Trade-Induced Production Upgrading and the Global Imbalances

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Abstract

This paper analyzes theoretically how trade integration affects financial capital flows in a world with heterogeneous financial development. In the presence of financial frictions and sector-specific minimum investment requirements, the static gains from trade trigger the cross-sector investment reallocation along the extensive margin, which may allow the more financially developed country (North) to gradually offshore low-return production activities and eventually upgrade fully in high-return activities. Trade-induced production upgrading in North leads to the international interest rate reversal and re-reversal sequentially, which first reduces and then amplifies the upstream financial flows. This finding complements that of Antras and Caballero (2009)

By weakening the extensive-margin responses of sectoral investment, wealth inequality dampens the dynamic gains from trade and the degree of production upgrading.

Keywords: dynamic gains from trade, extensive margin, financial frictions, global imbalances, production upgrading, wealth inequality

JEL Classification: F11, F41

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The recent globalization has two prominent features. First, emerging economies (especially China and other emerging Asian economies) have witnessed large current account surpluses, while advanced economies (notably, the United States and the United Kingdom) have incurred persistent current account deficits in the past two decades. Accordingly, financial flows have been upstream from the poor to the rich countries (Prasad, Rajan, and Subramanian, 2006). Such global imbalances differ significantly from the previous episodes and are in stark contrast to the predictions of neoclassical theories. Second, technological progress and the removal of trade barriers have dramatically reduced the costs of transportation, communication, and coordination since the 1990s, which accelerates international fragmentation of production (Baldwin, 2016; Grossman and Rossi-Hansberg, 2006; Timmer et al., 2014). Nowadays, international production, trade, and investments are organized within global value chains, which have transformed the landscape of global production networks and influenced trade policies.

Trade and financial flows have been analyzed separately in the literature and economists have put little research effort on their interactions. Recent works suggest that such a separation is not always innocuous (Eaton et al., 2016; Ghironi and Melitz, 2005; Jin, 2012; Ju, Shi, and Wei, 2014). In a seminal contribution to this literature, Antras and Caballero (2009) show that, if North (the rich countries) is more financially developed than South (the poor countries), the autarkic interest rate is higher in North than in South, which explains upstream financial capital flows as an equilibrium outcome. Deepening trade integration reverses the North-South interest rate differential, which leads to downstream financial flows and resolves the global imbalances. However, the recent global imbalances actually emerged and accelerated in parallel to worldwide trade liberalization, which seems at odds with their predictions.

Many emerging economies (e.g., the Asian Tigers, Brazil, Mexico, Argentina) that were integrated into the world trading system before 1990 were relatively small in terms of their respective share in global trade and did not have substantial impacts on the industrial composition in advanced economies. It is perfectly legitimate that Antras and Caballero (2009) treat South as a small open economy and study the impacts of trade integration on the return to capital there. However, a key feature of the recent wave of globalization is the rise of emerging economies in global trade (Hanson, 2012). In particular, China’s share of world merchandise exports rose from 1.8% in 1990 to 8.6% in 2007 and its impacts on advanced economies are unprecedented in terms of scale and speed. For example, China’s exports to the U.S. tripled within six years after its entry to the World Trade Organization (2000-2006), which reduced U.S. manufacturing price indexes by 7.6% (Amiti et al., 2018). Although imports from China significantly reduced the U.S. manufacturing jobs (Acemoglu et al., 2016; Autor, Dorn, and Hanson, 2016), the global export expansion of U.S. products also created a considerable number of jobs (Feenstra, Ma, and Xu, 2017). Overall, the U.S. manufacturing sector witnessed a decline in jobs by 19% and a rise in labor productivity by 34% between 2000 and 2007, implying that the jobs offshored are relatively low-value-added. Similar patterns exist in the United Kingdom, France, and some other European countries (Bloom, Draca, and van Reenen, 2016; 1

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1The non-farm business sector and the non-financial corporations in the U.S. only recorded the rise in labor productivity by 20% and 18%, respectively. Data Source: Manufacturing, Durable Manufacturing and Non-durable Manufacturing Sectors, the U.S. Bureau of Labor Statistics, retrieved on 26 January 2019.
Malgouyres, 2017; Pessoa, 2018). Obviously, one should not ignore the substantial impacts of the recent trade globalization on the industrial composition in advanced economies.

This paper proposes that trade-driven production upgrading in North has profound implications to financial capital flows. There are three major findings. First, whether deepening trade integration resolves or amplifies the global imbalances depends on the degree of specialization in North. If trade induces North to partially offshore low-return production activities, the interest rate in North is determined by low-return activities and falls below the world interest rate, a result Antras and Caballero (2009) call “the interest rate reversal”. Thus, North witnesses financial capital outflows, which helps resolve the global imbalances. However, if North eventually upgrades fully in high-return activities, the interest rate in North is determined by high-return activities and exceeds the world interest rate, a result I call “the interest rate re-reversal”. Then, North may receive even more financial capital inflows than before, which amplifies the global imbalances. It complements the findings of Antras and Caballero (2009).

Second, whether trade eventually leads to full specialization in North depends on the magnitude of the static and dynamic gains from trade. This paper features the extensive margin as the channel through which trade triggers cross-sector investment reallocation and leads to production upgrading in North. Due to financial frictions, agents can finance only a fraction of their investment with loans and use their net wealth to cover the rest. Thus, only those with sufficiently high net wealth can meet the minimum investment requirements (MIR, hereafter) and invest in a particular sector, while others have to lend out their net wealth and/or invest in the sector with no MIR. For simplicity, the sector with the MIR is called sector 1 and those who can invest there are called entrepreneurs, while the sector with no MIR is called sector 0.

If the level of financial development is sufficiently low, the borrowing constraints are so tight that the mass of entrepreneurs is inefficiently low and so is the investment in sector 1, while the opposite applies to sector 0. The cross-sector investment distortion implies that the output price and the investment return are higher in sector 1 than in sector 0. As North is more financially developed than South, it has comparative advantage in sector 1. If the country size of South is sufficiently large (e.g., China and emerging Asia), the world prices under trade integration are determined to a large extent by the autarkic prices in South. The larger the North-South differences in financial development, the larger the North-South differences in the autarkic prices, the larger the static gains from trade for North.

The static gains raises national income and allows more agents in North to overcome the MIR and invest in sector 1, which enhances North’s comparative advantage and allows North to specializes further towards sector 1 in the next period. Here, trade triggers a dynamic, virtuous cycle through which the rise in national income and the cross-sector investment reallocation reinforce each other over time. The reallocation effect depends negatively on the degree of wealth inequality in North and the level of financial development in South. The reallocation effect competes with the decreasing marginal-revenue-of-capital effect, which determines the dynamic gains from trade. If the static and the dynamic gains are sufficiently large, North may fully upgrade to the high-return sector (sector 1) in the long run.

In the two-sector setting, the interest rate re-reversal occurs when North fully offshores the low-return sector. In subsection 4.1 and appendix A.2, I analyze a multi-sector setting where
sectors are ranked in terms of the MIR. One may interpret “sectors” as production stages or tasks. If the borrowing constraints are sufficiently tight, the tasks with the higher MIR are subject to more severe underinvestment, which leads to the higher rates of return. If cross-task heterogeneity in the MIR and cross-country heterogeneity in financial development are sufficiently large, trade may allow North to sequentially offshore the low-MIR, low-return tasks and upgrade to the high-MIR, high-return tasks over time. Once the lowest-return task is offshored, the interest rate is decoupled from (coupled with) the rate of return in this task (the next lowest-return task). Here, *trade may create an inverse sawtooth pattern for the interest rate in North so that the interest rate reversal and re-reversal may occur recurrently over time.*

**Related Literature**  This paper belongs to the literature explaining the global imbalances as an equilibrium response to cross-country differences in financial development (Caballero, Farhi, and Gourinchas, 2008; Gourinchas and Rey, 2014; Ju and Wei, 2010; Mendoza, Quadrini, and Rios-Rull, 2009; von Hagen and Zhang, 2014). Although the U.S. started running current account deficits in the early 1980s (Faruqee and Lee, 2009), upstream financial flows emerged at the world level from the late 1990s (Gourinchas and Rey, 2014), parallel to the recent wave of trade globalization. Given cross-country differences in financial development, I propose a mechanism through which deepening trade integration has a non-monotonic effects on the return to capital in North. As a result, trade-driven production upgrading in North has non-trivial implications to the patterns of international financial flows.

Obstfeld and Rogoff (2001) argue that trade costs have the potential to resolve qualitatively several puzzles in international macroeconomics, which is confirmed by Eaton et al. (2016) in a quantitative, multi-country model. Alessandria and Choi (2019) find that two-thirds of the fluctuations in the U.S. trade balance are driven by changes in trade barriers. Reyes-Heroles (2017) finds that the substantial decline in trade frictions contributes significantly to global trade imbalances. The substantial declines in trade costs deepen trade integration, which may induce North to fully specialize in high-return activities and widen North-South interest rate differential in my model. Hence, upstream financial flows allow North to run trade deficits, consistent with the findings of Reyes-Heroles (2017).

Grossman and Rossi-Hansberg (2006) and Baldwin (2016) show that the recent wave of offshoring and supply-chain trade has changed the composition of world trade and transformed the industrial structures in advanced economies as well as in emerging economies. In subsection 4.2, I put my mechanism in the context of supply-chain trade and explain intuitively that trade-driven production upgrading along the value chain may amplify the global imbalances.

Baldwin (1992) argues that trade has large dynamic effects on output and welfare via human and physical capital accumulation. In a quantitative, multi-country model with capital accumulation, Ravikumar, Santacreu, and Sposi (2018) find that the dynamic gains from trade liberalization are substantially larger than the static gains. My paper offers a mechanism through which wealth distribution in North and the level of financial development in South jointly determine the dynamic gains from trade and the degree of production upgrading in North.

Income and wealth inequality has been rising almost everywhere across the world (Atkinson, Piketty, and Saez, 2011; Piketty, 2014). The literature has documented extensively the impacts of trade on income inequality (Dabla-Norris et al., 2015; Helpman et al., 2017; Jau-
motte, Lall, and Papageorgiou, 2013), while the impacts of inequality on trade patterns are addressed mainly by the demand-based theory featuring non-homothetic preferences (Fajgelbaum, Grossman, and Helpman, 2011; Fajgelbaum and Khandelwal, 2016; Fieler, 2011). My paper offers a supply-based theory in which wealth inequality weakens the extensive-margin responses of sectoral investment. Accordingly, the dynamic gains from trade are dampened and so is the degree of production upgrading in North.

Kletzer and Bardhan (1987) show that better access to capital becomes a source of comparative advantage. It was then followed by a strand of theoretical literature on financial development and international trade (Antras and Caballero, 2009; Beck, 2002; Chesnokova, 2007; Ju and Wei, 2005). Matsuyama (2005) introduces sector-specific borrowing constraints in a static model and shows that trade allows the rich (poor) country to fully specialize in the sector with tighter (looser) borrowing constraints. Wynne (2005) argues that a country’s wealth can be a determinant of comparative advantage when access to credit differs across sectors, i.e., wealthier nations exhibit a comprehensive advantage towards goods produced in sectors facing more severe financial frictions. Ju and Wei (2011) point out that, in the countries with low-quality institutions, the quality of financial system is an independent source of comparative advantage. Building upon this literature, I analyze the joint determination of trade and financial flows.

Jin (2012) integrates factor-proportions-based trade and financial flows in an OLG model and shows that capital tends to flow to countries that are more specialized in capital-intensive industries. Ju, Shi, and Wei (2014) embed two tradeable sectors with different factor intensity in a small-open-economy setting and show that the current account adjustment to exogenous shocks depends on factor market flexibility. Instead of introducing sector-specific financial frictions or factor intensity, I focus on a real friction, i.e., the sector-specific MIR. See the Introduction of Zhang (2017) for a detailed literature review on the MIR.

The rest of the paper is structured as follows. Section 1 sets up the model and section 2 analyzes the autarkic equilibrium. Section 3 discusses the dynamics of aggregate income and the interest rate under trade integration. Section 4 discusses the robustness of my results under alternative settings. Section 5 concludes with some final remarks. Online appendix includes relevant materials and technical proofs.

1 The Model Setting

The world consists of two countries, North and South. I first describe the economic setting of North and then use the asterisk superscript to denote the variables and the parameters in South.

Every period, a continuum of agents are born in North and they live for two period, young and old. In each generation, the population size is constant at one and agents are indexed by $j \in [0, 1]$. Agent $j$ is endowed with $l_j = (1 - \theta) \epsilon_j$ units of labor when young, where $\epsilon_j \in (1, \infty)$ follows the Pareto distribution, $G(\epsilon_j) = 1 - \epsilon_j^{-\theta}$ and $\theta \in (0, 1)$. As agents only consume when

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2 The inverse of $\theta$ is the tail index of the Pareto distribution. The literature to feature the income and wealth distribution (Atkinson, Piketty, and Saez, 2011; Gabaix, 2009; Jones, 2015). The top tail of income distribution is very well approximated by a Pareto distribution (Kuznets and Jenks, 1953; Piketty and Saez, 2003).
old, they supply the labor endowment inelastically to the market and hence, the aggregate labor supply is constant at \( L = \int_{1}^{\infty} l_j \, dG(e_j) = 1 \) every period.

There are two sectors in each country, indexed by \( s \in \{0, 1\} \). In period \( t \), \( K_{s,t} \) units of physical capital and \( L_{s,t} \) units of labor are hired in sector \( s \) to produce \( Y_{s,t} \) units of good \( s \). Physical capital is sector-specific and fully depreciates after use. Sectoral outputs are tradable, while labor and physical capital are not. \( V_{0,t} \) units of good 0 and \( V_{1,t} \) units of good 1 are combined to produce \( Y_t \) units of final goods which are used for consumption and investment.\(^3\) The markets for goods and productive factors are competitive and the final goods serve as the numeraire. There is no uncertainty in the model economy. Let \( w_t \) denote the wage rate. Let \( p_{s,t} \) and \( q_{s,t} \) denote respectively the price of good \( s \) and the rental price of capital in sector \( s \).

\[
Y_{s,t} = \left( \frac{K_{s,t}}{\alpha} \right)^{\alpha} \left( \frac{L_{s,t}}{1 - \alpha} \right)^{1 - \alpha}, \quad q_{s,t} K_{s,t} = \alpha p_{s,t} Y_{s,t}, \quad w_t L_{s,t} = (1 - \alpha)p_{s,t} Y_{s,t}, \quad \tag{1}
\]

\[
Y_t = \left( \frac{V_{1,t}}{\eta} \right)^{\eta} \left( \frac{V_{0,t}}{1 - \eta} \right)^{1 - \eta}, \quad p_{t,1} V_{1,t} = \eta Y_t, \quad p_{0,t} V_{0,t} = (1 - \eta)Y_t. \quad \tag{2}
\]

where \( \alpha, \eta \in (0, 1) \). In order to feature explicitly the impacts of trade on the interest rate, I exclude international capital flows so that domestic investment is funded by domestic saving and national income is equal to domestic output.

In period \( t \), domestic saving \( w_t L \) is invested in the two sectors, which yields \( K_{s,t+1} \) units of physical capital for sector \( s \) in period \( t + 1 \). Let \( \delta_t \) and \( \zeta_{t+1} \) denote respectively the fractions of domestic saving (\( w_t L \)) and labor (\( L \)) allocated for the production of good 1 in period \( t + 1 \).

\[
K_{1,t+1} = \delta_t w_t L, \quad L_{1,t+1} = \zeta_{t+1} L, \quad \tag{3}
\]

\[
K_{0,t+1} = (1 - \delta_t) w_t L, \quad L_{0,t+1} = (1 - \zeta_{t+1}) L. \quad \tag{4}
\]

Let \( \chi_t \equiv \frac{p_{0,t}}{p_{1,t}} \) and \( \mu_t \equiv \frac{q_{0,t}}{q_{1,t}} \) denote the sectoral price ratio and the sectoral rental-price-of-capital ratio, respectively. Combine equations (1) and (3)-(4) to get

\[
\chi_t = \mu_t^{\alpha}, \quad \delta_t = \frac{\zeta_{t+1}}{(1 - \zeta_{t+1})(\frac{1}{\mu_t} - 1)}. \quad \tag{5}
\]

Due to the frictionless labor market and cross-sector labor mobility, the wage rate equals the marginal revenue of labor across sectors. If the sectoral investment were also frictionless, the marginal revenue of capital (MRK, hereafter) would also equalize across sectors, \( q_{0,t+1} = q_{1,t+1} \) or equivalently \( \mu_{t+1} = 1 \). Then, the output price would equalize across sectors \( \chi_{t+1} = 1 \), while domestic saving and labor would be allocated in equal proportions across sectors \( \delta_t = \zeta_{t+1} \).

As shown below, individual investment is subject to financial frictions and sector-specific MIR, which may distort the sectoral investment and the two price ratios.

Agent \( j \) born in period \( t \) has three options to save its labor income \( n_{j,t} = w_t l_j \), i.e., investing in the two sectors and lending to the credit market. By investing \( k_{j,0,t+1} > 0 \) units of final goods in period \( t \), the agent gets \( k_{j,0,t+1} \) units of physical capital for sector 0 in period \( t + 1 \) and hence, the rate of return to its investment is \( q_{0,t+1} \). It gets the same one-to-one investment output

\[3\] Under autarky, domestic absorption is equal to domestic output at the sectoral level, \( V_{s,t} = Y_{s,t} \), while it is not the case with trade flows, \( V_{s,t} \neq Y_{s,t} \).
for sector 1, if its investment size meets the MIR, \( k_{j,1,t+1} \geq m > 0 \); otherwise, it gets zero output. Except for the MIR, the two sectors are symmetric in terms of the investment and the production technologies. Due to financial frictions, agent \( j \) can finance only up to a fraction \( \lambda \) of its investment in sector 1 with loans and has to cover the gap with its own funds \( n_{j,t} \),

\[
k_{j,1,t+1} - n_{j,t} \leq \lambda k_{j,1,t+1},
\]

where \( \lambda \in (0, 1) \) measures the level of financial development.\(^4\) If the borrowing constraints are binding in equilibrium, the mass of agents who can meet the MIR is inefficiently low and so is the total investment in sector 1, while the opposite applies to sector 0. The cross-sector investment distortion leads to the sectoral rate-of-return wedge, \( q_{1,t+1} > q_{0,t+1} \).

