Trade-Induced Sectoral Upgrading and Upstream Financial Flows

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Abstract

This paper shows that, in a world with heterogeneous financial development, deepening trade integration may have non-monotonic impacts on the direction of financial flows. By inducing the more financially developed country (North) to specialize towards the financially constrained, high-return sectors, trade integration reduces the return to capital in the unconstrained, low-return sectors, which reverses the upstream financial flows, as predicted by Antras and Caballero (2009, Journal of Political Economy). However, if the cross-country difference in financial development and the investment elasticity in North are sufficiently large, North may offshore the low-return sectors and specialize fully in the high-return sectors. In this case, upstream financial flows may not be dampened or reversed in the long run. Our findings offer an alternative perspective of understanding the persistent current account imbalances between emerging Asia and the United States.

Keywords: extensive margin, financial frictions, investment elasticity, production upgrading, upstream financial flows, wealth inequality

JEL Classification: F11, F41

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The recent globalization has two prominent features. First, emerging economies (especially China and other emerging Asia countries) have witnessed large current account surplus, while advanced economies (notably, the United States and the United Kingdom) have incurred persistent current account deficits in the past two decades.\(^1\) In the net term, capital flows have been upstream from the poor to the rich countries (Prasad, Rajan, and Subramanian, 2006), which is in stark contrast to the predictions of neoclassical theories. Second, technological progress and the removal of trade barriers have dramatically reduced the costs of transportation, communication, and coordination since the 1990s, which accelerates the integration of emerging economies into the world trading system and stimulates international fragmentation of production (Baldwin, 2016; Grossman and Rossi-Hansberg, 2006; Timmer et al., 2014). In figure 1, the left panel shows the current account imbalances of relevant economies, while the right panel shows the rising shares of emerging economies\(^2\) in the world trade in manufactures. The upward trend of emerging economies’ trade share is mainly driven by that of emerging Asia (especially China).

\[\text{Figure 1: Current Account Imbalances and World Trade Shares in 1980-2018}\]

Trade and financial flows have been analyzed separately in the literature and economists have put little research effort on their interactions. Recent works suggest that such a separation is not always innocuous (Eaton et al., 2016; Ghironi and Melitz, 2005; Jin, 2012; Ju, Shi, and Wei, 2014). In a seminal contribution to this literature, Antras and Caballero (2009) show that, if North (the rich countries) is more financially developed than South (the poor countries), the autarkic interest rate is higher in North than in South and hence, upstream financial flows are an equilibrium outcome. Besides, North has a comparative advantage in the financially constrained, high-return sector, while South has a comparative advantage in the unconstrained, low-return sector. By inducing South to specialize towards the unconstrained sector, trade integration raises the return to capital and enhances the incentive for capital to flow South (or reduce net capital outflows), which helps dampen or even reverse the upstream financial flows at the global scale.

\(^1\)Although the U.S. current account has already been in deficit since the early-1980s (Faruqee and Lee, 2009), upstream financial flows emerged at the world level from the late-1990s (Gourinchas and Rey, 2014).

\(^2\)According to the classification of IMF World Economic Outlook, emerging Asia refers to China, India, Indonesia, Malaysia, the Philippines, Thailand, and Vietnam, while emerging economies refers to the 7 emerging Asian countries and the other 16 countries, including Argentina, Bangladesh, Brazil, Bulgaria, Chile, Colombia, Hungary, Mexico, Pakistan, Peru, Poland, Romania, South Africa, Turkey, Ukraine, Venezuela.
level. Many emerging economies (e.g., the Asian Tigers, Brazil, Mexico, Argentina) that were integrated into the world trading system before 1990 were relatively small in terms of their respective share in global trade and hence, they did not have substantial impacts on the industrial structure in advanced economies. It makes perfect sense that Antras and Caballero (2009) treat South as a small open economy which takes the world prices as given.

