Thus, there must exist at least one stable steady state under free trade. In the following, I derive the threshold conditions for the steady-state properties under free trade.

First, derive the condition under which free trade leads to multiple steady states in North. If there exists an unstable steady state M, it must hold that $w_M \in (0, \bar{w})$ and

$$\frac{\partial w_{t+1}}{\partial w_t}|_{w_M} = \alpha + \alpha \frac{\frac{1-\theta}{\theta}}{\frac{\mu^*}{(1-\mu^*)\delta_M} + 1} > 1, \quad \Rightarrow \quad \frac{\alpha}{1-\alpha} - \frac{\theta}{1-\theta} > \frac{\theta}{1-\theta} \frac{\mu^*}{(1-\mu^*)\delta_M}. \tag{59}$$

For $\theta \geq \alpha$, condition (59) does not hold. In this case, the law of motion for wage in logarithm has the slope less than unity so that there exists a unique, stable steady state under free trade.

In the following, I focus on the case of $\theta \in (\underline{\theta}, \alpha)$. Combine condition (59) with $\delta_M = \left(\frac{w_M}{\bar{w}}\right)^{\frac{\theta}{1-\theta}} < 1$ to get an upper bound for μ^*,

$$\frac{\mu^*}{1-\mu^*} \frac{\$}{1-\$} < \delta_M < 1, \quad \Rightarrow \quad \mu^* \leq \hat{\mu}^*_T \equiv 1 - \$, \quad \text{where } \$ \equiv \frac{\frac{1}{\alpha} - 1}{\frac{1}{\theta} - 1} \in (\eta, 1). \tag{60}$$

According to equations (53)-(54), μ_A is an increasing function of λ. As South has the economic same structure as North, equations (53)-(54) also specify μ^*_A as an increasing function of $\lambda^* \in [0, \tilde{\lambda}_A)$. Combine (60) and (53) to get the upper bound for λ^*,

$$\lambda^* \leq \hat{\lambda}^*_T \equiv 1 - \left(\frac{Z}{\hat{Z}}\right)^{\rho(1-\theta)}, \quad \text{where } \hat{Z} \equiv (1-\$)^{\$-\eta}(1-\eta\$)^{1-\$} < 1. \tag{61}$$

In order to ensure $\hat{\lambda}^*_T \geq 0$ and the existence of multiple steady states, $Z \leq \hat{Z}$. $\hat{Z}$ is a function of θ, as shown by the curve between region M and UF in the right panel of figure 13.

Given $\{\theta, Z\}$ in region M of the right panel of figure 13, derive the condition in the $\{\lambda^*, \lambda\}$ space under which the law of motion for wage is tangent with the 45° line at steady state M,

$$\frac{\partial w_{t+1}}{\partial w_t}|_{w_M} = \alpha + \alpha \frac{\frac{1-\theta}{\theta}}{\frac{\mu^*}{(1-\mu^*)\delta_M} + 1} = 1, \quad \Rightarrow \quad \delta_M = \frac{\$}{1-\$} \frac{\mu^*}{(1-\mu^*)}. \tag{62}$$

Combine equations (18), (27) with (62) and get a threshold value of $\hat{\lambda}_T$ as a function of μ^*,

$$w_M = \left[\frac{1}{\rho}(\mu^*)^\eta \left(1 + \frac{1-\mu^*}{\mu^*}\delta_M\right)\right]^\rho = \bar{w}\delta_M^{\frac{\theta}{1-\theta}} = (1-\hat{\lambda}_T)^{\frac{1}{1-\theta}} \frac{\mathfrak{m}}{1-\theta} \left[\frac{\$}{1-\$} \frac{\mu^*}{(1-\mu^*)}\right]^{\frac{\theta}{1-\theta}} \tag{63}$$

$$\hat{\lambda}_T = 1 - \left\{ \frac{Z}{\left(\frac{\$}{(1-\mu^*)\eta}\right)^{\$} (\mu^*)^{\$-\eta}(1-\$)^{1-\$}} \right\}^{\rho(1-\theta)}. \tag{64}$$

For each $\lambda^* \in [0, \hat{\lambda}^*_T]$, use equation (53) to solve for μ^*_A and then combine $\mu^* = \mu^*_A$ with (64) to solve for $\hat{\lambda}_T$. The curve between region M and UFA in the left panel of figure 14 shows $\hat{\lambda}_T$ as a function of $\lambda^* \in [0, \hat{\lambda}^*_T]$.

Second, derive the condition under which North abandons sector 1 before reaching the stable steady state T. Given μ^*, use equation (27) to get,

$$w_T = \left[\frac{1}{\rho}(\mu^*)^{\eta-1}\right]^{\rho} \geq \bar{w} = \frac{\mathfrak{m}}{1-\theta}(1-\lambda)^{\frac{1}{1-\theta}}, \;\Rightarrow\; \lambda \geq \tilde{\lambda}_T = 1 - \left[\frac{Z\eta^{\mathfrak{s}}}{(\mu^*)^{1-\eta}}\right]^{\rho(1-\theta)}. \tag{65}$$

Given $\lambda \in [0, \tilde{\lambda}_A]$, combine equation (65) with (55) to get an upper bound for μ^*,

$$\tilde{\lambda}_T \leq \tilde{\lambda}_A \quad \Rightarrow \quad \mu^* < \tilde{\mu}_T^* = \eta^{\frac{\mathfrak{s}}{(1-\eta)}}. \tag{66}$$

By the same logic as mentioned above, combine (66) and (53) to get an upper bound for λ^*,

$$\lambda^* < \tilde{\lambda}_T^* \equiv 1 - \left(\frac{Z}{\tilde{Z}}\right)^{\rho(1-\theta)}, \;\text{ where }\; \tilde{Z} \equiv \eta^{\frac{\mathfrak{s}(\mathfrak{s}-\eta)}{1-\eta}}(1-\eta+\eta^{\frac{\mathfrak{s}}{1-\eta}+1})^{1-\mathfrak{s}}. \tag{67}$$

In order to ensure $\tilde{\lambda}_T^* \geq 0$, $Z \leq \tilde{Z}$. $\tilde{Z}$ is a function of θ and shown by the curve between region UF and UP in the right panel of figure 13.

Given $\{\theta, Z\}$ in region UF and M in the right panel of figure 13, for each $\lambda^* \in [0, \tilde{\lambda}_T^*]$, use equation (53) to solve for μ_A^* and then combine $\mu^* = \mu_A^*$ with (65) to solve for $\tilde{\lambda}_T$. For $\{\theta, Z\}$ in region UF of the right panel of figure 13, the curve between region UF2 and UP2 in the middle panel of figure 14 shows $\tilde{\lambda}_T$ as a function of $\lambda^* \in [0, \hat{\lambda}_T^*]$; for $\{\theta, Z\}$ in region M of the right panel of figure 13, the curve denoted by $\tilde{\lambda}_T$ in the middle panel of figure 14 shows $\tilde{\lambda}_T$ as a function of $\lambda^* \in [0, \hat{\lambda}_T^*]$.

□