Under autarky, sector 0 is always active and hence, \( K_{0,t+1} > 0 \) holds strictly. As all agents can freely invest in sector 0 and lend to the credit market, the two options are perfect substitutes and the interest rate is coupled with the rate of return in sector 0. However, as shown in section 3, trade may induce North to offshore sector 0 and specialize fully in sector 1, i.e., \( K_{0,t+1} = 0 \). In that case, the interest rate is coupled with the rate of return in sector 1.

\[
r_t = \begin{cases} q_{0,t+1}, & \text{if } K_{0,t+1} > 0 \text{ or equivalently } \delta_t < 1; \\ q_{1,t+1}, & \text{if } K_{0,t+1} = 0 \text{ or equivalently } \delta_t = 1. \end{cases}
\]

Consider first the case where \( K_{0,t+1} > 0 \) and the borrowing constraints are binding, \( q_{1,t+1} > r_t = q_{0,t+1} \). Given the rate-of-return spread, agent \( j \) invests its labor income in sector 1 and borrows to the limit, as long as it can meet the MIR,

\[
k_{j,1,t+1} = \frac{n_{j,t}}{1 - \lambda} = w_t \frac{1 - \theta}{1 - \lambda} \epsilon_j \geq m, \quad \Rightarrow \quad \epsilon_j \geq \epsilon_t \equiv \frac{m}{w_t} \frac{1 - \lambda}{1 - \theta}.
\]

Thus, the agents with \( \epsilon_j \geq \epsilon_t \) can meet the MIR and are called entrepreneurs, with the mass \( \tau_t = \epsilon_t^{-\frac{1}{\theta}} \). When old, they get the investment return, repay the debt, and consume the rest, \( c_{j,t+1}^e = q_{1,t+1} k_{j,1,t+1} - r_t (k_{j,1,t+1} - n_{j,t}) \). The agents with \( \epsilon_j \in [1, \epsilon_t) \) cannot meet the MIR and are called households. When young, they invest \( k_{j,0,t+1} \) in sector 0 and lend out \( n_{j,t} - k_{j,0,t+1} \) for the same rate of return \( q_{0,t+1} = r_t \); when old, they consume \( c_{j,t}^h = n_{j,t} r_t \).

In period \( t \), the fraction of domestic saving allocated in sector 1 is

\[
\delta_t = \int_{c_t^e}^{c_t^h} \frac{k_{j,1,t+1} dG(\epsilon)}{w_t L} = \frac{\epsilon_t^{-\frac{1}{\theta}}}{1 - \lambda} = \frac{\tau_t^{-\frac{1}{\theta}}}{1 - \lambda} = \left( \frac{w_t}{w} \frac{1 - \lambda}{1 - \theta} \right)^{\frac{1}{\theta}}, \quad \text{where } w \equiv \frac{m(1 - \lambda)}{1 - \theta}.
\]

Due to the binding borrowing constraints and sector-specific MIR, the mass of entrepreneurs \((\tau_t)\) becomes endogenous and so does the fraction of domestic investment in sector 1 \((\delta_t)\).

The markets for credit, sector-specific capital, labor, and final goods clear,

\[
\int_{c_t^e}^{c_t^h} (k_{j,1,t+1} - n_{j,t}) dG(\epsilon) = \int_{c_t^h}^{c_t^h} (n_{j,t} - k_{j,0,t+1}) dG(\epsilon),
\]

\[
K_{1,t} = \delta_{t-1} w_{t-1} L, \quad K_{0,t} = (1 - \delta_{t-1}) w_{t-1} L, \quad L_{0,t} + L_{1,t} = L,
\]

\[
\int_{c_t^e}^{c_t^h} c_{j,t}^e dG(\epsilon) + \int_{c_t^e}^{c_t^h} c_{j,t}^h dG(\epsilon) + K_{1,t+1} + K_{0,t+1} = Y_t.
\]

\(^4\)Antras and Caballero (2009) give some micro-foundations for this borrowing constraint. The credit markets would be perfect and the borrowing constraints would be slack if \( \lambda = 1 \).
In the case of $q_{1,t+1} = q_{0,t+1}$, the sectoral rate of return equalizes and hence, individual agents who can meet the MIR do not have the incentive to invest their entire labor income in sector 1 or to borrow to the limit. Despite the indeterminacy at the individual level, the equilibrium allocation can be solved easily at the aggregate level. See the proof of proposition 1 for detailed characterization.

Under autarky, the markets for sectoral outputs clear domestically,

$$V_{s,t} = Y_{s,t}.$$  \hfill (13)

**Definition 1.** Under autarky, a market equilibrium is a set of choices of agents $\{n_{j,t}, k_{j,s,t+1}\}$ and aggregate variables $\{Y_t, Y_{s,t}, K_{s,t}, L_{s,t}, V_{s,t}, q_{s,t}, w_t, r_t, \varepsilon_t\}$, satisfying equations (1)-(2), (7)-(8), (10)-(11), and (13), where $s \in \{0, 1\}$.\(^5\)

Trade integration aligns the sectoral price ratio to the world level and trade is balanced,

$$\chi_t = \mu_t, \quad \chi_t^* (V_{0,t} - Y_{0,t}) + (V_{1,t} - Y_{1,t}) = 0.$$  \hfill (14)

**Definition 2.** Under free trade, a market equilibrium is a set of choices of agents $\{n_{j,t}, k_{j,s,t+1}\}$ and aggregate variables $\{Y_t, Y_{s,t}, K_{s,t}, L_{s,t}, V_{s,t}, q_{s,t}, w_t, r_t, \varepsilon_t\}$, satisfying equations (1)-(2), (7)-(8), and (10)-(11), where $s \in \{0, 1\}$, while the sectoral price ratio $\chi_t$ is determined at the world level by equation (14).

Without international factor mobility, domestic investment is financed by domestic saving in period $t$, $\sum_{s=0}^{1} K_{s,t+1} = w_t L$. According to equations (1)-(2), the return to aggregate investment is $\sum_{s=0}^{1} q_{s,t+1} K_{s,t+1} = \frac{\alpha}{1 - \alpha} w_{t+1} L$ in period $t + 1$. Define the social rate of return,

$$\Upsilon_t = \delta_t q_{1,t+1} + (1 - \delta_t) q_{0,t+1} = \rho \frac{w_{t+1}}{w_t}, \quad \text{where} \quad \rho \equiv \frac{\alpha}{1 - \alpha}.$$  \hfill (15)

According to equations (1)-(2), $w_t = (1 - \alpha) \frac{\Upsilon_t}{T}$ holds, implying that the wage rate can serve as a proxy of national income. From now on, I use the law of motion for wage to analyze the model dynamics and the steady-state properties.

## 2 The Autarkic Equilibrium

Under autarky, the sectoral share of labor input is efficient and equal to the sectoral share in the final goods production, $\zeta_{t+1} = \eta$. In the benchmark case of perfect credit markets $\lambda = 1$, the borrowing constraints are slack. Thus, the rate of return equalizes across sectors and so does the output price, $\chi_{t+1} = \mu_{t+1} = 1$. Combine it with equation (5) to get $\delta_t = \zeta_{t+1} = \eta$. The Cobb-Douglas production function implies the decreasing marginal revenue of capital (DMRK, hereafter). Thus, the law of motion for wage is concave,

$$w_{t+1} = \left( \frac{w_t}{\rho} \right)^\alpha, \quad \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \frac{(1 - \alpha)}{\rho} < 1, \quad \text{where} \quad \rho \equiv \frac{\alpha}{1 - \alpha}.$$  \hfill (16)

\(^5\)According to the Walras’ law, if the markets for labor, credit, sector-specific physical capital, and sectoral outputs clear, the final good market must clear. In this sense, equation (12) is redundant.
The DMRK is a convergence force that drives North to a unique steady state with the wage rate \( w_B = \rho^{-\rho} \). Subscript \( B \) refers to the benchmark case. The smaller the \( \alpha \), the stronger the DMRK effect, the faster the convergence.

Consider then the case of imperfect credit markets \( \lambda \in (0,1) \). Let \( \tilde{\delta}_t \equiv \min\{\left(\frac{w_t}{\rho}\right)^{-\frac{1}{\rho-1}}, 1\} \) denote the maximum possible share of domestic investment in sector 1 when all entrepreneurs borrow and invest to the limit. If \( w_t \geq \tilde{w}_A \equiv \eta^{\frac{1}{\rho-\sigma}} \bar{w} \), the efficient allocation is feasible, \( \tilde{\delta}_t \geq \eta \). In equilibrium, the investment in sector 1 is efficient \( \delta_t = \zeta_{t+1} = \eta \leq \tilde{\delta}_t \), the rate of return equalizes across sectors \( \mu_{t+1} = 1 \), and the borrowing constraints are slack. The income dynamics are still characterized by equation (16).

If \( w_t < \tilde{w}_A \), the efficient allocation is infeasible, \( \tilde{\delta}_t < \eta \). In equilibrium, the investment in sector 1 is below the efficient level, \( \delta_t = \tilde{\delta}_t < \zeta_{t+1} = \eta \), which leads to the sectoral rate-of-return wedge \( \mu_{t+1} < 1 \). Thus, the borrowing constraints are binding. Given \( w_t \), the income dynamics are characterized by \( \{\delta_t, \mu_{t+1}, w_{t+1}\} \) satisfying equations (9), (17)-(18),\(^6\)

\[
\begin{align*}
w_{t+1} &= \left(\frac{w_t}{\rho} \Gamma_t^{\alpha}\right)^{\alpha}, \quad \text{where} \quad \Gamma_t \equiv \frac{\mu_{t+1}^{\eta - 1}}{1 - \eta (1 - \mu_{t+1})} < 1, \quad \text{and} \quad \frac{\partial \Gamma_t}{\partial \mu_{t+1}} > 0, \quad (17) \\
\mu_{t+1} &= \frac{\eta - 1}{\frac{\delta_t - 1}{\delta_t}} \Rightarrow \frac{\partial \mu_{t+1}}{\partial \delta_t} = \frac{\partial \mu_{t+1}}{\partial \delta_t} > 0, \quad (18) \\
\frac{\partial \ln w_{t+1}}{\partial \ln w_t} &= \left[1 - \left(1 - \alpha\right)\right] \left[1 + \frac{\partial \ln \Gamma_t}{\partial \ln \mu_{t+1}} \frac{\partial \ln \mu_{t+1}}{\partial \ln \delta_t} \frac{\partial \ln \delta_t}{\partial \ln w_t}\right] \quad \text{DMRK effect} \\
&= \left[1 - \left(1 - \alpha\right)\right] \left[1 + \eta (1 - \mu_{t+1}) \left(\frac{1}{\theta} - 1\right)\right] \quad \text{investment composition effect} \vphantom{\frac{1}{2}} \tag{19}
\end{align*}
\]

\( \Gamma_t \) measures the aggregate investment efficiency. The distortion on sectoral investment \( (\mu_{t+1} < 1) \) leads to the aggregate inefficiency \( (\Gamma_t < 1) \).\(^7\) A rise in national income affects domestic investment in two ways. First, it allows agents to invest more so that sectoral investment rises on the intensive margin and the DMRK effect dampens the initial income change. Second, it allows more agents to meet the MIR and hence, domestic investment shifts towards sector 1 on the extensive margin, which improves the aggregate efficiency and amplifies the initial income change. The DMRK effect depends negatively on the capital share \( (\alpha) \), while the investment composition effect depends negatively on the degree of wealth inequality \( (\theta) \). If the former dominates the latter at any steady state, there is a unique steady steady. Let \( X_A \) denote the steady-state value of variable \( X_t \) under autarky.

**Proposition 1.** Let \( \tilde{\theta} \equiv \frac{\alpha}{\frac{\alpha}{1 + (\alpha - 1) (1 - \eta)} + \eta} \times \frac{1}{\eta} \), \( Z \equiv \frac{1}{\rho (1 - \eta) \Gamma^\theta} \), and \( \tilde{\lambda}_A \equiv 1 - Z \rho^{(1-\theta)} \).

For \( \tilde{\theta} \in [\tilde{\theta}, 1) \) and \( \lambda \in [0, \tilde{\lambda}_A) \), there is a unique, stable steady state under autarky where the borrowing constraints are binding and sectoral investment is inefficient.

Figure 1 shows \( \tilde{\theta} \) in the \( \{\alpha, \theta\} \) space and \( \tilde{\lambda}_A \) in the \( \{\lambda, Z\} \) space. For \( \{\alpha, \theta\} \) in region \( U \) of the left panel and \( \{\lambda, Z\} \) in region \( UB \) (US) of the right panel, there exists a unique, autarkic steady state where \( w_A < \tilde{w}_A (w_A > \tilde{w}_A) \) and the borrowing constraints are binding (slack).\(^8\)

\(^6\)See the proof of proposition 1 for technical derivation.

\(^7\)In the benchmark case, \( \mu_{t+1} = 1 \) and the aggregate efficiency index is constant at \( \Gamma_t = 1 \).

\(^8\)See the proof of Proposition 1 for the laws of motion for wage in case UB and US, respectively. The proof of
North and South are inherently identical, except that North is more financially developed. Besides, the population share of North in the world economy is negligible.

**Assumption 1.** \( \theta \in (\bar{\theta}, 1), 0 < \lambda^* < \lambda < \bar{\lambda}_A, \) and \( \frac{L}{L_{t+T}} \to 0. \)

Given assumption 1, there is a unique, autarkic steady state in each country and the borrowing constraints are binding there. The higher the level of financial development, the larger the mass of entrepreneurs, the less distorted the sectoral investment, the closer the sectoral price ratio to its efficient level and the higher the income level. Thus, the income level is higher in North than in South \( w_A > w_A^* \), while good 1 (0) is cheaper in North (South), \( \chi_A^* < \chi_A < 1 \). Here, the level of financial development becomes a determinant of comparative advantage.

Under autarky, the interest rate is coupled with the rate of return in sector 0,

\[
 r_t = q_{0,t+1} = \Upsilon_t[1 - \eta(1 - \mu_{t+1})] < \Upsilon_t. \tag{20}
\]

In the autarkic steady state, the social rate of return is constant at \( \Upsilon_A = \rho \) and the interest rate depends on the sectoral rate-of-return ratio \( \mu_A \). The higher the level of financial development, the smaller the sectoral investment distortion and the sectoral rate-of-return wedge, the higher the rate of return in sector 0, the higher the interest rate.

**Lemma 1.** \( w_A^* < w_A < w_B, \chi_A^* < \chi_A < 1, \mu_A^* < \mu_A < 1, \) and \( r_A^* < r_A < \rho. \)

Start from the autarkic steady state. If agents can borrow/lend abroad, financial flows are upstream, which widens the initial North-South income gap. Here, the “global imbalances” arise as an equilibrium response to cross-country differences in financial development.

### 3 Can Trade Integration Resolve the Global Imbalances?

The world economy is initially at the autarkic steady state. In period 0, the two countries announce that goods 0 and 1 will be freely traded from period 1 on. Due to its negligible world

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9 By definition, the composite parameter \( Z \) is independent of the level of financial development and the population size. Thus, \( Z \) takes the same value for both countries.

10 The final good is tradeable and serves as the vehicle for international borrowing/lending.

11 If free trade is announced and implemented in period 0, \( \chi_0 \) is immediately aligned with \( \chi_A^* \), which affects the investment return of those born in period \( t = -1 \) unexpectedly. In the two-period, OLG setting, announcing the
population share, North is a small open economy. Under trade integration, the world prices are determined by the autarkic prices in South, $\chi^* = \chi_A$. It seems extreme to model North as a small open economy, especially if North refers to countries like the United States. However, the rise of emerging economies in global trade significantly affects the domestic price level and industrial composition in advanced economies, as highlighted at the beginning of this paper. This paper focuses on the qualitative impacts of trade integration on the aggregate dynamics in North. Modeling North as a small open economy substantially simplifies the analysis.\textsuperscript{12}

3.1 Specialization as An Amplification Mechanism

Under trade integration, the sectoral price ratio in North falls to the world level, $\chi_t = \chi^* = \chi_A^* < \chi_A < 1$, and so does the sectoral rental-price-of-capital ratio, $\mu_t = \chi_t^\frac{1}{\alpha} = (\chi_A^*)^{\frac{1}{\alpha}} = \mu_A^* < \mu_A < 1$. As long as $\delta_t < 1$, sector 0 is active and, according to equations (5) and (7), the borrowing constraints are binding so that $\delta_t$ is determined by equation (9).

Lemma 2. For $w_1 < \bar{w}$ ($w_1 \geq \bar{w}$), North specializes partially (fully) in sector 1, $\delta_t < 1$ ($\delta_t = 1$).

The law of motion for wage is piecewise and characterized by equation (21).

$$w_{t+1} = \left(\frac{w_t}{\rho_t} \Gamma_t\right)^\alpha, \text{ where } \Gamma_t = (\mu_t)^n \left[1 + \left(\frac{1 - \mu_t}{\mu_t}\right)\delta_t\right] \text{ and } \delta_t = \min\left\{\frac{w_t}{\bar{w}}^{\frac{1-\eta}{\eta}}, 1\right\}. \quad (21)$$

In period 0, North is in the autarkic steady state, $w_0 = w_A < \bar{w}_A < \bar{w}$. The announcement of free trade does not change the sectoral investment, $\delta_0 = \left(\frac{w_0}{\bar{w}}\right)^{\frac{1-\eta}{\eta}} = \delta_A$. In period 1, trade induces North to specialize towards sector 1 along the labor margin. The larger the heterogeneity in financial development $\lambda - \lambda^*$, the larger the international sectoral price differential $\chi_A^* - \chi^*$, the larger the static gains from trade.

$$\frac{\partial \ln w_1}{\partial \ln \chi_1} = -\frac{\chi_A^* - (\chi^*)^{\frac{1}{\alpha}}}{\chi_A^{\frac{1}{\alpha}} + (\chi^*)^{\frac{1}{\alpha}}} < 0. \quad (22)$$

The static gains from trade $w_1 > w_0$ allow more agents to overcome the MIR and invest in sector 1, $\delta_1 = \left(\frac{w_1}{\bar{w}}\right)^{\frac{1-\eta}{\eta}} > \delta_0$, which enhances North’s comparative advantage and further induces North to specialize towards sector 1 along the labor margin in period 2. Here, trade triggers a dynamic, virtuous cycle through which the rise in national income and the cross-sector resource reallocation reinforce each other over time.