The recent wave of globalization has a distinctive feature of integrating emerging economies in the world trading system (Hanson, 2012). As shown in the right panel of figure 1, emerging Asia’s share of world trade in manufactures rose from 4% in 1990 to 20% in 2010, while China’s share alone rose from 1.8% in 1990 to 15% in 2010. The impacts on advanced economies are unprecedented in terms of scale and speed. For example, China’s exports to the U.S. tripled within six years after its entry to the World Trade Organization (2000-2006), which reduced U.S. manufacturing price indexes by 7.6% (Amiti et al., 2018). Although imports from China significantly reduced the U.S. manufacturing jobs (Acemoglu et al., 2016; Autor, Dorn, and Hanson, 2016), the global export expansion of U.S. products created a considerable number of jobs (Feenstra, Ma, and Xu, 2017). Overall, the U.S. manufacturing sector had a decline in jobs by 19% and a rise in labor productivity by 34% between 2000 and 2007, implying that the jobs offshored are relatively low value-added. Similar patterns exist in the United Kingdom, France, and some other European countries (Bloom, Draca, and van Reenen, 2016; Malgouyres, 2017; Pessoa, 2018). Thus, one should not ignore the impacts of the recent trade globalization on domestic prices and industrial structure in advanced economies.

By featuring the impacts of trade on the industrial structure in North, we revisit the results of Antras and Caballero (2009) and find that trade integration may have non-monotonic impacts on the direction of financial flows. If the international price differentials are sufficiently large, trade induces North to specialize fully in the financially dependent, high-return sectors. Due to the borrowing constraints, the high-return sectors cannot absorb the entire domestic saving in the short run so that North witnesses financial outflows and domestic investment falls. If the static gains from trade are so large as to dominate the fall in domestic investment, trade raises aggregate income in North, which stimulates the investment in the high-return sectors. If the latter eventually exceeds domestic saving, North witnesses financial inflows in the long run.

In our model, the financial flow re-reversal requires three conditions. First, South should be sufficiently large so that North-South trade can have substantial impacts on North’s industrial structure. Second, the North-South difference in financial development should be sufficiently large so that international price differentials are large enough to ensure full specialization in North. Third, the investment demand in North should be sufficiently elastic with respect to the income rises so that domestic investment can eventually exceed domestic saving. As the integration of emerging Asia (especially, China) into the world trading system has fundamentally triggered the industrial upgrading in advanced economies (particularly, the U.S.), upstream financial flows may not be dampened or reversed in the long run. By exploring the case of full

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3The non-farm business sector and the non-financial corporations in the U.S. only recorded the rise in labor productivity by 20% and 18%, respectively. Data Source: Manufacturing, Durable Manufacturing and Non-durable Manufacturing Sectors, the U.S. Bureau of Labor Statistics, retrieved on 26 January 2019.

4If the international price differentials are small, trade only induces North to specialize partially in the high-return sectors, which reduces the return to capital in the low-return sectors and weakens the incentive for capital to flow North. Then, upstream financial flows are reversed, as predicted by Antras and Caballero (2009).
specialization in North, we complement the findings of Antras and Caballero (2009).

To be specific, we feature North as a small open economy in an overlapping-generation model with three key assumptions. In the presence of the borrowing constraints and the sector-specific minimum investment requirements (MIR), only the agents with sufficiently high net wealth can meet the MIR and invest in a particular sector (sector 1) and they are called “entrepreneurs”, while others just invest in the sector with no MIR (sector 0) and/or lend out their net wealth. If the borrowing constraints are binding in equilibrium, the return to capital is higher in sector 1 than in sector 0. Given individual wealth heterogeneity, the higher the level of financial development, the less tight the borrowing constraints, the larger the mass of entrepreneurs and the fraction of domestic investment in sector 1, the lower the relative price of good 1. The level of financial development becomes a determinant of comparative advantage in our model. The larger the North-South difference in financial development, the larger the static gains from trade, the more likely trade induces North to specialize fully in sector 1 as well as raises aggregate income in the short run.

The higher aggregate income raises the average net wealth and allows more agents to become entrepreneurs, which amplifies the investment expansion along the extensive margin. The smaller the wealth dispersion, the more responsive the mass of entrepreneurs with respect to income changes, the larger the extensive margin effect and the investment elasticity, the larger the investment expansion and the dynamic gains from trade, the more likely domestic investment exceeds domestic saving and North witnesses financial inflows in the long run.