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = [1 - (1 - \alpha)] \frac{\partial \ln \Gamma_t}{\partial \ln \delta_t} \frac{\partial \ln \delta_t}{\partial \ln w_t}, \quad (23)$$

where $\frac{\partial \ln \Gamma_t}{\partial \ln \delta_t} = \frac{1}{1 + \frac{\mu_t^\alpha}{(1-\mu_t)\delta_t}}$, and $\frac{\partial \ln \delta_t}{\partial \ln w_t} = \begin{cases} \frac{1}{\sigma} - 1, & \text{if } w_t < \bar{w}; \\ 0, & \text{if } w_t \geq \bar{w}. \end{cases} \quad (24)$

\textsuperscript{12}Alternatively, one can introduce a parameter representing the relative country size and consider explicitly the impacts of North on the world prices. It only complicates the analysis, without changing my results qualitatively.
The reallocation effect depends on two factors. First, the lower the $\theta$, the smaller the wealth inequality, the larger the mass of entrepreneurs responds to the gains from trade, the larger the cross-sector investment reallocation $\frac{\partial \ln \delta_t}{\partial \ln w_t}$. Second, the lower the $\lambda^*$, the lower the $\mu^*_A$, the larger the sectoral rate-of-return differential in South, the larger North gains from specializing in the high-return sector $\frac{\partial \ln \Gamma_t}{\partial \ln \delta_t}$. As long as the reallocation effect dominates the DMRK effect, the virtuous cycle goes on over time until the mass of entrepreneurs becomes so large that sector 0 vanishes in North, i.e., $w_t \geq \bar{w}$ and $\delta_t = 1$. From then on, aggregate income is driven by the DMRK effect. Let $X_T$ denote the steady-state value of variable $X_t$ under trade integration.

**Proposition 2.** If the degree of wealth inequality in North and the level of financial development in South are sufficiently low and the North-South heterogeneity in financial development is sufficiently large, trade integration induces North to converge towards a unique steady state where North specializes fully in sector 1.

Let case UF1 denote the case described in proposition 2. In figure 2, the left panel of shows the threshold value $\tilde{\theta}$ in the $\{\theta, Z\}$ space, while the right panel shows the threshold values $\tilde{\lambda}_T$ and $\lambda^*_T$ in the $\{\lambda^*, \lambda\}$ space. Case UF1 arises for $\{\theta, Z, \lambda^*, \lambda\}$ in region UF1, i.e., $\theta \in [\theta, \tilde{\theta})$, $\lambda^* < \tilde{\lambda}_T$, and $\lambda > \bar{\lambda}_T$. In the following, I focus on case UF1 and analyze the dynamic responses of national income and the interest rate under trade integration.$^{13}$

![Figure 2: Threshold Values for the Free-Trade Equilibrium](image)

In figure 3, the solid (dashed) curve in the left panel shows the law of motion for wage under trade (autarky), while the solid curve in the right panel shows the impulse responses of wage under trade.$^{14}$ In a numerical example, for period $t \leq 6$, $w_t < \bar{w}$ and $\delta_t < 1$ hold, implying that sector 0 is active and the income dynamics are driven by the reallocation effect and the DMRK effect. For period $t \geq 7$, $w_t > \bar{w}$ and $\delta_t = 1$ hold, implying that North fully offshores sector 0 and the income dynamics are driven purely by the DMRK effect.

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$^{13}$Appendix A.1 offers a detailed description of the dynamic and steady-state properties under trade integration.

$^{14}$For illustration clarity, the axes in the left panel are scaled in logarithm and so is the vertical axis in the right panel, while the horizontal axis in the right panel shows the period index. This scaling applies to figures 4, 6, 7.
3.2 Interest Rate Reversal and Re-reversal under Trade Integration

Lemma 3. Under trade integration, the interest rate in North is a piecewise function of the income level, depending on whether the low-return sector is fully offshored.

1.) For \( w_t \in (0, \bar{w}) \), the mass of entrepreneurs is so small that their total debt capacity is less than the entire household saving. In equilibrium, sector 0 is still active, the interest rate is aligned with the rate of return in sector 0, and the borrowing constraints are binding.

\[
    r_t = q_{0,t+1} = \frac{\gamma_t}{1 + \left(\frac{1 - \mu^*}{\mu^*}\right) \delta_t} < \Upsilon_t, \quad \frac{\partial \ln r_t}{\partial \ln w_t} = - \left(1 - \alpha\right) \left[1 + \frac{\frac{1}{\eta} - 1}{1 + \left(\frac{1 - \mu^*}{\mu^*}\right) \delta_t}\right] < 0. \tag{25}
\]

2.) For \( w_t \geq \bar{w} \), the mass of entrepreneurs is so large that their total debt capacity exceeds the entire household saving. In equilibrium, sector 0 vanishes, the interest rate is aligned with the rate of return in sector 1, and the borrowing constraints are slack.

\[
    r_t = q_{1,t+1} = \frac{\gamma_t}{w_t}, \quad \frac{\partial \ln r_t}{\partial \ln w_t} = - \left(1 - \alpha\right) < 0. \tag{26}
\]

In figure 4, the left panel shows the interest rate as a piecewise function of the income level, while the right panel shows the impulse responses of the interest rate under trade integration.\(^{15}\)

In period 1, sector 0 is active \( \delta_0 = \delta_A < \eta \) and trade-driven labor reallocation \( \zeta_1 > \zeta_A \) raises the capital-labor ratio in sector 0, which reduces marginal product of capital in sector 0.

\[
    K_{0,1} = \frac{1 - \delta_0}{1 - \zeta_1} w_0 L_0 = \left[1 + \delta_0 \left(\frac{1}{\mu^*} - 1\right)\right] w_0 > K_{0,0} = \left[1 + \delta_A \left(\frac{1}{\mu_A} - 1\right)\right] w_A.
\]

Besides, the price of good 0 also falls in period 1. Overall, the MRK in sector 0 falls in period 1 and so does the interest rate in period 0.

\[
    q_{0,1} = p_{0,1} \left(\frac{K_{0,1}}{\rho L_{0,1}}\right)^{\alpha-1} < q_{0,0} = p_{0,0} \left(\frac{K_{0,0}}{\rho L_{0,0}}\right)^{\alpha-1} \Rightarrow r_0 < r_A.
\]

---

\(^{15}\)As noted in footnote 14, the axes of the left panel are in logarithm and so is the vertical axis in the right panel.
Furthermore, the reallocation effect dominates the financial development effect so that the interest rate in North is even lower than the world interest rate in period 0, \( r_0 < r_A^* < r_A \), a result Antras and Caballero (2009) calls the interest rate reversal.

In period \( t \in \{1, \ldots, 6\} \), \( w_t < \bar{w} \) and sector 0 is active. As the sectoral input of labor is frictionless and that of investment is frictional, trade triggers a disproportional resource reallocation, which raises the capital-labor share rises and reduces the marginal product of capital in sector 0. Besides, the gains from trade raise national income and the size of domestic investment. Overall, the reallocation effect and the investment size effect jointly reduce the MRK in sector 0. Thus, the interest rate also falls over time.

\[
\frac{K_{0,t+1}}{L_{0,t+1}} = \frac{1 - \delta_t}{1 - \zeta_{t+1}} \frac{w_t L}{L} = \left[ 1 + \delta_t \left( \frac{1}{\mu^*} - 1 \right) \right] \text{relocation effect} \quad \text{investment size effect} \quad w_t .
\]

In period \( t = 7 \), \( w_t > \bar{w} \) and North fully offshores sector 0. Thus, the interest rate is decoupled from (coupled with) the rate of return to sector 0 (1), shown by an interest rate jump in the right panel of figure 4. I call it the interest rate re-reversal. From then on, the interest rate is driven by the DMRK effect and falls over time until North reaches the new steady state \( T \) where the interest rate is even higher than its autarkic value, \( r_T = \Upsilon_T = \rho > r_A > r_A^* \).

To sum up, the world economy witnesses sequentially the interest rate reversal and re-reversal when North converges from the autarkic to the new steady state. Whether deepening trade integration resolves the global imbalances depends on the timing of financial integration.\textsuperscript{16}

\textbf{Proposition 3.} In case UF1, the interest rate in North has a non-monotonic pattern along the convergence path under trade integration. In the new steady state, \( r_T > r_A > r_A^* \).

For the parameter configurations outside region UF1 of figure 2, the static and the dynamic gains are too weak to ensure that North fully offshores sector 0 in the long run. Hence, trade integration just leads to the interest rate reversal, as predicted by Antras and Caballero (2009).

\textsuperscript{16}In the example analyzed above, trade integration resolves (amplifies) the global imbalances if financial flows are allowed in period \( t \in \{0, \ldots, 6\} \) (\( t \geq 7 \)).
In my model, trade integration may have non-monotonic impacts on industrial composition in North, which then affects the patterns of financial flows. Undoubtedly, various factors, e.g., globalization, technology progress, industrial policies, and etc., may induce North to upgrade along the value chain. No matter what the causes are, if the shifts in industrial composition change fundamentally the way the interest rate is determined, the global imbalances may arise.

4 Extensions and Discussions

4.1 Recurrent Interest Rate Reversal and Re-Reversal

In this subsection, I show in a three-sector setting that the interest rate reversal and re-reversal arise recurrently if trade allows North to sequentially offshore low-return sectors and upgrade gradually to high-return sectors. See appendix A.2 for the analysis of a multi-sector model.

Suppose that there are three sectors in each country, indexed by \( s \in \{0, 1, 2\} \) and ranked in ascending order with respect to the MIR, i.e., \( m_0 < m_1 < m_2 \). Let \( \delta_{s,t} \) and \( \zeta_{s,t+1} \) denote the respective shares of domestic saving and labor input used for producing good \( s \) in period \( t + 1 \), \( Y_{s,t+1} = \left( \frac{\delta_{s,t} m_0 L}{\alpha} \right)^{1-\alpha} \left( \frac{\zeta_{s,t+1} L}{1-\alpha} \right)^{1-\alpha} \). Sectoral outputs are combined for the production of final goods, \( Y_{t+1} = \Pi_{s=0}^{2} \left( \frac{V_{s,t+1}}{\eta_s} \right)^{\eta_s} \), where \( \eta_s \) denotes the sectoral share in the final good production and \( \sum_{s=0}^{2} \eta_s = 1 \). Let \( \xi_{s,t} \equiv \frac{m_s}{w_t \bar{L}} \). According to equation (8), the agents with \( \epsilon_j \geq \xi_{s,t} \) can meet the MIR and invest in sector \( v \geq s \).

4.1.1 The Autarkic Equilibrium

Under autarky, the sectoral labor input is efficient at \( \zeta_{s,t+1} = \eta_s \). If credit markets were perfect, the same would also apply to the sectoral investment, \( \delta_{s,t} = \zeta_{s,t+1} = \eta_s \) and the rate of return would equalize across sectors, \( q_{s,t+1} = r_t \). If credit markets are imperfect, the binding borrowing constraints may depress the mass of investors in the MIR sectors, which distorts the sectoral investment and leads to the sectoral rate-of-return differentials.

Assumption 2. \( \gamma = \frac{m_1}{m_2} < \left( \frac{\eta_2}{\eta_1 + \eta_2} \right)^{\theta} \).

Let \( \bar{w}_s \equiv \frac{m_s(1-\lambda)}{1-\theta}, \bar{w}_{1,A} \equiv \bar{w}_1 \left[ 1 + \frac{m_0}{m_1} \left( 1 - \gamma \frac{1-\theta}{\theta} \right) \right]^{-\frac{\theta}{1-\theta}} < \bar{w}_1 \), and \( \bar{w}_{2,A} \equiv \bar{w}_2 \eta_2^{\frac{\theta}{1-\theta}} < \bar{w}_2 \). For \( w_t < \bar{w}_{1,A} \), the mass of investors in sector 1 and 2 is inefficiently low and so is the aggregate credit demand. Thus, the rate-of-return spread, \( r_t < q_{s,t+1} \), arises in sector \( s \in \{1, 2\} \) and the borrowing constraints are binding there. The sectoral investment shares are

\[
\delta_{2,t} = \frac{\int_{z_{2,t}}^{\infty} \frac{(1-\theta) \epsilon_j w_t}{1-\lambda} dG(\epsilon_j)}{w_t L} = \left( \frac{w_t}{\bar{w}_2} \right)^{\frac{1-\theta}{\theta}},
\]

\[
\delta_{1,t} = \frac{\int_{z_{1,t}}^{\infty} \frac{(1-\theta) \epsilon_j w_t}{1-\lambda} dG(\epsilon_j)}{w_t L} = \left( \frac{w_t}{\bar{w}_1} \right)^{\frac{1-\theta}{\theta}} - \left( \frac{w_t}{\bar{w}_2} \right)^{\frac{1-\theta}{\theta}},
\]

\[
\delta_{0,t} = 1 - (\delta_{1,t} + \delta_{2,t}) = 1 - \left( \frac{w_t}{\bar{w}_1} \right)^{\frac{1-\theta}{\theta}}.
\]
Assumption 2 and \( w_t < \bar{w}_{1,A} \) jointly ensure the descending sectoral capital-labor ratio, which implies the ascending pattern of the sectoral rate of return. Under autarky, all sectors are active and the interest rate is determined by the lowest-return sector (sector 0).

\[
\frac{\delta_{0,t}}{\eta_0} > \frac{\delta_{1,t}}{\eta_1} > \frac{\delta_{2,t}}{\eta_2}, \quad \Rightarrow \quad r_t = q_{0,t+1} < q_{1,t+1} < q_{2,t+1}.
\]

(30)

A rise in national income raises the investment shares of sector 1 and 2 in equal proportions through the extensive margin, while that of sector 0 falls.\(^{17}\)

Let \( \theta_3 \equiv \frac{\alpha}{1 + (1 - \theta) \eta_0}, \quad \Pi_3 \equiv \left\{ \begin{array}{ll} \frac{1 - \theta}{\eta_2} \left[ \frac{\gamma - \theta}{\eta_1} \right]^{1 - \eta_2} & \\
\frac{\eta_2 - (1 - \theta)}{\eta_1 (1 - \gamma)} & \
\end{array} \right\} \), and \( Z_3 \equiv \frac{1}{\left[ \frac{\eta_2 (\gamma - \theta)}{\eta_1} \right]^{1 - \theta}} \).

**Proposition 4.** For \( \theta \in (\theta_3, 1) \), \( Z_3 < \frac{1}{\theta} \), and \( \lambda < \tilde{\lambda}_3 \equiv 1 - (\Pi Z_3)^{\rho(1 - \theta)} \), there is a unique, autarkic steady state where the borrowing constraints are binding in sector 1 and 2.

![Figure 5: Threshold Values for the Autarkic Steady State in the 3-Sector Setting](image_url)

In figure 5, the left panel shows \( \theta_3 \) in the \( \{\alpha, \theta\} \) space, while the right panel shows \( \tilde{\lambda}_3 \) in the \( \{\lambda, Z_3\} \) space. For \( \{\alpha, \theta\} \) in region U of the left panel and \( \{\lambda, Z_3\} \) in region UB12 of the right panel, there is a unique, autarkic steady state with the binding borrowing constraints in sector 1 and 2. Let \( X_{s,A} \) denote the autarkic steady-state value of variable \( X_{s,t} \). Let \( \chi_{s,t} \equiv \frac{p_{s,t}}{p_{2,t}} \) and \( \mu_{s,t} \equiv \frac{q_{s,t}}{q_{2,t}} \) denote the relative output price and the relative rental price of capital in sector \( s \) with respect to sector 2.

**Lemma 4.** Given \( \theta > \theta_3 \) and \( Z_3 < \frac{1}{\theta} \), for \( 0 < \lambda^* < \lambda < \tilde{\lambda}_3 \), \( \chi^*_0,A < \chi_{0,A} < \chi^*_1,A = \chi_{1,A} < 1 \), \( \mu^*_0,A < \mu_{0,A} < \mu^*_1,A = \mu_{1,A} < 1 \), and \( r^*_A < r_A < \rho \).

\(^{17}\)For \( w_t \in (\bar{w}_{1,A}, \bar{w}_{2,A}) \), the mass of investors in sector 2 is still so low that the rate-of-return spread \( r_t < q_{2,t+1} \) keeps the borrowing constraints binding there. Thus, the investment share of sector 2 is still specified by equation (27). Meanwhile, the mass of investors in sector 1 is so large that the rate-of-return spread vanishes \( q_{1,t+1} = r_t = q_{0,t+1} \) and the borrowing constraints are slack there. Thus, the capital-labor ratio equalizes in sector 0 and 1 at \( \delta_{1,t} = \delta_{2,t} = \frac{1 - \delta_{3,t}}{\eta_0 + \eta_1} \). A rise in national income raises the investment share of sector 2 through the extensive margin, while that of sector 0 and 1 falls in equal proportions.

For \( w_t > \bar{w}_{2,A} \), the mass of investors in sector 1 and 2 are so large that the rate of return equalizes across the three sectors and the borrowing constraints are slack \( q_{s,t+1} = r_t \) for \( s \in \{0, 1, 2\} \). Thus, the sectoral investment shares are efficient at \( \delta_{s,t} = \eta_s \), independent of the change in national income.
At the autarkic steady state, the borrowing constraints are binding in sector $s \in \{1, 2\}$. According to equations (27)-(28), the relative capital-labor ratio in sector 1 is independent of the level of financial development, $\frac{\delta_{1,A}}{q_{s,t}} = \left( \gamma^{-\frac{1-a}{a}} - 1 \right) \frac{w_{s,t}}{\eta_{s,t}} > 1$. Thus, $\mu_{1,A} = \mu_{1,A}^* < 1$ and $\chi_{1,A} = \chi_{1,A}^* < 1$. Besides, the higher the level of financial development, the less severe the overinvestment in sector $0$, the higher the relative output price and the relative rate of return in sector $0$, i.e., $\chi_{0,A}^* < \chi_{0,A}$ and $\mu_{0,A}^* < \mu_{0,A}$. Thus, South has a comparative advantage in sector $0$, while North has a comparative advantage in sector $s \in \{1, 2\}$.