To sum up, the borrowing constraints and the sector-specific MIR jointly endogenize the extensive margin of sectoral investment, while wealth heterogeneity matters for the sectoral investment elasticity. The North-South difference in financial development and the investment elasticity in North jointly determine the impacts of trade integration on financial flows.

In the two-sector setting, the financial flow re-reversal occurs if North fully specializes in the high-return sector and domestic investment eventually exceeds domestic saving. In section 4 and appendix B, we embed this mechanism in a multi-sector setting where sectors are ranked in terms of the MIR. In the sector with a higher MIR, the mass of investors and the sectoral investment are smaller and hence, the sectoral rate of return is also higher. Under the three conditions mentioned above, trade may allow North to sequentially offshore the low-MIR, low-return sectors and upgrade to the high-MIR, high-return sectors over time. Once the lowest-return sector is offshored, the investment demand in that sector vanishes and the fall in domestic investment reduces financial inflows in the short run. If the static and dynamic gains from trade are sufficiently large, aggregate income rises over time, which stimulates domestic investment and leads to financial inflows. Then, the next lowest-return sector may be offshored. If so, the financial flow re-reversal occurs again. Thus, trade-driven sequential sectoral upgrading in North may make the financial flow re-reversal a recurrent phenomenon.

**Related Literature** Our paper belongs to the literature explaining upstream financial flows as an equilibrium outcome in a world with heterogeneous financial development (Caballero, Farhi, and Gourinchas, 2008; Gourinchas and Rey, 2014; Ju and Wei, 2010; Mendoza, Quadrini, and Rios-Rull, 2009; von Hagen and Zhang, 2014). By revisiting the results of Antras and Caballero

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5Zhang (2017) offers a detailed literature review on the MIR.
(2009), we find that deepening trade integration may not necessarily dampen or reverse upstream financial flows, if it induces North to upgrade its industrial structure and specialize fully in the high-return sectors. Our findings offer an alternative perspective for understanding the persistent current account imbalances between emerging Asia and the United States when the former has changed the world trading system substantially in the recent decades.

Kletzer and Bardhan (1987) show that better access to capital can be a source of comparative advantage. It was then followed by a literature on financial development and international trade (Beck, 2002; Chesnokova, 2007; Ju and Wei, 2005, 2011). Our model shares some key features with Wynne (2005) and Matsuyama (2005). Wynne (2005) argues that, when access to credit differs across sectors, wealthier nations have the comparative advantage in sectors facing more severe financial frictions. In a Ricardian model with a continuum of tradable goods and the sector-specific borrowing constraints, Matsuyama (2005) shows that trade allows the more (less) financially developed country to specialize fully in the sectors with the tighter (looser) borrowing constraints. Our model features the sector-specific MIR, while investors in all sectors are subject to the same borrowing constraints. Thus, the extensive margin of sectoral investment is endogenous, which is crucial for full specialization and the financial flow re-reversal in North.

Jin (2012) integrates factor-proportions-based trade and financial flows in an OLG model and shows that capital tends to flow to countries that are more specialized in capital-intensive industries. In a dynamic Heckscher-Ohlin model where sectors differ in factor intensity, Ju, Shi, and Wei (2018) find that trade reform and liberalization lead to capital outflows for the developing, labor-abundant country and capital inflows for the developed, capital-abundant country, which contributes to upstream financial flows. Besides, they find that both financial frictions and labor market frictions tend to dampen the current account responses to trade liberalization. Ju, Shi, and Wei (2014) show that, in the case of a labor productivity shock, the labor market rigidity dampens the responses of intra-temporal trade (goods trade), leading to large and persistent current account adjustments. In our two-sector model, labor is perfectly mobile across sectors, while the individual borrowing constraints in sector 1 slow down the cross-sector investment reallocation. Upon trade integration, labor is reallocated towards sector 1 at a faster rate than capital in North, which raises the capital-labor ratio and reduces the return to capital in the unconstrained sector. Essentially, the re-reversal of financial flow is driven by the asymmetric labor and capital reallocation,\(^6\) which reflects the relative rigidity of financial and labor markets.