4.1.2 The Free-Trade Equilibrium

In period 0, the world economy is at the autarkic steady state with $\mu_{0,A}^* < \mu_{1,A}^* < \mu_{2,A} = 1$, and it is announced that sectoral outputs will be freely traded from period 1 on. From period $t = 1$ on, the relative sectoral prices in North are aligned to the world levels, $\chi_{s,t} = \chi_{s,A}^*$, and so are its relative sectoral rates of return, $\mu_{s,t} = \mu_{s,A}^*$. Thus, the sectoral rate of return is strictly ascending in North, $q_{0,t+1} = q_{2,t+1} \mu_{0,A}^* < q_{1,t+1} = q_{2,t+1} \mu_{1,A}^* < q_{2,t+1}$ and hence, agents in North always invest in the sector with the highest MIR that they can afford.

Similar as in the two-sector setting, the static gains from trade trigger a dynamic, virtuous cycle through which the rise in national income and the cross-sector investment reallocation along the extensive margin reinforce each other over time.

For $w_t < \bar{w}_1$, the mass of investors in sector 1 and 2 is so low that they cannot borrow and invest the entire domestic saving. Thus, sector 0 is still active and the interest rate is coupled with the rate of return there, $r_t = q_{0,t+1}$. Given the ascending sectoral rates of return, the borrowing constraints are binding in sector 1 and 2. Thus, the sectoral investment shares are still featured by equations (27)-(29).

For $w_t \in (\bar{w}_1, \bar{w}_2)$, the mass of investors in sector 1 and 2 is so high that the entire domestic saving is invested in sector 1 and 2, while sector 0 is inactive, $\delta_{0,t} = 0$. The interest rate is coupled with the rate of return in sector 1 and the borrowing constraints are slack there, while the mass of investors in sector 2 is still so low that the borrowing constraints are binding there, $r_t = q_{1,t+1} < q_{2,t+1}$. Then, $\delta_{2,t}$ is still featured by equation (27), while $\delta_{1,t} = 1 - \delta_{2,t}$.

For $w_t > \bar{w}_2$, the mass of investors in sector 2 is so large that they fully absorb domestic saving $\delta_{2,t} = 1$, while other sectors are inactive $\delta_{0,t} = \delta_{1,t} = 0$.

Similar as in the two-sector setting, whether trade leads to the interest rate re-reversal depends critically on the magnitude of the static and the dynamic gains from trade, which depends positively on the heterogeneity in financial development ($\lambda - \lambda^*$) and negatively on the level of financial development in South ($\lambda^*$) and the degree of wealth inequality in North ($\theta$).

Proposition 5. Trade may induce North to offshore sequentially the low-MIR, low-return sectors and upgrade to the high-MIR, high-return sectors. Along the convergence path, the interest rate has an inverse sawtooth wave pattern with respect to national income.

For the moderate values of $\lambda - \lambda^*$, $\lambda^*$, and $\theta$, figure 6 shows the dynamics of national income and the interest rate in North, where the axis notations are identical as in figures 3 and 4. Given $w_0 = w_A < \bar{w}_1$, North specializes partially towards sector 1 and 2 in period $t = 1$, which leads
Figure 6: The Case of Moderate Heterogeneity in Financial Development

to the interest rate reversal in period 0, \( r_0 < r_A^* < r_A \), by the logic as mentioned in subsection 3.2. Then, the rise in national income reduces the interest rate in North until period \( t = 20 \). In period \( t = 21 \), North fully offshores sector 0, leading to the interest rate re-reversal, \( r_{21} > r_A^* \). Eventually, North converges to the new steady state with \( w_T \in (\bar{w}_1, \bar{w}_2) \) and \( r_T > r_A^* \).

For a sufficiently large heterogeneity in financial development and the sufficiently low values of \( \lambda^* \) and \( \theta \), figure 7 shows the dynamics of national income and the interest rate in North. In comparison with the previous case, the static and the dynamic gains from trade are much larger, which allows North to converge eventually to a new steady state with \( w_T > \bar{w}_2 \). During the convergence process, North witnesses the interest rate reversal and re-reversal twice, when North fully offshores sector 0 and 1 sequentially in period 6 and 26, respectively.

The interest rate reversal and re-reversal can be recurrent if trade allows North to sequentially offshore the low-return sectors and fully upgrade to the high-return sectors over time.

### 4.2 Supply-Chain Trade and the Global Imbalances

In this subsection, I use the mechanism featured above to explain intuitively that the rise of supply-chain trade may contribute to the global imbalances.
In manufacturing industries, upstream activities (such as R&D, product design, or the manufacturing of key parts and components) and downstream activities (such as marketing, brand building, and customer service) constitute a large share of value-added, while the intermediate production stages (such as component fabrication and final assembly) account for a small value-added share (Kimura, 2003). Stan Shih, the founder of Acer, introduced the smile curve to feature such an “U-shaped” value-added pattern along the production chain (Baldwin, Ito, and Sato, 2014). Compared with those of upstream and downstream activities, the value-added shares of fabrication and assembly activities have declined substantially in the OECD countries since the 1970s (Baldwin and Lopez-Gonzalez, 2015; Gereffi, 1999; Koopman, Wang, and Wei, 2014). These facts can be justified in my model as follows.

Fabrication and assembly are involved intensively with standardized, routine tasks that require mainly the input of tangible investment, while upstream and downstream activities are more involved with knowledge-intensive, non-routine tasks that require mostly the input of intangibles. Compared to tangible investment, intangibles are subject to higher MIR and more severe financial frictions. In my model, investment can be tangible or intangible, while “sectors” can be interpreted broadly as production stages or tasks. My model predicts that the output price and the investment return are higher in upstream and downstream activities than
in fabrication and assembly. One may use the smile curve to feature the rate-of-return pattern across production stages.  

Before moving on to the smile curve analysis, I first distinguish two ways of ranking tasks in the production chain. Suppose that capital $K_s$ and labor $L_s$ are hired to complete various tasks, where subscript $s$ denotes the task index. In the left panel of figure 8, the rightward pointing arrow shows the direction of the production process and tasks are ranked in sequential order, which is common in the literature of global value chain (Antras and Chor, 2013). As my model features cross-task heterogeneity in the MIR and the rate of return, I rank tasks in ascending order in terms of the MIR, as shown by the upward pointing arrow in the right panel. A task may show up in different positions in the two panels. For example, if the production process starts with R&D which require the largest MIR, the R&D shows up as “Task a” in the left panel and “Task 4” in the right panel; if assembly activities take place in the middle of the production process and do not require the MIR, the assembly activities show up as “Task c” in the left panel and “Task 0” in the right panel.

Without loss of generality, I assume that fabrication and assembly activities do not require the MIR, while upstream and downstream activities require roughly the same MIR. Thus, the former are regarded as “task 0”, while the latter as “task 1” in my two-sector setting.

Let us use the smile curve to feature the sectoral rate-of-return pattern. As advanced economies (North) are more developed than emerging economies (South) in financing intangibles (Corrado et al., 2013), the sectoral rate-of-return differential is smaller and the smile curve is flatter in North than in South. In the upper panel of figure 9, variables in North (South) are denoted without (with) the asterisk superscript. The interest rate $r$ is coupled with the MRK in the lowest-return production stages/tasks that are active, $r = \min\{MRK_s || K_s > 0\}$. At the autarkic steady state, the interest rate is higher in North than in South, $r_A > r^*_A$. If allowed, financial flows are upstream from South to North. Besides, North (South) has a comparative advantage in upstream and downstream (fabrication and assembly) activities.

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18 The original smile curve shows the value-added pattern along the production chain. Since the production stages/tasks that account for the high value-added shares usually have the high MRK, I use the smile curve to show the MRK pattern along the value chain.

19 Although tasks may differ in many other dimensions (e.g., capital intensity, riskiness), I focus only on the task-specific MIR in this paper.

20 One can further disaggregate upstream and downstream activities into multiple stages, according to their respective MIR. Then, one just follows the analysis in the multi-sector setting as shown in appendix A.2.
Consider North as a small open economy. In the presence of high trade costs, the incentive of offshoring is small and a marginal decline in trade costs only allows North to offshore part of fabrication and assembly activities, leading to a “falling jaw” of its smile curve, as shown in the middle panel of figure 9. It is consistent with the dynamics of value-added shares in OECD countries since the 1970s (OECD, 2013). As the interest rate in North is still coupled with the MRK in fabrication and assembly, the interest rate reversal occurs, i.e., \( r_T = MRK_0 < r^*_A < r_A \), consistent with the prediction of Antras and Caballero (2009).

Since the 1990s, technological progress and world-wide economic liberalization have substantially reduced the costs of transportation, communication, and coordination, leading to the boom of supply-chain trade. Vertically and horizontally linked production stages/tasks have been increasingly conducted in different countries. In my model, a substantial decline in trade costs may allow North to offshore all fabrication/assembly and specialize fully in upstream/downstream activities. If so, the smile curve in North witnesses a “missing jaw” and...
the interest rate is coupled with the MRK in upstream and downstream activities. As shown in the lower panel of figure 9, supply-chain trade leads to the interest rate re-reversal, which amplifies the initial interest rate differential, \( r_T = MRK_1 > r_A > r_A^* \).

4.3 Endogenous Responses of \( \delta_t \) through Two Channels

Let \( W_t^e \) and \( W_t \) denote respectively entrepreneurial wealth and national wealth. In the case of the binding borrowing constraints, the fraction of domestic investment in sector 1, \( \delta_t = \frac{W_t^e}{W_t} = \frac{1}{1-\lambda} \frac{W_t^e}{W_t^e} \), is proportional to the leverage multiplier \( \frac{1}{1-\lambda} \) and the entrepreneurial wealth share \( \frac{W_t^e}{W_t} \). The former is constant by assumption, while the latter is higher if the mass of entrepreneurs is higher (the extensive margin) and/or if the average wealth of entrepreneurs rises relative to the national average (the intensive margin). As shown below, \( \delta_t \) may respond to the static gains from trade through these two margins.

In my model, agents live for two periods; each agent invests its entire labor income when young and consumes when old. Due to the absence of wealth accumulation at the individual level, agent \( j \)'s wealth relevant for investment is just its labor income, \( n_{j,t} = w_1 (1 - \theta) \epsilon_j \). One can decompose along two margins the impacts of the static gains from trade on the entrepreneurial wealth share in North, \( \lambda_t \).

\[
\frac{W_t^e}{W_t} - \frac{W_t^{e}}{W_t} = \frac{\int_{\xi_1}^{\infty} n_{j,1} dG(\epsilon_j)}{w_1 L} - \frac{\int_{\xi_1}^{\infty} n_{j,1} dG(\epsilon_j)}{w_A L} + \frac{\int_{\xi_1}^{\infty} n_{j,1} dG(\epsilon_j)}{w_1 L} \tag{31}
\]

Consider the agents who are born in period 1 and would meet the MIR at the autarkic steady state, i.e., \( \epsilon_j \geq \xi_A \). As The static gains from trade affect the wealth of these “existing” entrepreneurs, \(^{21}\) \( n_{j,1} = w_1 (1 - \theta) \epsilon_j \), and the national wealth, \( w_1 L \), in equal proportions through the wage rate, the national wealth share of “existing” entrepreneurs is unaffected and hence, the intensive margin is mute. Due to financial frictions and sector-specific MIR, the cutoff value \( \xi_t \) and the mass of entrepreneurs \( \tau_t = \xi_t^{-\frac{1}{2}} \) are endogenous. Thus, the static gains from trade raise \( \frac{W_t^e}{W_t} \) purely along the extensive margin, which then raises \( \delta_t = \frac{1}{1-\lambda} \frac{W_t^e}{W_t} \) and allows North to offshore sector 0.

In the static model of Antras and Caballero (2009), the extensive margin of entrepreneurial wealth is mute, due to two assumptions: (1) there is no MIR; (2) only a fixed mass \( \tau \) of agents can invest in the constrained sector and are called entrepreneurs. In the case of the homogeneous labor endowment, \( l_j = 1 \), one can embed this static model into the two-period OLG setting and the wealth share of entrepreneurs is constant at \( \frac{W_t^e}{W_t} = \frac{\tau w_1 L}{w_1 L} = \tau \). Under assumption 1 of Antras and Caballero (2009), the borrowing constraints are so tight that the investment share of the constrained sector is inefficiently low, \( \delta = \frac{\tau}{1-\lambda} < \eta \). As \( \delta \) is constant

^{21}Referring these agents as “existing” entrepreneurs is purely for ease of exposition. It may be misleading, because they are born in period 1 and did not “exist” in period 0.
and does not respond to the static gains from trade, the unconstrained sector is always active, 
\[ \frac{K_{0,t+1}}{w_0L} = 1 - \delta > 1 - \eta > 0 \] and the interest rate re-reversal never happens. It confirms that the endogenous extensive margin of entrepreneurial wealth in my two-period OLG model is key to the interest rate re-reversal.

Antras and Caballero (2009) embed their static model into a continuous-time setting with two key assumptions: (1) agents are born at a constant rate per unit of time and die at the same rate, (2) agents save all their (labor and investment) income and consume only when they die. Due to the law of large numbers, the mass of entrepreneurs is constant and hence, the extensive margin of entrepreneurial wealth is mute. Different from the two-period OLG setting, agents accumulate wealth over their lifetime. Due to the privilege of investing in the constrained sector at a higher rate of return, entrepreneurs accumulate wealth at a faster speed than others under autarky and hence, their wealth share \( \frac{W_t}{W_t} \) becomes endogenous on the intensive margin. By aligning the sectoral price ratio at the world level, trade raises (reduces) the sectoral rate-of-return differential in North (South), which induces entrepreneurs to accumulate wealth at a faster (slower) speed than in the autarkic steady state. In this case, trade affects the entrepreneurial wealth share along the intensive margin and \( \delta_t \) responds accordingly.

Antras and Caballero (2009) focus on the interest rate response to trade flows for the less financially developed country. As both sectors are always active there, the interest rate reversal always holds. They do not analyze explicitly whether and under what conditions trade can induce the more financially developed country to offshore the low-return sector.\(^{22}\) In contrast, I highlight explicitly the extensive margin as a key channel through which the entrepreneurial wealth share responds to the static gains and show that trade may lead to the interest rate re-reversal if it substantially changes the sectoral composition in the more financially developed country.\(^{23}\) This result complements the findings of Antras and Caballero (2009).

5 Conclusion

This paper highlights trade-driven production upgrading as a novel mechanism through which deepening trade integration may lead to the interest rate reversal and re-reversal over time in a world with heterogeneous financial development. Whether deepening trade integration resolves or amplifies upstream financial capital flows depends critically on the degree of production upgrading, which complements the findings of Antras and Caballero (2009).

One can interpret “sectors” in my model as tasks and apply this mechanism to supply-chain trade in a multi-task setting. Given the cross-task heterogeneity in the MIR, the interest rate reversal and re-reversal can be recurrent phenomena over time when North gradually offshores low-return activities and sequentially upgrades to high-return activities. This mechanism helps explain the parallel rises in supply-chain trade and the global imbalances in the recent decades.

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\(^{22}\)Proposition 2 and figure 2 specify the conditions for North to fully specialize in the high-return sector.

\(^{23}\)Allowing wealth accumulation at the individual level in my model would strengthen my results through the additional, positive intensive margin effect.
References


A.1 The Free-Trade Equilibrium

Given $\theta \in (\bar{\theta}, 1)$ and $\lambda^*, \lambda \in (0, \bar{\lambda}_A)$, there are four cases under free trade, $\chi^* = \chi^*_A$.

- **Case M:** free trade leads to multiple steady states in North, i.e., two stable steady states denoted by H and L, and one unstable steady state denoted by M. The steady states are ranked in terms of income level, $w_H > \bar{w} > w_M > w_L > w_A$.

- **Case UF1:** North converges to a unique steady state where it fully specializes in sector 1;

- **Case UP1:** North converges to a unique steady state where it partially specializes in sector 1;

- **Case UP0:** North converges to a unique steady state where it partially specializes in sector 0;

Define some threshold values:

$$
\hat{Z} \equiv (1 - g)\eta - \eta (1 - \eta g) 1^{-\alpha} \quad \text{and} \quad \hat{\lambda}^*_T \equiv 1 - \left( \frac{Z}{\bar{Z}} \right)^{\rho(1-\theta)}, \quad \text{where} \quad g \equiv \frac{\theta}{\rho(1-\theta)}
$$

$$
\tilde{Z} \equiv \eta \frac{(g-\eta)}{1-\eta} (1 - \eta + \eta g \eta (1-\alpha)) 1^{-\alpha} \quad \text{and} \quad \tilde{\lambda}^*_T \equiv 1 - \left( \frac{Z}{\bar{Z}} \right)^{\rho(1-\theta)}
$$

In the following, I first show the threshold conditions in the $\{\theta, Z, \lambda^*, \lambda\}$ space that characterize the four cases. Then, I show the law of motion for wage in each case.

**Case M:** under free trade, multiple steady states arise if four conditions are satisfied simultaneously,

- $\theta \in (\bar{\theta}, \alpha)$, i.e., $\{\alpha, \theta\}$ in region M of the upper-left panel of figure 10, and

- $Z < \hat{Z}$, i.e., $\{\theta, Z\}$ in region M of the upper-right panel of figure 10, and

- $\lambda^* < \hat{\lambda}^*_T$ and $\lambda \in (\hat{\lambda}^*_T, \tilde{\lambda}^*_T)$, i.e., $\{\lambda^*, \lambda\}$ in region M of the lower-left panel of figure 10.

**Case UF1:** section 3 focuses on case UF1 which arises if four conditions are satisfied jointly, i.e.,

- given $\theta \in (\bar{\theta}, 1)$ and $Z < \hat{Z}$, $\{\theta, Z\}$ are in region M of the upper-right panel of figure 10 and $\{\lambda^*, \lambda\}$ are in region UF1 of the lower-left panel of figure 10.

- given $\theta \in (\bar{\theta}, 1)$ and $Z \in (\tilde{Z}, \hat{Z})$, $\{\theta, Z\}$ are in region UF of the upper-right panel of figure 10 and $\{\lambda^*, \lambda\}$ are in region UF1 of the lower-middle panel of figure 10.

**Case UP1 and Case UP0:** given $\theta \in (\bar{\theta}, 1)$, free trade induces North to move from the autarkic steady state to a unique steady state where it specializes partially in sector 1 (0) if parameters are in region UP1 (UP0) of the lower panels of figure 10 which correspond to $\{\theta, Z\}$ in the three regions of the upper-right panel of figure 10.

In figure 11, the solid (dashed) curve in each panel shows the law of motion for wage under free trade (autarky) in the each case, respectively.

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24 The proof of proposition 2 provides the technical derivation for these threshold values.
A.2 The Multi-Sector Model

This subsection analyzes the $S$-sector model with $S \geq 2$. In period $t$, a fraction $\delta_{s,t}$ of domestic saving $w_tL$ is allocated in sector $s \in \{0, 1, ..., S - 1\}$, which yields $K_{s,t+1} = \delta_{s,t}w_tL$ units of capital in period $t + 1$. They are hired together with $\zeta_{s,t+1}L$ units of labor to produce $Y_{s,t+1}$ units of good $s$. $V_{s,t+1}$ units of good $s$ are combined with the goods from other sectors to produce $Y_{t+1}$ units of final goods which serves as the numeraire and is used for consumption and investment. Let $\eta_s$ denote the sectoral share in the production of final goods and $\sum_{s=0}^{S-1} \eta_s = 1$. Let $\gamma_{t+1} \equiv \left( \frac{w_tL}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{L}{1-\alpha} \right)^{1-\alpha}$ denote the maximum possible output in a particular sector if domestic saving and labor are fully hired in that sector.

$$Y_{s,t+1} = \left( \frac{\delta_{s,t}w_tL}{\alpha} \right)^{\alpha} \left( \frac{\zeta_{s,t+1}L}{1-\alpha} \right)^{1-\alpha} = \delta_{s,t}^{\alpha} \zeta_{s,t+1}^{1-\alpha} \gamma_{t+1}^{\eta_s}, \quad Y_{t+1} = \prod_{s=0}^{S-1} \left( \frac{V_{s,t+1}}{\eta_s} \right)^{\eta_s}. \quad (34)$$

$$Y_{s,t+1} = \left( \frac{w_{t+1}\zeta_{s,t+1}L\eta_s}{\alpha} \right)^{\frac{1}{1-\alpha}} = \frac{w_{t+1}\zeta_{s,t+1}L\eta_s}{\alpha} = Y_{t+1}. \quad (35)$$

In the absence of international mobility of labor and capital, use the domestic labor market clearing condition to derive the production possibility frontier (PPF, hereafter),

$$\zeta_{s,t+1} = \left( \frac{V_{s,t+1}}{\gamma_{t+1}} \right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad \sum_{s=0}^{S-1} \zeta_{s,t+1} = 1, \quad \Rightarrow \quad \sum_{s=0}^{S-1} \left( \frac{V_{s,t+1}}{\gamma_{t+1}} \right)^{\frac{1}{1-\alpha}} = \gamma_{t+1}^{\frac{1}{1-\alpha}},$$

which holds under autarky as well as under free trade.
Sectors are ranked in terms of the sector-specific MIR, \( m_0 \equiv 0, m_{s-1} < m_s \) for \( s \in \{1, ..., S - 1\} \). For notational simplicity, define \( m_S \equiv \infty \). For \( z \in \{0, ..., S - 1\} \), define two series of threshold values,

\[
\bar{w}_z = \frac{m_z}{1 - \theta} \left( 1 - \lambda \right)^{\frac{1}{1 - \gamma}}, \quad \text{and} \quad \bar{w}_{z,A} = \frac{\bar{w}_z}{1 + \sum_{s=1}^{z-1} \frac{\eta_s}{\eta_z} \left[ 1 - \left( \frac{m_s}{m_{s+1}} \right)^{\frac{1-\theta}{\theta}} \right]^{\frac{1}{1-\gamma}}}.
\]

Let \( a \equiv \min\{v : \delta_{v,t} > 0\} \) denote the lowest sector index among all active sectors. As all goods are essential for the production of final goods, \( a = 0 \) must hold under autarky. Under trade, North may abandon low-index sectors so that \( 0 \leq a \leq S - 1 \) holds.

### A.2.1 The Autarkic Equilibrium: \( a = 0 \)

**Assumption 3.** \( \frac{m_{s+1} \theta^s + m_{s+2} \theta^s - m_s \theta^s - m_{s+1} \theta^s}{m_s \theta^s - m_{s+1} \theta^s} < \frac{\eta_{s+1}}{\eta_s} \) holds for \( s \in \{1, ..., S - 2\} \).

**Lemma 5.** Under autarky, for \( w_t \in [\bar{w}_{z,A}, \bar{w}_{z+1,A}] \), the borrowing constraints are slack and the rate of return equalizes in sector \( s \in \{0, ..., z\} \), while the borrowing constraints are binding and the sectoral rate of return is ascending in sector \( s \in \{z + 1, ..., S - 1\} \).
Under autarky, the law of motion for wage is a piecewise function.\(^{25}\) For \(w_t \in [\bar{w}_{z,A}, \bar{w}_{z+1,A})\),

\[
w_{t+1} = \left(\frac{w_t \Gamma_t}{\rho}\right)^\alpha, \quad \Gamma_t = \left(\frac{\delta_{w,t}}{\eta_w}\right)^\eta_{w} \Pi_{v=z+1}^{S-1} \left(\frac{\delta_{v,t}}{\eta_v}\right)^{\eta_v}
\]

where \(\delta_{v,t} = w_t^{\frac{1-\theta}{\sigma}} \kappa_v\), \(\kappa_v \equiv \bar{w}_v^{\frac{1-\theta}{\sigma}} - \bar{w}_{v+1}^{\frac{1-\theta}{\sigma}}\) for \(v \in \{z + 1, ..., S - 1\}\),

\[
\delta_{w,t} = w_t^{\frac{1-\theta}{\sigma}} \kappa_{w,t}, \quad \kappa_{w,t} \equiv w_t^{\frac{1-\theta}{\sigma}} - \bar{w}_{z+1}^{\frac{1-\theta}{\sigma}}\]

Let us analyze the sectoral rate-of-return pattern. Define \(\mu_{s,t+1} \equiv \frac{q_{s,t+1}}{q_{S-1-t,i+1}}\) as the rate of return in sector \(s\) relative to that in sector \(S - 1\). Consider the case of \(w_t \in [\bar{w}_{z,A}, \bar{w}_{z+1,A})\).

- In sector \(s \in \{0, ..., z\}\), the borrowing constraints are slack. The rate of return equals among these sectors \(q_{s,t+1} = q_{z,t+1} = r_t\) and so does the capital-labor ratio. Combine them with (38)\(^{26}\)

\[
\mu_{s,t+1} = \mu_{z,t+1} = \mu_{w,t+1} = \frac{\delta_{s-1,t}}{\zeta_{s-1,t+1}^{1+0}} = \frac{-\frac{1-\theta}{\eta} \sum_{v=0}^{S} \eta_v}{m_{S-1} \frac{1-\theta}{\sigma} w_t - m_{z+1} \eta_{S-1}}.
\]

- In sector \(s \in \{z + 1, ..., S - 1\}\), the borrowing constraints are binding. Combine (36) and (38)

\[
\mu_{s,t+1} = \frac{\delta_{S-1,t}}{\zeta_{s,t+1}^{1+0}} = \frac{-\frac{1-\theta}{\eta} \sum_{v=0}^{S} \eta_v}{m_s - \frac{1-\theta}{\sigma} m_{S-1} \eta_{S-1}}.
\]

Assumption 3 ensures the ascending relative sectoral rate of return, \(\mu_{s,t+1} < \mu_{s+1,t+1}\).

\[\text{Figure 12: Patterns of Relative Sectoral Rate-of-Return}\]

Given \(w_t \in (\bar{w}_{z,A}, \bar{w}_{z+1,A})\), the left panel of figure 12 shows the relative sectoral rate-of-return, with the sector index on the horizontal axis.\(^{27}\) According to equations (39) and (40), \(\mu_{s,t+1}\) is constant at \(\mu_{zL,t+1}\) in sector \(s \in \{0, ..., z_L\}\) and ascending in sector \(s \in \{z_L + 1, ..., S - 1\}\).

- Given \(\lambda\), if the income level rises to be in the interval of \(w_t \in (\bar{w}_{zH,A}, \bar{w}_{zH+1,A})\), the relative rate of return in sector \(s \in \{z_H + 1, ..., S - 1\}\) does not change, while that in sector \(s \in \{0, ..., z_H\}\) shifts upwards and becomes flat, with \(\mu_{s,t+1} = \mu_{zH,t+1}\). See the right panel of figure 12.

\(^{25}\)See the proof of Lemma 5 for derivation.

\(^{26}\)As shown in the proof of Lemma 5, \(\delta_{s,t} = \sum_{v=0}^{z} \delta_{v,t}\) and \(\zeta_{s,t+1} = \sum_{v=0}^{z} \zeta_{v,t+1}\). The sectoral capital-labor ratio equalizes among sector \(s \in \{0, ..., z\}\), \(\zeta_{s,t+1} = \delta_{s,t} / \zeta_{s,t+1} = \delta_{s,t} / \zeta_{s,t+1}\).

\(^{27}\)The exact shape of the curve depends on the parameter values of \(m_s\) and \(\eta_s\).
Given $w_t$, a rise in the level of financial development $\lambda$ reduces the threshold values $\bar{w}_{z,A}$, which affects the relative sectoral rate of return in the same way as the rise in the income level.

The rise in $w_t$ and/or $\lambda$ allows each agent to raise the investment. In sector $s \in \{z_H + 1, \ldots, S - 1\}$, some agents who previously invest in sector $s - 1$ can overcome the MIR and invest in sector $s$, while some agents who previously invest in sector $s$ can overcome the MIR and invest in sector $s + 1$. Given the Pareto distribution of labor endowment, the sectoral investment share rises in equal proportions in these sectors and hence, the relative sectoral rate of return stays constant. Meanwhile, as domestic investment shifts towards sector $s \in \{z_H + 1, \ldots, S - 1\}$, the overinvestment problem in sector $s \in \{0, \ldots, z_L\}$ is ameliorated so that the relative rate of return in these sectors gets closer to the frictionless level.

Let $\tilde{\lambda}_z$ denote a series of threshold values, where $z \in \{0, \ldots, S - 1\}$.

**Proposition 6.** For $1 - \eta_0 > \frac{1 - \alpha}{\theta - 1}$ and $\lambda \in (\tilde{\lambda}_z, \tilde{\lambda}_{z+1})$, there exists a unique steady state under autarky with the steady-state wage rate $w_A \in [\bar{w}_{z,A}, \bar{w}_{z+1,A})$.

The world consists of two countries, North and South, which are inherently identical, except that North has a negligible population size $L \ll L^*$ and is more financially developed, $\lambda^* < \lambda < \tilde{\lambda}_{S-1}$. The two countries are initially at their respective autarkic steady state.

![Figure 13: Steady-State Patterns of the Relative Sectoral Output Prices under Autarky](image)

Let $\chi_{s,t} = \frac{p_{s,t}}{p_{S-1,t}}$ denote the relative output price in sector $s$ with respect to sector $S - 1$. Since $\chi_{s,t} = \mu_{s,t}$, the relative sectoral price has the same pattern as the relative sectoral rate of return. The solid (dashed) curve in figure 13 shows the steady-state pattern of the relative sectoral price in North (South). In North, with $w_A \in (\bar{w}_{z,A}, \bar{w}_{z+1,A})$, the borrowing constraints are slack in sector $s \in \{0, \ldots, z\}$ so that the relative sectoral price is equalized at $\chi_{s,A} = \chi_{z,A}$; the borrowing constraints are binding in sector $s \in \{z + 1, \ldots, S - 1\}$ and $\chi_{s,A}$ is ascending in the sector index. South is less financially developed and the autarkic steady-state wage is $w_A^* \in (\bar{w}_{z,A}^*, \bar{w}_{z+1,A}^*)$, where $z^* < z$. The relative sectoral price in South $\chi_{s,A}^*$ is identical as in North for sector $s \in \{z + 1, \ldots, S - 1\}$ and lower than in North for sector $s \in \{0, \ldots, z\}$. Thus, North has the comparative advantage in the sector with the binding borrowing constraints $s \in \{z + 1, \ldots, S - 1\}$, while South has the comparative advantage in sector $s \in \{0, \ldots, z\}$.

### A.2.2 The Free-Trade Equilibrium: $a \geq 0$

Given $L \ll L^*$, North is a small open economy and the world prices under trade integration are determined by the autarkic prices in South, $\chi_s^* = \chi_{s,A}^*$. Next, we analyze the impacts of free trade on

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28Equation (83) in the proof of proposition 6 specifies the exact form of $\tilde{\lambda}_z$. 

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the sectoral investment in North. Free trade directly aligns the relative sectoral prices to their respective world levels $\chi_{s,t+1} = \chi^*_s A^*$, which implicitly aligns the relative sectoral rates of return to their respective world levels, $\mu_{s,t+1} = \mu^*_s A^*$. Under free trade, the law of motion for wage is a piecewise function.

For $w_t \in (\bar{w}_z, \bar{w}_{z+1})$ and $z \in \{z^* + 1, \ldots, S - 1\}$, the mass of investors in sector $s \in \{z, \ldots, S - 1\}$ is so large that domestic saving is fully invested there, while sector $s \in \{0, \ldots, z - 1\}$ vanishes, $\delta_{s,t} = 0$. The borrowing constraints are slack in sector $z$ and binding in sector $s \in \{z + 1, \ldots, S - 1\}$,

$$w_{t+1} = \left(\frac{w_t}{\rho} \Gamma_t\right)^\alpha, \quad \text{where } \Gamma_t = \frac{\mu^*_{s,t} \delta_{s,t} + \sum_{z = z+1}^{S-1} \mu^*_{v,t} \delta_{v,t}}{\Pi_{t = 0}^{S-1} (\mu^*_v A^*_v)^{\eta_v}},$$

(41)

$$\delta_{z,t} = \frac{1-\theta}{\alpha} \kappa_{z,t}, \quad \kappa_{z,t} = \frac{\mu^*_{t} - \bar{w}_{z+1}^\frac{1-\theta}{\alpha}}{w_t^\frac{1-\theta}{\alpha}},$$

(42)

$$\delta_{v,t} = \frac{1-\theta}{\alpha} \kappa_{v,t}, \quad \kappa_{v,t} = \frac{\mu^*_{t} - \bar{w}_{v+1}^\frac{1-\theta}{\alpha}}{w_t^\frac{1-\theta}{\alpha}}, \quad \text{for } v \in \{z + 1, \ldots, S - 1\}.$$  

(43)

**Corollary 1.** Trade integration may allow North to sequentially offshore the low-index sectors and upgrade to the high-index sectors.

The conditions for sequential production upgrading are qualitatively similar as in proposition 2.