Obstfeld and Rogoff (2001) argue that trade costs have the potential to resolve qualitatively several puzzles in international macroeconomics, which is confirmed by Eaton et al. (2016) in a quantitative, multi-country model. Alessandria and Choi (2019) find that two-thirds of the fluctuations in the U.S. trade balance are driven by changes in trade barriers. Reyes-Heroles (2017) finds that the substantial decline in trade frictions contributes to global trade imbalances. In our model, the substantial declines in trade frictions and costs deepen North-South trade integration so that North may offshore low-return production activities and upgrade fully in high-return activities. Along the convergence path, the inflows of cheap foreign funds allows North to finance

\(^6\)Besides financial frictions, one can also introduce labor market frictions into our model and study how the relative rigidity in the financial and labor markets may affect the current account responses to trade integration. Consider an extreme case where labor is perfectly immobile across sectors. When trade induces North to specializes towards sector 1, the investment reallocation reduces the capital-labor ratio and raises the return to capital in sector 0. In this case, upstream financial flows are amplified rather than dampened.
its trade deficits, consistent with the findings of Reyes-Heroles (2017).

Baldwin (1992) argues that trade has large dynamic effects on output and welfare via human and physical capital accumulation. In a multi-country model with capital accumulation, Ravikumar, Santacreu, and Sposi (2018) find that the dynamic gains from trade liberalization are substantially larger than the static gains. In our model, the investment elasticity in the high-return sectors determines the dynamic gains from trade which are crucial for the re-reversal of financial flows in North. Market frictions and regulatory requirements that weaken the investment elasticity in the high-return sectors may slow down the production upgrading in North, which reduces the magnitude or even the possibility of upstream financial flows.

Income and wealth inequality has been rising almost everywhere across the world (Atkinson, Piketty, and Saez, 2011; Piketty, 2014). The literature has documented the impacts of trade on income inequality (Dabla-Norris et al., 2015; Helpman et al., 2017; Jaumotte, Lall, and Papa-georgiou, 2013), while the impacts of inequality on trade patterns are addressed mainly by the demand-based theory with non-homothetic preferences (Fajgelbaum, Grossman, and Helpman, 2011; Fajgelbaum and Khandelwal, 2016; Fieler, 2011). We offer a supply-side theory in which wealth inequality weakens the investment elasticity and the dynamic gains from trade in North.

The rest of the paper is structured as follows. Section 1 sets up the model. Section 2 analyzes the equilibrium allocations under autarky versus under financial integration. Section 3 shows that trade may lead to the reversal and the re-reversal of financial flows in the two-sector setting. Section 4 extends our mechanism in a three-sector setting and shows that trade may induce North to sequentially offshore the low-return sectors and upgrade to the high-return sectors, leading to the recurrent reversal and re-reversal of financial flows. Section 5 concludes with final remarks. Appendices include some supporting materials and proofs.

1 The Model

The model features a small open economy (country N) in a two-sector, overlapping-generation framework with three key assumptions: (1) the sector-specific MIR for capital formation, (2) the borrowing constraints, and (3) the heterogeneity in individual net wealth.

In country N, a continuum of agents indexed by \( j \in [0, 1] \) are born every period and live for two periods, young and old. The population size of each generation is constant and normalized at unity. Agents only consume when old.\(^7\) When young, agent \( j \) supplies its labor \( l_j = (1 - \theta)\epsilon_j \) inelastically to the market. \( \epsilon_j \in (1, \infty) \) follows the Pareto distribution,\(^8\) where \( G(\epsilon_j) = 1 - \epsilon_j^{-\theta} \) and \( \theta \in (0, 1) \) denote respectively the cumulative distribution function and the inverse of the shape parameter. The aggregate labor supply is constant at \( L = \int_1^\infty l_j dG(\epsilon_j) = 1. \)

\(^7\)Given that agents are endowed with labor only when young and they consume only when old, domestic saving is interest-inelastic, which simplifies the credit supply dynamics and allows us to focus on the dynamics of entrepreneurial credit demand and investment. The overlapping-generation framework ensures that some agents cannot overcome the borrowing constraints by accumulating their savings over time.

\(^8\)Pareto distribution is widely used in the literature to feature the income and wealth distribution (Atkinson, Piketty, and Saez, 2011; Gabaix, 2009; Jones, 2015). The top tail of income distribution is very well approximated by a Pareto distribution (Kuznets and Jenks, 1953; Piketty and Saez, 2003). Besides, assuming the Pareto distribution for the individual labor endowment allows us to get the analytical solution.
There are two sectors, indexed by \( s \in \{0, 1\} \). In period \( t \), \( K_{s,t} \) units of physical capital and \( L_{s,t} \) units of labor are hired to produce \( Y_{s,t} \) units of goods in sector \( s \). Physical capital fully depreciates after use. \( V_{0,t} \) units of good 0 and \( V_{1,t} \) units of good 1 are combined to produce \( Y_t \) units of final goods.\(^9\) Sectoral and final goods are internationally tradable, while labor and physical capital are not. Final goods can be consumed contemporaneously or converted into sector-specific physical capital which is available for production in period \( t+1 \). Let \( M_t \) denote aggregate investment. Let \( \delta_t \) and \( \zeta_{t+1} \) denote respectively the fractions of aggregate investment and labor inputs for the production of good 1 in period \( t+1 \).\(^10\) There is no uncertainty in the model. The markets for goods and productive factors are competitive. The final good is chosen as the numeraire. Let \( w_t \) denote the wage rate. Let \( p_{s,t} \) and \( q_{s,t} \) denote respectively the price of good \( s \) and the rental price of capital in sector \( s \). To sum up,

\[
Y_{s,t} = \left( \frac{K_{s,t}}{\alpha} \right)^{1-\alpha} \left( \frac{L_{s,t}}{1-\alpha} \right)^{1-\alpha}, \quad q_{s,t}K_{s,t} = \alpha p_{s,t}Y_{s,t}, \quad w_tL_{s,t} = (1 - \alpha)p_{s,t}Y_{s,t},
\]

\[
Y_t = \left( \frac{V_{1,t}}{\eta} \right)^{1-\eta} \left( \frac{V_{0,t}}{1-\eta} \right)^{1-\eta}, \quad p_{1,t}V_{1,t} = \eta Y_t, \quad p_{0,t}V_{0,t} = (1 - \eta)Y_t.
\]

\[
K_{1,t+1} = \delta_tM_t, \quad L_{1,t+1} = \zeta_{t+1}L, \quad K_{0,t+1} = (1 - \delta_t)M_t, \quad L_{0,t+1} = (1 - \zeta_{t+1})L.
\]

where \( \alpha, \eta \in (0, 1) \). Let \( \chi_t = \frac{p_{0,t}}{p_{1,t}} \) and \( \mu_t = \frac{q_{0,t}}{q_{1,t}} \) denote the sectoral output price ratio and the sectoral rental-price-of-capital ratio, respectively. Combine equations (1) and (3) to get

\[
\chi_t = \mu_t^\alpha, \quad \delta_t = \frac{\zeta_{t+1}}{1 + (1 - \zeta_{t+1})(\mu_{t+1} - 1)}.
\]

Due to the frictionless labor market, the wage rate equalizes the marginal revenue of labor (MRL, hereafter) across sectors. If the sectoral investment were also frictionless, the marginal revenue of capital (MRK, hereafter) would also equalize across sectors. If the sectoral price, \( \chi_{t+1} = 1 \). In this case, investment and labor would be allocated across sectors in equal proportions, \( \delta_t = \zeta_{t+1} \).

In the following, we discuss the implications of the three assumptions mentioned above.