**B Technical Proofs**

**Proof of Proposition 1**

*Proof.* Combine equations (1) and (3)-(4) to get (5), which implicitly describes the production possibility frontier (PPF, hereafter) in period $t + 1$ and reflects the domestic supply of sectoral outputs.

$$q_{1,t+1} \frac{\delta_{t} w_t L}{\alpha} = p_{1,t+1} \chi_{1,t+1} = \frac{w_{t+1} \chi_{t+1} L}{1 - \alpha}, \quad w_{t+1} = \frac{\delta_{t} w_t}{\rho \chi_{t+1}},$$

$$q_{0,t+1} \frac{(1 - \delta_{t}) w_t L}{\alpha} = p_{0,t+1} \chi_{0,t+1} = \frac{w_{t+1} (1 - \chi_{t+1}) L}{1 - \alpha}, \quad w_{t+1} = \frac{(1 - \delta_{t}) w_t}{\rho (1 - \chi_{t+1})},$$

$$\mu_{t+1} = \frac{q_{0,t+1}}{q_{1,t+1}} = \frac{\delta_{t}}{1 - \delta_{t}}, \quad \chi_{t+1} = \frac{(1 - \delta_{t}) \mu_{t+1} + \delta_{t}}{\mu_{t+1}},$$

(44)

$$Y_{1,t+1} = \left(\frac{\delta_{t} w_t L}{\alpha}\right)^\alpha \left(\frac{\chi_{t+1} L}{1 - \alpha}\right)^{1 - \alpha} = \frac{p_{1,t+1} Y_{1,t+1}}{q_{1,t+1}^{1 - \alpha} w_{t+1}^{1 - \alpha}},$$

(45)

$$Y_{0,t+1} = \left[\frac{1 - \delta_{t}}{\alpha} w_t L \right]^{\alpha} \left[\frac{1 - \chi_{t+1} L}{1 - \alpha}\right]^{1 - \alpha} = \frac{p_{0,t+1} Y_{0,t+1}}{q_{0,t+1}^{1 - \alpha} w_{t+1}^{1 - \alpha}},$$

(46)

$$p_{1,t+1} = (q_{1,t+1})^{1 - \alpha} w_{t+1}^{1 - \alpha}, \quad p_{0,t+1} = (q_{0,t+1})^{1 - \alpha} w_{t+1}^{1 - \alpha}, \quad \Rightarrow \chi_{t+1} = \mu_{t+1}^{1 - \alpha}.$$  

(47)

Combine (1) and (3)-(4) to derive the marginal rate of transformation (MRT, hereafter),

$$\ln Y_{1,t+1} = \alpha \ln \delta_{t} + (1 - \alpha) \ln \chi_{t+1} + \ln Y_{t+1}, \quad \text{where } Y_{t+1} = \left[\frac{w_t L}{\alpha}\right]^\alpha \left[\frac{L}{1 - \alpha}\right]^{1 - \alpha}.$$  

$$\ln Y_{0,t+1} = \alpha \ln (1 - \delta_{t}) + (1 - \alpha) \ln (1 - \chi_{t+1}) + \ln Y_{t+1}$$

$$\frac{\partial \ln Y_{1,t+1}}{\partial \ln \chi_{t+1}} = \frac{\partial \ln \delta_{t}}{\partial \ln \chi_{t+1}} + (1 - \alpha), \quad \frac{\partial \ln Y_{0,t+1}}{\partial \ln \chi_{t+1}} = -\frac{\delta_{t} \alpha}{1 - \delta_{t}} \frac{\partial \ln \delta_{t}}{\partial \ln \chi_{t+1}} + \frac{-\chi_{t+1} (1 - \alpha)}{1 - \chi_{t+1}}$$

$$MRT_{0,1} = -\frac{\partial Y_{1,t+1}}{\partial Y_{0,t+1}} = \frac{\partial \ln Y_{1,t+1}}{\partial \ln Y_{0,t+1}} + \frac{\partial \ln Y_{1,t+1}}{\partial \ln Y_{0,t+1}} = \frac{\alpha \frac{\partial \ln \delta_{t}}{\partial \ln \chi_{t+1}} + (1 - \alpha)}{\alpha \frac{\partial \ln \delta_{t}}{\partial \ln \chi_{t+1}} + (1 - \alpha)} \frac{Y_{1,t+1}}{Y_{0,t+1}}.$$  

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In the case of efficient sectoral investment, $\mu_{t+1} = 1$ and combine it with equation (44) to get $\zeta_{t+1} = \delta_t$ and $\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} = 1$. In the case of inefficient sectoral investment, $\mu_{t+1} < 1$ and, given the predetermined $\delta_t$, $\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} = 0$. Thus, the (absolute) gradient of the PPF is

$$MRT_{0,1} = \frac{1 - \zeta_{t+1}}{\zeta_{t+1}} \frac{Y_{1,t+1}}{Y_{0,t+1}}. \quad (48)$$

Equation (2) specifies the isoquant with the (absolute) gradient of

$$MRS_{0,1} = -\frac{\partial V_{1,t+1}}{\partial V_{0,t+1}} = \frac{1 - \eta}{\eta} \frac{V_{1,t+1}}{V_{0,t+1}}. \quad (49)$$

In the autarkic equilibrium, the PPF and the isoquant are tangent and the markets for sectoral outputs clear domestically. Combine equations (13), (48), and (49) to get

$$MRS_{0,1} = MRT_{0,1}, \quad \Rightarrow \frac{1 - \eta}{\eta} = \frac{1 - \zeta_{t+1}}{\zeta_{t+1}}, \quad \Rightarrow \zeta_{t+1} = \eta. \quad (50)$$

Under autarky, the sectoral share of domestic labor is efficient and equal to the sectoral share in the production of final goods. Whether this also applies to the investment share depends on how far financial frictions distort the maximum share of domestic investment in sector 1.

$$\delta_t = \begin{cases} \tilde{\delta}_t < \zeta_{t+1} = \eta, & \text{if } \tilde{\delta}_t < \eta; \\ \zeta_{t+1} = \eta, & \text{if } \tilde{\delta}_t \geq \eta. \end{cases} \quad (51)$$

Combine equations (50)-(51) with (1)-(4) and (5) to get (17). If the sectoral investment is efficient, $\mu_{t+1} = 1$ and the law of motion for wage degenerates into (16). If the sectoral investment is inefficient, equation (9) specifies the fraction of domestic investment in sector 1 as a function of $w_t$. Combine equations (5) and (50) to get (18) specifying the sectoral return wedge $\mu_{t+1}$ as a function of $\delta_t$.

Next, derive the condition for the unique, stable steady state. For $w_t \geq \bar{w}_A$, the law of motion for wage in logarithm is linear with the slope less than unity, as shown by equation (16). If there exists a steady state with $w_{A,t} \geq \bar{w}_A$, it is stable. For $w_t < \bar{w}_A$, combine equations (9), (17), and (18)

$$\ln \delta_t = \frac{1 - \theta}{\theta} (\ln w_t - \ln \bar{w}), \quad \frac{\partial \ln \delta_t}{\partial \ln w_t} = \frac{1 - \theta}{\theta},$$
$$\ln \mu_{t+1} = \ln \delta_t - \ln(1 - \delta_t) + \ln \frac{1 - \eta}{\eta}, \quad \frac{\partial \ln \mu_{t+1}}{\partial \ln \delta_t} = \frac{1}{1 - \delta_t},$$
$$\ln \Gamma_t = \eta \ln \mu_{t+1} - \ln(1 - \eta + \eta \mu_{t+1}), \quad \frac{\partial \ln \Gamma_t}{\partial \ln \mu_{t+1}} = \eta(1 - \mu_{t+1})(1 - \delta_t).$$

Combine them with equation (17) to get (19). The condition for the stable steady state is.

$$\frac{\partial w_{t+1}}{\partial w_t} \bigg|_{w_{t+1}=w_t} < 1, \quad \Rightarrow \quad \mu_A > 1 - \frac{g}{\eta}, \quad \text{where} \quad g \equiv \frac{1}{\alpha} - \frac{1}{1 - \beta}. \quad (52)$$

Given $\mu_{t+1} \in (0, 1]$, a sufficient condition for inequality (52) to hold is

$$1 - \frac{g}{\eta} \leq 0, \quad \Rightarrow \quad \theta \geq \theta_{\text{th}} \equiv \frac{\eta \alpha}{\eta \alpha + (1 - \alpha)}.$$
Combine \( w_{t+1} = w_t = w_A \) with equations (9) and (17)-(18) to get \( \mu_A < 1 \) as a function of \( \lambda \),

\[
\begin{align*}
    w_A &= \left( \frac{1}{\rho} \frac{\eta \mu_A}{1 - \eta + \eta \mu_A} \right)^{\frac{\ell}{1-\rho}} = \delta_{\ell}^{\frac{\ell}{1-\rho}} \tilde{w} = \left[ \frac{\eta \mu_A}{1 - \eta + \eta \mu_A} \right]^{\frac{\ell}{1-\rho}} \frac{m}{1 - \theta} (1 - \lambda)^{\frac{1}{1-\rho}} \\
    \Rightarrow \quad Z &= \frac{1}{\rho} \frac{\eta \mu_A}{(1 - \theta)^{\frac{\ell}{1-\rho}}} = (1 - \lambda)^{\frac{\ell}{1-\rho} - \eta} (1 - \eta + \eta \mu_A)^{\frac{\ell}{1-\rho} - 1}, \\
    \frac{\partial \ln \mu_A}{\partial \ln \lambda} &= \frac{1}{(1 - \delta_A)[1 - (1 - \mu_A)\frac{\eta}{\xi}]}.
\end{align*}
\]

Given \( \theta > \theta, g > \eta \) and hence, \( (1 - \mu_A)\frac{\eta}{\xi} < 1 \), and hence, \( \frac{\partial \ln \mu_A}{\partial \ln \lambda} > 0 \). Let \( \tilde{\lambda}_A \) denote the threshold value that ensures \( \tilde{\lambda}_A = \eta \) and then \( \mu_A = 1 \). Combine them with equation (53) to get

\[
\tilde{\lambda}_A = 1 - Z(1 - \theta)\rho^{\frac{1}{1-\rho}}. 
\]

In figure 14, the dashed curves show the law of motion for wage under autarky in the case of perfect credit markets \( \lambda = 1 \), while the solid curves show those in the case where \( \{\alpha, \theta\} \) are in region U and \( \{\lambda, Z\} \) in region UB and US of figure 1, respectively. For \( w_t \in (0, \bar{w}_A) \), the solid curve lies below the dashed curve and the gap reflects the efficiency losses, \( (1 - \Gamma_\theta^\alpha) \left( \frac{w_t}{\bar{w}} \right)^\alpha \).

![Figure 14: Laws of Motion for Wage under Autarky: \( \theta \geq \theta \)](image)

Finally, in the case of \( \theta < \theta \), i.e., \( \{\alpha, \theta\} \) in region M of the left panel of figure 1, \( g < \eta \). Derive the threshold condition for multiple steady states, i.e., the existence of unstable steady state M. In the case of \( \lambda = \hat{\lambda}_A \), the law of motion for wage is tangent with the 45° line at \( w_M \in (0, \bar{w}_A) \),

\[
\frac{\partial w_{t+1}}{\partial w_t} \bigg|_{w_M} = 1, \quad \Rightarrow \quad \mu_M = 1 - \frac{g}{\eta}. 
\]

Combine equations (56) and (53) to get the threshold condition,

\[
\hat{\lambda}_A = 1 - \left[ \frac{Z(1 - \theta)}{(1 - g)^{\frac{\ell}{1-\rho}}(1 - \frac{\eta}{\xi})} \right]^{\frac{\ell}{1-\rho}}. 
\]

In the case of \( \theta \in (0, \theta) \), figure 15 shows two threshold values, \( \hat{\lambda}_A \) and \( \tilde{\lambda}_A \), in the \( \{\lambda, Z\} \) space. The solid curves in the three panels of figure 16 show the law of motion for wage, given \( \{\lambda, Z\} \) in the three regions of figure 15, respectively, while the dashed curves show the laws of motion for wage in the benchmark case of \( \lambda = 1 \). The steady state properties in the three cases are summarized as follows.
For \{\lambda, Z\} in region \(N\), the economy converges to the steady state with the income level at zero, \(w_A = 0\). In this case, there does not exist a non-trivial steady state.

- For \{\lambda, Z\} in region \(MB\), there is an unstable steady state \(M\) and a stable steady state \(A\) with \(0 < w_M < w_A < w_B\), and the borrowing constraints are binding in steady state \(A\).
- For \{\lambda, Z\} in region \(MS\), there is an unstable steady state \(M\) and a stable steady state \(A\) with \(0 < w_M < w_A = w_B\), and the borrowing constraints are slack in steady state \(A\).

\[\text{Proof of Lemma 1}\]

\[\text{Proof.}\] Under autarky, the markets for sectoral outputs clear domestically, \(V_{s,t} = Y_{s,t}\), implying \(K_{1,t+1} > 0\) and \(r_t = q_{0,t+1}\). According to (1)-(2), \(w_{t+1}^\frac{\eta}{1-\eta} (q_{0,t+1})^\alpha (q_{1,t+1})^\alpha = 1\). Combine them to get \(w_{t+1} = \left(\frac{1}{r_t \mu_{t+1}}\right)^\frac{\eta}{\rho}\). Combine it with (17) to get (20).

For \(\theta > \tilde{\theta}\), there exists a unique steady state with \(w_A = \left(\frac{\Gamma_A}{\rho}\right)^\frac{\eta}{\rho}\) where

\[\Gamma_A = \frac{\mu_A^\eta}{1-\eta + \eta \mu_A} \quad \text{and} \quad \frac{\partial \ln \Gamma_A}{\partial \ln \mu_A} = \frac{\eta (1-\delta_A)}{\eta (1-\delta_A)} > 0, \text{ given } \lambda \in [0, \tilde{\lambda}_A]. \tag{58}\]

In the autarkic steady state, \(w_{t+1} = w_t\) implies that \(Y_A = \rho\) and \(r_A = \rho [1 - \eta + \eta \mu_A]\). According to equation (54), \(\frac{\partial w_A}{\partial \lambda_A} > 0\) for \(\theta > \tilde{\theta}\). Thus, \(\frac{\partial r_A}{\partial \lambda_A} > 0\), \(\frac{\partial w_A}{\partial \lambda_A} > 0\), \(\frac{\partial \ln \chi_A}{\partial \lambda_A} > 0\). Given \(\lambda^* < \lambda < \tilde{\lambda}_A\), it holds that \(Y_A^* < Y_A\), \(\chi_A^* < \chi_A < 1\), and \(r_A^* < r_A < \rho\). \(\square\)
Proof of Lemma 2

Proof. For \( w_t < \bar{w} \), the maximum possible share of investment in sector 1 is less than unity. In this case, sector 0 is active \( K_{0,t+1} > 0 \) and the interest rate coupling gives \( r_t = q_0t+1 \). Thus, the borrowing constraints are binding in North, \( \mu_{t+1} = \frac{q_{1,t+1}}{q_{1,t+1}} = \frac{r_t}{q_{1,t+1}} = \mu_A^* < 1 \) and \( \delta_t = \delta_t < 1 \) and \( \zeta_{t+1} = \frac{1}{(\frac{1}{\mu_A^*})} \in (\delta_t, 1) \).

For \( w_t \geq \bar{w} \), North offshores sector 0, \( K_{0,t+1} = 0 \), implying the interest rate decoupling from the rate of return in sector 0. The credit market competition leads to the interest rate coupling with the rate of return in sector 1. Combine them with equations (14), (1) and (3)-(5) to get (21).

Proof of Proposition 2

Proof. For \( \theta \in (\theta, 1) \), there exists a unique, autarkic steady state. Under trade integration, the static gains raise national income in North so that the law of motion for wage lies strictly above the one under autarky. For \( w_t > \bar{w} \), the law of motion for wage in logarithm has a slope of \( \alpha < 1 \). Thus, there exists at least one stable steady state. Next, I analyze the steady-state properties under free trade.

First, derive the condition under which trade leads to multiple steady states in North. If there exists an unstable steady state \( M \), \( w_M \in (0, \bar{w}) \) should hold and

\[
\frac{\partial w_{t+1}}{\partial w_t} \bigg|_{M} = \alpha + \alpha \frac{\frac{1-\theta}{\theta} \frac{\mu}{(1-\mu)^{\delta_M}} + 1}{\left(\frac{1}{\mu^*} - \frac{1}{\mu^*} + 1\right)} > 1, \quad \Rightarrow \quad \frac{\alpha}{1-\alpha} - \frac{\theta}{1-\theta} > \frac{\theta}{1-\theta} \frac{\mu^*}{(1-\mu^*)\delta_M}.
\]  

(59)

For \( \theta \geq \alpha \), condition (59) does not hold and the law of motion for wage in logarithm has the slope less than unity. Thus, there exists a unique, stable steady state. Next, I focus on the case of \( \theta \in (\theta, \alpha) \).

Combine condition (59) with \( \delta_M = \left(\frac{w_M}{\bar{w}}\right)^{\frac{1}{\theta}} < 1 \) to get an upper bound for \( \mu^* \),

\[
\frac{\mu^*}{1-\mu^*} - \frac{\theta}{1-\theta} < \delta_M < 1, \quad \Rightarrow \quad \mu^* \leq \mu_T \equiv 1 - \frac{\theta}{1-\theta} \frac{\mu^*}{(1-\mu^*)}, \quad \text{where } \frac{1-\theta}{1-\theta} = (\eta, 1).
\]  

(60)

According to equations (53)-(54), \( \mu_A \) is an increasing function of \( \lambda \). As South has the same economic structure as North, equations (53)-(54) also specify \( \mu_A \) as an increasing function of \( \lambda^* \in [0, \lambda_A] \). Combine (60) and (53) to get the upper bound for \( \lambda^* \),

\[
\lambda^* \leq \lambda_T \equiv 1 - \left(\frac{Z}{\hat{Z}}\right)^{\theta(1-\theta)}, \quad \text{where } \hat{Z} \equiv (1-g)^{\gamma-\eta}(1-\eta g)^{1-g} < 1.
\]  

(61)

\( Z \leq \hat{Z} \) ensures \( \lambda_T \geq 0 \) and the existence of multiple steady states. \( \hat{Z} \) is a function of \( \theta \), as shown by the curve between region M and UF in the upper-right panel of figure 10.

Given \( \{\theta, Z\} \) in region M of the upper-right panel of figure 10, derive the condition in the \( \{\lambda^*, \lambda\} \) space under which the law of motion for wage is tangent with the 45° line at a steady state M,

\[
\frac{\partial w_{t+1}}{\partial w_t} \bigg|_{M} = \alpha + \alpha \frac{\frac{1-\theta}{\theta} \frac{\mu}{(1-\mu)^{\delta_M}} + 1}{\left(\frac{1}{\mu^*} - \frac{1}{\mu^*} + 1\right)} = 1, \quad \Rightarrow \quad \delta_M = \frac{g}{1-g} \frac{\mu^*}{1-\mu^*}.
\]  

(62)

Combine equations (21) with (62) and get a threshold value of \( \lambda_T \) as a function of \( \mu^* \),

\[
w_{M} = \left[\left(\frac{(\mu^*)^{\eta}}{\rho} \left(1 + \frac{1-\mu^*}{\mu^*} \delta_M\right)\right)^{\theta}\right] = \bar{w} \delta_M^{\frac{\rho}{\theta}} = \left(1 - \lambda_T\right)^{\frac{1}{1-\theta}} \frac{m}{1-\theta} \left(\frac{g}{1-g} \frac{\mu^*}{1-\mu^*}\right)^{\frac{\theta}{1-\theta}}.
\]

\[
\lambda_T = 1 - \left\{\frac{Z}{\left(\frac{1}{\mu^*}\right)^{\gamma-\eta}(1-g)^{1-g}}\right\}^{\theta(1-\theta)}.
\]  

(63)
For $\lambda^* \in [0, \hat{\lambda}_A^*]$, use (53) to solve for $\mu^*_A$. Then, combine $\mu^* = \mu^*_A$ with (63) to solve for $\hat{\lambda}_T$. The curve between region M and UF1 in the lower-left panel of figure 10 shows $\hat{\lambda}_T$ as a function of $\lambda^* \in [0, \hat{\lambda}_A^*]$. Given $\eta \in [0, \bar{\eta}]$, combine equation (64) with (55) to get an upper bound for $\lambda^*$,

$$\hat{\lambda}_T \leq \bar{\lambda}_A \quad \Rightarrow \quad \mu^* < \tilde{\mu}_T^* = \eta^{\frac{\epsilon}{1-\eta-\epsilon}}.$$

By the same logic as mentioned above, combine (65) and (53) to get an upper bound for $\lambda^*$,

$$\lambda^* < \tilde{\lambda}_T^* \equiv 1 - \left( \frac{Z_T}{Z} \right)^{\rho(1-\theta)}$$

where $Z \equiv \eta^{\frac{\epsilon(\eta-\epsilon)}{1-\eta-\epsilon}}(1 - \eta + \eta^{\frac{\epsilon}{1-\eta+1}})^{1-\epsilon}$.