**Cross-Sector Investment Distortion due to Financial Frictions and Sector-Specific MIR**

Agent \( j \) born in period \( t \) has three options to save its labor income \( n_{j,t} = w_tl_j \) over time, i.e., investing in the two sectors and lending to the credit market. By investing \( k_{j,0,t+1} \) units of final goods in period \( t \), the agent gets \( k_{j,0,t+1} \) units of physical capital for sector 0 in period \( t+1 \) and hence, the gross rate of return to its investment is \( q_{0,t+1} \). It gets the same one-for-one output of physical capital for sector 1, if its investment size meets the MIR, \( k_{j,1,t+1} \geq m > 0 \); otherwise, it gets zero output. Due to limited commitment, agent \( j \) can borrow up to a fraction \( \lambda \) of its investment in sector 1 at the gross interest rate \( r_1 \) and use its net wealth to cover the gap,

\[
k_{j,1,t+1} - n_{j,t} \leq \lambda k_{j,1,t+1},
\]

where \( \lambda \in (0, 1) \) measures the level of financial development.\(^11\)

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\(^9\)In the absence of international trade in sectoral goods, domestic absorption is equal to domestic output at the sectoral level, \( V_{s,t} = Y_{s,t} \). It is not the case, \( V_{s,t} \neq Y_{s,t} \), in the presence of trade flows in sectoral goods.

\(^10\)As the model dynamics are essentially driven by the endogenous, cross-sector allocation of investment and labor, we only need to focus on the relative sectoral inputs of investment and labor rather than on their levels.

\(^11\)Antras and Caballero (2009) offer the micro-foundations for this borrowing constraint.
Both sectoral goods are essential for the production of final goods. In the absence of trade in sectoral goods, sector 0 is active in country N and hence, the sector-0 rate of return is equal to the interest rate. As shown in section 3 and appendix A, trade in sectoral goods may induce country N to fully specialize in sector 1. If so, the interest rate exceeds the sector-0 rate of return in equilibrium, confirming that nobody invests in sector 0 in country N. The interest rate cannot exceed the sector-1 rate of return; otherwise, nobody would invest there. If the interest rate is below the sector-1 rate of return, the agent borrows to the limit for leveraged investment.

\[ r_t = \begin{cases} 
q_{0,t+1}, & \text{if } K_{0,t+1} > 0 \text{ or equivalently } \delta_t < 1; \\ 
\in (q_0,t+1, q_{1,t+1}], & \text{if } K_{0,t+1} = 0 \text{ or equivalently } \delta_t = 1.
\end{cases} \]  

(6)

Consider first the case where sector 0 is active and the borrowing constraints in sector 1 are binding, \( q_{1,t+1} > r_t = q_{0,t+1} \). Given the rate-of-return spread, agent \( j \) invests its entire labor income in sector 1 and borrows to the limit, as long as it can meet the MIR.

\[ k_{j,1,t+1} = \frac{n_{j,t}}{1 - \lambda} = \frac{w_t(1 - \theta)\epsilon_j}{1 - \lambda} \geq m, \quad \Rightarrow \quad \epsilon_j \geq \epsilon_t \equiv \frac{m(1 - \lambda)}{w_t(1 - \theta)}. \]  

(7)

The agents with \( \epsilon_j \geq \epsilon_t \) can meet the MIR and are called entrepreneurs, with the mass \( \tau_t = -\epsilon_j^\frac{1}{\theta} \). When old, they get the investment return, repay the debt, and consume the rest \( c_{j,t+1} = q_{1,t+1}k_{j,1,t+1} - r_t(k_{j,1,t+1} - n_{j,t}) \). The agents with \( \epsilon_j \in [1, \epsilon_t] \) cannot meet the MIR and are called households. When young, they invest \( k_{j,0,t+1} \) in sector 0 and lend out \( n_{j,t} - k_{j,0,t+1} \); when old, they consume \( c_{j,t} = n_{j,t}r_t \).

Wealth Heterogeneity and the Investment Elasticity in The Constrained Sector

The investment elasticity in the constrained sector (sector 1) with respect to aggregate income is key to income dynamics, trade flows, and the direction of financial flows in country N. Given the inelastic labor supply, the wage rate is proportional to aggregate income, \( w_t = \frac{(1 - \alpha)Y_t}{L} \), and determined by the preinstalled sectoral capital. Thus, we treat the wage rate as a predetermined variable and use it as a proxy for aggregate income.