$Z \leq \tilde{Z}$ ensures $\bar{\lambda}_T^* \geq 0$. $\tilde{Z}$ is a function of $\theta$ and shown by the curve between region UF and UP in the upper-right panel of figure 10.

Given $\{\theta, Z\}$ in region UF and M in the upper-right panel of figure 10, for each $\lambda^* \in [0, \hat{\lambda}_A^*]$, use (53) to solve for $\mu^*_A$ and then combine $\mu^* = \mu^*_A$ with (64) to solve for $\hat{\lambda}_T$.

For $\{\theta, Z\}$ in region M of the upper-right panel of figure 10, the curve denoted by $\tilde{\lambda}_T$ in the lower-left panel of figure 10 shows $\tilde{\lambda}_T$ as a function of $\lambda^* \in [0, \hat{\lambda}_A^*]$. For $\{\theta, Z\}$ in region UF of the upper-right panel of figure 10, the curve between region UF1 and UP1 in the lower-middle panel of figure 10 shows $\bar{\lambda}_T$ as a function of $\lambda^* \in [0, \hat{\lambda}_A^*]$.

**Proof of Lemma 3**

**Proof.** Following the approach shown in the proof of lemma 1 and taking into account equation (21), one can derive the interest rate as the piecewise function of national income characterized by (25)-(26). □

**Proof of Proposition 4**

**Proof.** Given the Cobb-Douglas production function at the aggregate level,

$$Y_{t+1} = \Pi^2_{s=0} \left( \frac{V_{s,t+1}}{\eta_s} \right)^{\eta_s}, \quad MRS_{x,s} = \frac{\eta_s V_{x,t+1}}{\eta_s V_{x,t+1}}$$

where $x, s \in \{0, 1, 2\}, s \neq x$. (66)

Following the proof of proposition 1, one can derive $MRT_{x,s} = \frac{\zeta_{s,t+1}}{\zeta_{s,t+1} - \zeta_{s,t+1}}$. Under autarky, the sectoral output markets clear domestically, $V_{s,t} = Y_{s,t}$. Combine it with $MRT_{x,s} = MRS_{x,s}$ to get the share of labor input in sector $s$ at $\zeta_{s,t+1} = \eta_s$. Given the Cobb-Douglas production function at the sectoral level,

$$\frac{q_{s,t+1} \delta_{s,t+1} w_{1,t+1}}{L} = \frac{p_{s,t+1} Y_{s,t+1} = \frac{w_{t+1} \delta_{s,t+1} L}{1 - \alpha}, \quad \frac{q_{s,t+1} \delta_{s,t+1}}{\zeta_{s,t+1}} = \frac{\rho w_{t+1}}{w_t} = \frac{q_{x,t+1} \delta_{x,t+1}}{\zeta_{x,t+1}}.$$ (67)

Thus, a sector’s rate of return is inversely related to its capital-labor ratio.

**First, solve for the sectoral investment shares $\delta_{s,t}$ in three cases.**

**Case 1:** if the borrowing constraints are binding in sector 1 and 2, the sectoral investment shares are specified by equations (27)-(29). Given $\zeta_{s,t+1} = \eta_s$, assumption 2 ensures $\frac{\delta_{1,t}}{\eta_1} > \frac{\delta_{2,t}}{\eta_2}$. Combine it with equation (67) to get $q_{1,t+1} < q_{2,t+1}$. Thus, the agents with $\epsilon_j \geq \epsilon_2$ borrow to the limit and invest in sector 2. Besides, $w_t < \tilde{w}_{1,t}$ ensures $\frac{\delta_{1,t}}{\eta_1} > \frac{\delta_{2,t}}{\eta_2}$. Combine it with equation (67) to get $q_{0,t+1} < q_{1,t+1}$. 36
Thus, the agents with \( \epsilon_j \in [\epsilon_1, \epsilon_2] \) borrow to the limit and invest in sector 1. Thus, assumption 2 and \( w_t < \bar{w}_{1,A} \) are the necessary and sufficient condition for this case to arise. **Case 2**: use the same logic to prove that \( w_t \in [\bar{w}_{1,A}, \bar{w}_{2,A}] \) ensure the binding (slack) borrowing constraints in sector 2 (1). In this case, \( \delta_{2,t} \) is still specified by equation (27), while the capital-labor ratio equalizes in sector 0 and 1, i.e., \( \delta_{s,t} = \frac{\eta_0}{\eta_0 + \eta_1} (1 - \delta_{s,t}) \) for \( s \in \{0, 1\} \). **Case 3**: use the same logic to prove that \( w_t > \bar{w}_{2,A} \) ensure the efficient sectoral investment shares, i.e., \( \delta_{s,t} = \eta_s \).

**Second, derive the conditions for the unique, autarkic steady state.**

Under autarky, \( V_{s,t} = Y_{s,t} \). Combine it with \( \zeta_{s,t+1} = \eta_s \) and equations (27)-(29) and (66) to get,

\[
\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \alpha + \alpha \frac{\partial \ln \Gamma_t}{\partial \ln w_t}, \quad \text{where} \quad \frac{\partial \ln \Gamma_t}{\partial \ln w_t} = \begin{cases} 
\frac{1}{\theta} \left( 1 - \frac{\eta_0}{\delta_{0,t}} \right), & \text{if } w_t < \bar{w}_1; \\
\frac{1}{\theta} \left( 1 - \frac{\eta_0 + \eta_1}{\delta_{2,t}} \right), & \text{if } w_t \in [\bar{w}_1, \bar{w}_2); \\
1, & \text{if } w_t > \bar{w}_2.
\end{cases}
\]

Given \( \theta > \bar{\theta}_3 \), \( \frac{\partial \ln w_{t+1}}{\partial \ln w_t} < 1 \) holds strictly. As the slope of the law of motion for wage at any steady state is less than unity, the autarkic steady state is unique. In the following, I focus on the case of \( \theta > \bar{\theta}_3 \).

**Finally, derive the threshold value \( \tilde{\lambda}_3 \) such that for \( \lambda < \tilde{\lambda}_3 \), the borrowing constraints are binding in sector 1 and 2 at the autarkic steady state.** As the law of motion for wage is a piecewise function with two kinks at \( w_t = \bar{w}_{1,A} \) and \( w_t = \bar{w}_{2,A} \), one can solve \( \tilde{\lambda}_3 \) by keeping the first kink point as a steady state, i.e., \( w_A = \bar{w}_{1,A} \) and \( \delta_{0,A} = \frac{\delta_{1,A}}{\eta_0 + \eta_1} = \frac{1 - \delta_{2,A}}{\eta_0} \). Let \( D_1 = \frac{1}{1 - \tilde{\lambda}_3} \). Thus, \( \bar{w}_2 = m_2 D_1 \) and \( \bar{w}_1 = \gamma \bar{w}_2 \). Combine them with \( w_A = \bar{w}_{1,A} \), the definition of \( \bar{w}_{1,A} \), and equations (27)-(29) and (68),

\[
\begin{align*}
\delta_{2,A} &= \left( \frac{w_A}{\bar{w}_2} \right)^{\frac{1-\theta}{\rho}} = \left( \frac{\bar{w}_{1,A}}{D_1} \right)^{\frac{1-\theta}{\rho}} m_2^{\frac{1-\theta}{\rho}}, \\
\delta_{0,A} + \delta_{1,A} &= 1 - \delta_{2,A} = \left( \frac{\bar{w}_{1,A}}{D_1} \right)^{\frac{1-\theta}{\rho}} m_2^{\frac{1-\theta}{\rho}} \left( \frac{\eta_0 + \eta_1}{\eta_1} \right) \left( \gamma^{\frac{1-\theta}{\rho}} - 1 \right), \\
\rho w_A^{\frac{1}{\rho}} &= \Gamma_A = \left( \frac{\delta_{0,A} + \delta_{1,A}}{\eta_0 + \eta_1} \right)^{\frac{\eta_1}{\eta_0 + \eta_1}} \left( \frac{\delta_{2,A}}{\eta_2} \right)^{\frac{\eta_2}{\eta_2}}, \\
\rho \left( \frac{\bar{w}_{1,A}}{D_1} \right)^{\frac{1}{\rho}} &= \left( \frac{\bar{w}_{1,A}}{D_1} \right)^{\frac{1-\theta}{\rho}} m_2^{\frac{1-\theta}{\rho}} \left( \gamma^{\frac{1-\theta}{\rho}} - 1 \right)^{1-\eta_2} \left( \frac{1}{\eta_2} \right)^{\eta_2}, \\
\left( \frac{\bar{w}_{1,A}}{D_1} \right)^{\frac{1}{\rho}} &= \frac{\bar{w}_{1,A} \gamma m_2}{\bar{w}_1} = \gamma^{\frac{1}{\eta_2}} \left( 1 + \frac{\eta_0}{\eta_1} \right)^{\frac{1}{\rho}} = D_2^{\frac{1}{\rho}} = \frac{\gamma m_2}{\bar{w}_1}^{\frac{1}{\rho}} \left( \gamma^{\frac{1-\theta}{\rho}} - 1 \right)^{\frac{1}{\eta_2}} \left( \frac{1}{\eta_2} \right)^{\eta_2},
\end{align*}
\]

(1 - \( \tilde{\lambda}_3 \))^{\frac{1}{\rho(1-\theta)}} = \mathbb{I} Z_3, \quad \Rightarrow \quad \tilde{\lambda}_3 = 1 - (\mathbb{I} Z_3) \rho(1-\theta),

where \( Z_3 \) and \( \mathbb{I} > 1 \) are defined in proposition 4.

**Proof of Lemma 4**
Proof. According to proposition 4, there is a unique steady state where \( w_A < \bar{w}_A \), and the borrowing constraints are binding in sector 1 and 2. Let \( \mathbb{H} \equiv \frac{\delta_{2,A}}{\eta_2} = \delta_{2,A}^{-1} - \gamma^{-\frac{1}{1-\rho}}. \) At the autarkic steady state, combine \( w_{t+1} = w_t = \bar{w} \) with equations (27)-(29) and (68),

\[
\frac{1}{\rho} w_A = \Gamma_A, \quad \rho \delta_{2,A}^{(\eta_2 - 1)} w_A = \delta_{2,A} \left( \frac{\mathbb{H}}{\eta_0} \right)^{\eta_0} \left( \gamma^{-\frac{1}{\eta_1}} - 1 \right) \eta_1 \left( \frac{1}{\eta_2} \right)^{\eta_2},
\]

\[
\bar{w}_2 = [\mathbb{H} + \gamma^{-\frac{1}{\eta_1}}] \rho^{\eta_1 - 1} \mathbb{H}^{\eta_0} \left( \gamma^{-\frac{1}{\eta_1}} - 1 \right)^{\eta_1},
\]

\[
1 = \left( \eta_0 + \frac{1}{\rho(\delta - 1)} - 1 \right) \frac{\delta_{0,A}}{\eta_0} \frac{\partial \ln \mathbb{H}}{\partial \ln \bar{w}_2},
\]

for \( \theta > 0, \) \( \eta_0 + \frac{1}{\rho(\delta - 1)} - 1 \) \( \delta_{0,A} > \eta_0 (1 - \delta_{0,A}), \) \( \Rightarrow \) \( \partial \ln \mathbb{H} \partial \ln \bar{w}_2 > 0, \)

\[
\frac{\partial \ln \bar{w}_2}{\partial \ln \lambda} = -\frac{\lambda}{(1 - \theta)(1 - \lambda)} < 0, \Rightarrow \frac{\partial \ln \mathbb{H}}{\partial \ln \bar{w}_2} = \frac{\partial \ln \mathbb{H}}{\partial \ln \bar{w}_2} < 0.
\]  

(69)

At the autarkic steady state, combine \( \zeta_{s,t+1} = \eta_s \) with equations (27)-(29) and (67),

\[
\mu_{0,A} \equiv \frac{q_{0,A}}{q_{2,A}} = \frac{\delta_{2,A}}{\eta_2}, \quad \frac{\partial \ln \mu_{0,A}}{\partial \ln \lambda} = \frac{\partial \ln \mathbb{H}}{\partial \ln \lambda} > 0,
\]

\[
\mu_{1,A} \equiv \frac{q_{1,A}}{q_{2,A}} = \frac{\delta_{2,A}}{\eta_1}, \quad \frac{\partial \ln \mu_{1,A}}{\partial \ln \lambda} = 0.
\]

Thus, given \( \lambda^* < \lambda < \bar{\lambda}, \) \( \mu_{0,A}^* < \mu_{0,A} \) and \( \mu_{1,A}^* = \mu_{1,A} \) hold. According to equation (30), the ascending sectorial rate of return in each country implies,

\[
\mu_{0,A}^* < \mu_{0,A} < \mu_{1,A}^* = \mu_{1,A} < \mu_{2,A}^* = \mu_{2,A} = 1.
\]  

(70)

For \( s \in \{0, 1, 2\}, \) use equation (67) to get

\[
q_{s,t+1} = \alpha p_{s,t+1} \left( \frac{\delta_{s,t}}{\zeta_{s,t+1}} \right)^{\alpha - 1} \varphi(t+1), \quad \mu_{s,t+1} \equiv \frac{q_{s,t+1}}{q_{2,t+1}} = \frac{\delta_{s,t}}{\eta_2^s}, \quad \frac{\partial \ln \mu_{s,t+1}}{\partial \ln \lambda} = \frac{\partial \ln \mathbb{H}}{\partial \ln \lambda} > 0,
\]

\[
\mu_{s,t+1} = \chi_{s,t+1} \left( \frac{\delta_{s,t+1}}{\delta_{s,t+1}} \right)^{\alpha - 1} = \chi_{s,t+1} \mu_{s,t+1}^{1-\alpha}, \Rightarrow \chi_{s,t+1} = \mu_{s,t+1}^{1/\alpha},
\]  

(72)

which holds under autarky as well as under free trade. Combine it with inequality (70) to get the cross-country patterns of the relative sectorial rates,

\[
\chi_{0,A}^* < \chi_{0,A} < \chi_{1,A}^* = \chi_{1,A} < \chi_{2,A}^* = \chi_{2,A} = 1.
\]

Combine \( r_t = q_{0,t+1} \) with equations (67) and (69) to get the steady-state interest rate,

\[
r_A = \frac{\rho}{\delta_{2,A}} = \rho \eta_0 \left( 1 + \frac{1}{\mathbb{H} \gamma^{-\eta}} \right), \quad \frac{\partial r_A}{\partial \mathbb{H}} < 0, \Rightarrow \frac{\partial r_A}{\partial \lambda} = \frac{\partial r_A}{\partial \mathbb{H}} \frac{\partial \mathbb{H}}{\partial \lambda} > 0, \Rightarrow r_A^* < r_A.
\]

Proof of Proposition 5
Proof. Free trade aligns the relative sectoral prices in North to the world levels, $\chi_{s,t} = \chi^*_s$, while equation (72) implies $\mu_{s,t} = \frac{1}{2} \lambda^*_{s,t} = (\chi^*_s)^{\frac{1}{2}} = \mu^*_s$. For expositional convenience, define $\bar{w}_0 \equiv 0$ and $\bar{w}_3 \equiv \infty$. As shown in subsection 4.1.2, trade may allow North to offshore the low-return sectors. Let $a \equiv \min\{z : \delta_{z,t} > 0\}$ denote the lowest sector index among all active sectors in North. For $w_t \in (\bar{w}_a, \bar{w}_{a+1})$, North offshores sector $u < a$ and specializes in sector $s \geq a$.

Combine $\mu_{s,t+1} = \mu^*_s$ with equation (71) and the labor market clearing condition,

$$\zeta_{s,t+1} = \frac{\mu^*_s}{\zeta_{s,t+1}} \delta_{t,s}, \quad \sum_{v=a}^{2} \zeta_{v,t+1} = 1 = \sum_{v=0}^{2} \mu^*_v \delta_{v,t}, \quad \delta_{s,t} = \sum_{v=a}^{2} \mu^*_v \delta_{v,t}$$

Let $\Gamma_t \equiv \sum_{i=0}^{2} \mu^*_t \delta_{t,i}$. Derive the law of motion for wage under free from the aggregate production function and the national income-expenditure identity,

$$Y_{t+1} = \Pi^2_{v=0} \left( \frac{V_{v,t+1}}{\eta_v} \right)^{\eta_v} = Y_{t+1}, \quad \Pi^2_{v=0} \eta_v = 1, \quad p_{s,t+1} = p_{2,t+1} \chi^*_s,$$

$$p_{s,t+1} = \frac{1}{\Pi^2_{v=0} (\chi^*_v)^{\eta_v}}, \quad p_{s,t+1} = \frac{\mu^*_s}{\Pi^2_{v=0} (\mu^*_v)^{\eta_v}} = \left[ \frac{\mu^*_s}{\Pi^2_{v=0} (\mu^*_v)^{\eta_v}} \right]^\alpha$$

$$w_{t+1} = \left( 1 - \alpha \right) \frac{Y_{t+1}}{L} = \left( 1 - \alpha \right) \sum_{v=a}^{2} \eta_v \Pi_{v,t+1} \eta_v = \left( \frac{\bar{w}_t}{\rho} \right)^\alpha.$$

According to lemma 4, all sectors are active in the autarkic steady state, $a = 0$, with $\mu^*_{0,A} < \mu_{0,A} < \mu^*_{1,A} < 1$. Given $w_0 = w_A$, the static gains from trade are

$$\frac{\partial \ln w_1}{\partial \ln \chi} = \frac{\partial \ln \Gamma}{\partial \ln \mu_1} = \frac{\mu_0^0 - \mu_{0,A}}{\ln \mu_0^0 + \ln \mu_{0,A}} < 0,$$

which is equivalent to equation (22) in the two-sector setting. Given $\lambda^* < \lambda < \lambda^*$, the larger the heterogeneity in financial development $\lambda - \lambda^*$, the larger the international relative price differential $\chi_{0,A} - \chi^*_0$ and the international relative rate-of-return differential in sector $0, \mu_{0,A} - \mu^*_0$, the larger the trade flows and the static gains. From period $t = 1$ on, the rise in national income triggers the sectoral investment reallocation towards the high-MIR, high-return sectors along the extensive margin, which improves aggregate efficiency indicator $\Gamma_t$ and raises national income in period $t + 1$.

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - (1 - \alpha) + \alpha \sum_{s=0}^{2} \frac{\partial \ln w_{t+1}}{\partial \ln \delta_{s,t}} \frac{\partial \ln w_t}{\partial \ln w_t} = \alpha + \frac{1 - \theta}{\left( \frac{1 - \theta}{\rho_0 - 1} - (1 - \frac{1 - \theta}{\rho_0 - 1}) \delta_{2,t} \right)}.$$

which is similar as equation (23) in the two-sector setting. For $w_t < \bar{w}_1$, the income dynamics are driven jointly by the DMRK effect and the investment reallocation effect. Given $\lambda$, the lower the $\lambda^*$, the lower the $\mu^*_0$, the stronger the investment reallocation effect; the lower the $\theta$, the larger the mass of investors in sector 1 and 2 responds to the gains from trade, the larger the reallocation effect.

Proof of Lemma 5

Proof. Under autarky, sectoral output is equal to sectoral absorption, $V_{s,t+1} = Y_{s,t+1}$, while the sectoral share of labor input is efficient and equal to the sectoral share in the production of final goods, $\zeta_{s,t} = \eta_s$, as shown in the proof of proposition 1. Combine these two conditions with equations (36) to get

$$q_{s,t+1} \delta_{s,t} = q_{v,t+1} \delta_{v,t}, \quad \text{for } s \neq v \text{ and } s, v \in \{0, 1, ..., S - 1\}.$$  

(73)
Step 1: derive the condition for the ascending sectoral rate of return.

Suppose that the borrowing constraints are binding in sector \( s \in \{z + 1, \ldots, S - 1\} \) and the sectoral rate of return is ascending, \( q_{s-1, t+1} < q_{s, t+1} \). Thus, agents always invest in the sector with the highest MIR they can afford and borrow to the limit. Let \( \delta_{s, t} \equiv \frac{1 - \lambda}{\eta_s - w_s} \). The investment share of sector \( s \) is

\[
\delta_{s, t} = \frac{\int_{\epsilon_s, t}^{s+1, t} \frac{1}{1-\lambda} dG(\epsilon)}{w_t L} = \frac{-\frac{1-\theta}{\eta_s} - \frac{1-\theta}{\eta_{s+1}}}{1 - \lambda} = \frac{1-\theta}{\eta_s} \kappa_s, \quad \text{where} \quad \kappa_s \equiv \frac{1-\theta}{\eta_s} - \frac{1-\theta}{\eta_{s+1}}.
\]

Combine \( \zeta_{s, t} = \eta_s \) with equation (73) to get the condition for the ascending sectoral rate of return,

\[
q_{s, t+1} < q_{s+1, t+1} \implies \frac{\delta_{s+1, t}}{\delta_{s, t}} < \frac{\zeta_{s+1, t} + 1}{\zeta_{s, t} + 1}, \quad \Leftrightarrow \quad \frac{m_s}{\eta_s} < \frac{m_{s+1}}{\eta_{s+1}}, \quad \text{for} \quad \eta_s, \eta_{s+1} > 0.
\]

which is guaranteed by assumption 3. \(^{29}\)

Step 2: derive the threshold values of \( \bar{w}_z \) and \( \bar{w}_{z, A} \).

Suppose that the borrowing constraints are slack in sector \( s \in \{0, \ldots, z\} \). The sectoral rate of return equalizes, \( q_{s, t+1} = r_t = q_{s, t+1} \) and so does the sectoral capital-labor ratio, \( \frac{\delta_{s, t}}{\zeta_{s, t} + 1} = \frac{\delta_{s, t}}{\zeta_{s, t} + 1} = \sum_{v=0}^{z} \delta_{v, t} \), according to equation (73). Technically, one can pool these sectors into a new one and call it sector \( u \) (unconstrained). Let \( \kappa_{u, t} \equiv \frac{w_t}{\bar{w}_z - \bar{w}_{t+1}} \). The investment and the labor shares of sector \( u \) are

\[
\delta_{u, t} \equiv \sum_{v=0}^{z} \delta_{v, t} = 1 - \sum_{v=z+1}^{S-1} \delta_{v, t} = 1 - \left( \frac{w_t}{\bar{w}_{z+1}} \right)^{1-\theta} = \frac{1-\theta}{\eta_u} \kappa_{u, t}, \quad \text{and} \quad (76)
\]

\[
\zeta_{u, t+1} \equiv \sum_{v=0}^{z} \zeta_{v, t+1} = \bar{u} \equiv \sum_{v=0}^{z} \eta_v. \quad (77)
\]

In the boundary case where the borrowing constraints are slack in sector \( s \in \{0, \ldots, z - 1\} \) and weakly binding in sector \( z \), the rate of return equalizes across these sectors

\[
q_{s, t+1} = q_{u, t+1} = q_{z, t+1} = r_t \implies \frac{\delta_{u, t}}{\zeta_{u, t}} = \frac{\delta_{z, t}}{\zeta_{z, t+1}}.
\]

Combine \( \zeta_{s, t} = \eta_s \) with (74) and (76)-(78) to get the threshold values \( \bar{w}_z \) and \( \bar{w}_{z, A} \) as specified in (37).

Step 3: derive the law of motion for wage under autarky.

Combine equations (34)-(36) with \( \zeta_{s, t+1} = \eta_s \) to get

\[
\bar{w}_{t+1} = \left( 1 - \alpha \right) \frac{Y_{t+1}}{L} = \left( 1 - \alpha \right) \frac{Y_{t+1}}{L} \Pi_{v=0}^{S-1} \left( \frac{\delta_{v, t}}{\eta_v} \right)^{\eta_v}. \quad (79)
\]

For sector \( s \in \{0, \ldots, z\} \), the sectoral capital-labor ratio equalizes and hence,

\[
\Pi_{v=0}^{z} \left( \frac{\delta_{v, t}}{\eta_v} \right)^{\eta_v} = \Pi_{v=0}^{z} \left( \frac{\delta_{v, t}}{\eta_v} \right)^{\eta_v} = \left( \frac{\delta_{v, t}}{\eta_v} \right)^{\eta_v}. \quad (80)
\]

Combine (74), (76), (79)-(80) to get the law of motion for wage as show by equation (38).

\[\square\]

**Proof of Proposition 6**

\(^29\)If assumption 3 does not hold for two neighboring sectors, the rate of return equalizes between them and so does the capital-labor ratio, according to equation (73). In that case, one can pool the two sectors together and redefine a new sector so as to maintain the ascending rate-of-return pattern. In other words, the sectors in this setting are specified in terms of their rate-of-return pattern rather than their physical characteristics.
Proof. Step 1: derive the sufficient condition for the unique steady state under autarky.
For \( w_t \in (\bar{w}_{z,A}, \bar{w}_{z+1,A}) \), the borrowing constraints are binding in sector \( b \in \{ z + 1, \ldots, S - 1 \} \) and slack in sector \( u \).\(^{30}\)

\[
\frac{\partial \ln \delta_{b,t}}{\partial \ln w_t} = \frac{1 - \theta}{\theta} > 0, \quad \text{and} \quad \frac{\partial \ln \delta_{u,t}}{\partial \ln w_t} = \frac{1 - \theta}{\theta} - \frac{1 - \theta}{\delta_{u,t}} < 0
\]  
\[ \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \alpha + \alpha \left[ \eta_u \frac{\partial \ln \delta_{u,t}}{\partial \ln w_t} + \sum_{v = z}^{S-1} \left( \eta_v \frac{\partial \ln \delta_{v,t}}{\partial \ln w_t} \right) \right] = \alpha + \alpha \left[ \frac{1 - \theta}{\delta_{u,t}} \left( 1 - \frac{\eta_u}{\theta} \right) \right]. \]  

(81)

(82)

There exists a unique steady state if \( \frac{\partial \ln w_{t+1}}{\partial \ln w_t} < 1 \) or equivalently, \( 1 - \frac{\eta_u}{\theta} < \frac{1 - \frac{\theta}{\eta_u}}{\frac{\theta}{\eta_u}} \). As \( 1 - \frac{\eta_u}{\theta} < 1 - \frac{\eta_u}{\theta} \), the sufficient condition for the unique steady state is \( 1 - \eta_0 < \frac{1 - \frac{\theta}{\eta_u}}{\frac{\theta}{\eta_u}} \).

Step 2: derive the threshold values \( \lambda_z \).

Let \( \mathbb{D} = \left( (1 - \lambda) \frac{1 - \theta}{1 - \rho} \right) \) and then \( \tilde{w}_s = \mathbb{D} m_s \). Let us derive the condition under which the steady-state wage rate is equal to a threshold value under autarky, \( w_A = \bar{w}_{z,A} \). In that case, the borrowing constraints are slack in sector \( s \in \{ 0, \ldots, z - 1 \} \), weakly binding in sector \( s = z \), and strictly binding in sector \( s \in \{ z + 1, \ldots, S - 1 \} \).

\[
\delta_{u,A} = \left( \frac{w_A}{\bar{w}_z} \right)^{\frac{1 - \theta}{\rho}} \left( \frac{w_A}{\bar{w}_z} \right)^{\frac{1 - \theta}{\rho}} - 1 = \left( \frac{\bar{w}_{z,A}}{\bar{w}_z} \right)^{\frac{1 - \theta}{\rho}} - 1, \]

\[
\delta_{u,A} = \left( \frac{\bar{w}_z,A}{\mathbb{D}} \right)^{\frac{1 - \theta}{\rho}} m_z - \frac{1 - \theta}{\rho} - \frac{1 - \theta}{\rho}, \quad \eta_u = \sum_{s=0}^{z-1} \eta_s
\]

\[
\delta_{v,A} = \left( \frac{\bar{w}_z,A}{\mathbb{D}} \right)^{\frac{1 - \theta}{\rho}} m_v - \frac{1 - \theta}{\rho} - \frac{1 - \theta}{\rho}, \quad \text{for } v \in \{ z, \ldots, S - 1 \}
\]

\[
\rho w_A^\frac{1}{\rho} = \Gamma_A = \left( \frac{\delta_{u,A}}{\eta_u} \right)^{\eta_s} \Pi_{v=z}^{S-1} \left( \frac{\delta_{v,A}}{\eta_v} \right)^{\eta_v}, \quad \text{and } w_A = \bar{w}_{z,A}
\]

\[
\rho \bar{w}_A^\frac{1}{\rho} = \left( \frac{\delta_{u,A}}{\eta_u} \right)^{\frac{1 - \theta}{\rho}} \Omega_z, \quad \text{with } \Omega_z = \left( \frac{\frac{1 - \theta}{\rho} - \frac{1 - \theta}{\rho}}{\eta_z} \right)^{\eta_z} \Pi_{v=z}^{S-1} \left( \frac{\frac{1 - \theta}{\rho} - \frac{1 - \theta}{\rho}}{\eta_v} \right)^{\eta_v}
\]

\[
\mathbb{D}^\frac{1}{\rho} = \left( \frac{\delta_{u,A}}{\eta_u} \right)^{\frac{1 - \theta}{\rho}} \frac{1}{\rho} \Omega_z = \left\{ \begin{array}{ll}
1 + \frac{m_z}{\eta_z} & 1 - \left( \frac{\frac{1 - \theta}{\rho}}{\eta_z} \right) \\
1 + \frac{m_z}{\eta_z} & 1 - \left( \frac{\frac{1 - \theta}{\rho}}{\eta_z} \right)
\end{array} \right\} \frac{1 - \theta}{\rho} \Omega_z.
\]

Combine it with \( \mathbb{D} = \left( (1 - \lambda) \frac{1 - \theta}{1 - \rho} \right) \) to get the threshold value

\[
\tilde{\lambda}_z \equiv 1 - (1 - \theta)^{1 - \theta} \left( \frac{\Omega_z}{\rho} \right)^{\rho(1-\theta)}.
\]  

(83)

For \( z \in \{ 1, \ldots, S - 1 \} \), there are \( S - 1 \) threshold values \( \tilde{\lambda}_z \) splitting the interval of \([0, 1]\) into \( S \) sub-intervals. Let \( \tilde{\lambda}_0 = 0 \) and \( \tilde{\lambda}_S = 1 \). For \( \lambda \in [\tilde{\lambda}_z, \tilde{\lambda}_{z+1}] \), the borrowing constraints are slack in sector \( s \in \{ 0, \ldots, z \} \) and binding in sector \( s \in \{ z + 1, \ldots, S - 1 \} \) in the autarkic steady state where \( w_A \in [\bar{w}_{z,A}, \bar{w}_{z+1,A}] \).

\(^{30}\)See the proof of lemma 5 for the definition of sector \( u \).
Proof of Corollary 1

Proof. Step 1: derive the sectoral investment share under free trade

For \( z \in \{z^*, ..., S-1\} \), the relative rate of return in sector \( s \in \{z, ..., S-1\} \) is ascending in South \( \mu_{s,A}^* < \mu_{s+1,A}^* \) under autarky and so is that in North under free trade. Agents in North prefer to invest in the sector with the highest MIR that they can afford. As the leverage ratio is identical across sectors, the total investment share of sector \( v \in \{z+1, ..., S-1\} \) is

\[
\sum_{v=z+1}^{S-1} \delta_{v,t} = \left( \frac{w_t}{\bar{w}_{s+1}} \right)^{\frac{1}{\rho_s}}.
\]

Given \( w_t \in (\bar{w}_z, \bar{w}_{z+1}) \), \( \sum_{v=z+1}^{S-1} \delta_{v,t} < 1 < \sum_{v=z+1}^{S-1} \delta_{v,t} \) implies that

- sector \( v \in \{z+1, ..., S-1\} \) is active, the borrowing constraints are binding, \( q_{v,t+1} > r_t \), and the sectoral investment share is specified by equation (43);
- sector \( z \) is active, the borrowing constraints are slack, \( q_{z,t+1} = r_t \), and the sectoral investment share is specified by equation (42);
- sector \( s \in \{0, ..., z-1\} \) is inactive, \( \delta_{s,t} = 0 \).

Step 2: derive the sectoral capital-labor ratio and output price under free trade

For \( s \in \{z, ..., S-1\} \), use the definition of the relative sectoral rate of return to get

\[
\mu_{s,t+1} = \frac{q_{s,t+1}}{q_{S-1,t+1}} = \frac{\frac{\delta_{S-1,t}}{\zeta_{S-1,t+1}}}{\delta_{s,t}} = \mu_{s,A}, \quad \Rightarrow \quad \zeta_{s,t+1} = \frac{\mu_{s,A} \delta_{s,t}}{\delta_{S-1,t}} \sum_{v=z}^{S-1} \zeta_{v,t+1} = 1, \quad \delta_{S-1,t} = \sum_{v=z}^{S-1} \mu_{v,A}^* \delta_{v,t}, \quad \zeta_{s,t+1} = \frac{\mu_{s,A}^* \delta_{s,t}}{\mu_{S,A}^*}, \quad \Rightarrow \quad \frac{\delta_{s,t}}{\zeta_{S-1,t+1}} = \frac{\sum_{v=z}^{S-1} \mu_{v,A}^* \delta_{v,t}}{\mu_{s,A}^*}.
\]

Use the production function of final goods to calculate the sectoral output price,

\[
Y_{t+1} = \prod_{v=0}^{S-1} \left( \frac{V_{v,t+1}}{\eta_v} \right)^{\eta_v}, \quad V_{v,t+1} / \eta_v = Y_{t+1} / p_{v,t+1}, \quad \prod_{v=0}^{S-1} \left( \frac{V_{v,t+1}}{\eta_v} \right)^{\eta_v} = p_{S-1,A} \prod_{v=0}^{S-1} \left( \chi_{v,A} \right)^{\eta_v} = 1 \Rightarrow p_{S-1,A} = \prod_{v=0}^{S-1} \left( \chi_{v,A} \right)^{\eta_v}, \quad p_s = \chi_{s,A} \prod_{v=0}^{S-1} \left( \chi_{v,A} \right)^{\eta_v} = \frac{\mu_{s,A}^*}{\prod_{v=0}^{S-1} \left( \mu_{v,A}^* \right)^{\eta_v}}.
\]

Step 3: derive the law of motion for wage under free trade

Under free trade, for \( w_t \in (\bar{w}_z, \bar{w}_{z+1}) \), sector \( s \in \{z, ..., S-1\} \) is active in period \( t+1 \), with the sectoral output value of

\[
p_{s}^* Y_{s,t+1} = \left[ \frac{\mu_{s,A}^*}{\prod_{v=0}^{S-1} \left( \mu_{v,A}^* \right)^{\eta_v}} \right]^{\alpha} \zeta_{s,t+1} \Gamma \prod_{v=0}^{S-1} \left( \chi_{v,A} \right)^{\eta_v} = \zeta_{s,t+1} \Gamma \prod_{v=0}^{S-1} \left( \chi_{v,A} \right)^{\eta_v} = \zeta_{s,t+1} \Gamma \prod_{v=0}^{S-1} \left( \mu_{v,A}^* \right)^{\eta_v}.
\]

Combine it with equation (36) to get the law of motion for wage,

\[
w_{t+1} = (1 - \alpha) \sum_{v=z+1}^{S-1} \frac{p_{v,t+1} Y_{v,t+1}}{\zeta_{v,t+1}} L = \left( \frac{w_t}{\rho_t} \right)^\alpha, \quad \text{where} \quad \Gamma_t \equiv \frac{\sum_{v=z+1}^{S-1} \mu_{v,A}^* \delta_{v,t}}{\prod_{v=0}^{S-1} \left( \mu_{v,A}^* \right)^{\eta_v}}.
\]