Let \( \bar{w} \equiv \frac{m(1 - \lambda)^{1-\theta}}{1 - \theta} \). If the borrowing constraints are binding, the total investment in sector 1 and its elasticity to aggregate income are respectively,

\[ K_{1,t+1} = \int_{L_t}^{\infty} k_{j,1,t+1}dG(\epsilon_j) = \frac{\epsilon_j^{1-\theta}}{1 - \lambda} w_tL = \frac{\tau_t^{1-\theta} w_tL}{1 - \lambda} = \left(\frac{w_t}{\bar{w}}\right)^{1-\theta} w_tL, \]  

\[ \frac{\partial \ln K_{1,t+1}}{\partial \ln w_t} = \frac{\partial \ln w_t}{\partial \ln w_t} + (1 - \theta) \frac{\partial \ln \tau_t}{\partial \ln w_t} = \frac{1}{\bar{w}} > 1. \]  

(8)  

(9)

A rise in aggregate income raises the wage rate and the individual wealth, which allows each entrepreneur to borrow and invest more as well as allows more agents to become entrepreneurs.

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12 As all agents can lend to the credit market and invest in sector 0, the two options are perfect substitutes.
13 If \( q_{0,t+1} = r_t \) holds, the household investment in sector 0 is indeterminate, \( k_{j,0,t+1} \in [0, n_{j,t}] \). If \( q_{0,t+1} < r_t \) holds, the household lends out its entire savings \( k_{j,0,t+1} = 0 \). In both cases, their savings return is \( c_{j,t}^{01} = n_{j,t}r_t \).
14 Due to the inelastic labor supply, exogenous variables can be characterized as the functions of \( w_t \). If the labor supply is assumed to be elastic, the wage rate becomes a jump variable, which makes the model less tractable.
Thus, the investment in sector 1 rises along the intensive margin and the 
extensive margin, respectively. Given \( \theta \in (0, 1) \), the investment elasticity exceeds unity, due to the 
extensive margin effect. The smaller the \( \theta \), the less dispersed the net wealth of young agents, 
the more responsive the mass of entrepreneurs to income change \( \frac{\partial \ln \tau_t}{\partial \ln w_t} = \frac{1}{\theta} \), the stronger the 
extensive margin effect, the more elastic the total investment in the constrained sector.

The fraction of domestic investment allocated in sector 1 is

\[
\delta_t = \frac{K_{1,t+1}}{M_t} = \left( \frac{w_t}{\bar{\theta}} \right)^{\frac{1-\theta}{\theta}} w_t L. \tag{10}
\]

Remark: the borrowing constraints and the sector-specific MIR jointly distort the cross-sector 
investment allocation \( \delta_t \) along the extensive margin, while the investment elasticity in the 
constrained sector is inversely related to the dispersion of individual net wealth \( \theta \).

In this model, we analyze three scenarios, (1) international autarky, (2) financial integration, 
and (3) dual integration.\(^{15}\)

Under international autarky, the markets for credit, labor, sector-specific capital and goods 
clear domestically.\(^{16}\) As young agents save all labor income, domestic saving is \( w_t L \).

\[
\int_{\varphi_t}^{\infty} (k_{j,1,t+1} - n_{j,t})dG(\epsilon_j) = \int_{\varphi_t}^{\infty} (n_{j,t} - k_{j,0,t+1})dG(\epsilon_j), \Rightarrow \sum_{s=0}^{1} K_{s,t+1} = M_t = w_t L, \tag{11}
\]

\[
L_{0,t} + L_{1,t} = L, \quad K_{1,t} = \delta_{t-1} M_{t-1}, \quad K_{0,t} = (1 - \delta_{t-1}) M_{t-1}, \tag{12}
\]

\[
V_{0,t} = Y_{0,t}, \quad V_{1,t} = Y_{1,t}. \tag{13}
\]

In the case of \( q_{1,t+1} = q_{0,t+1} \), the sectoral rate of return equalizes and hence, agents who can 
meet the MIR do not have the strong incentive to invest their entire labor income in sector 1 and 
borrow to the limit. Despite the indeterminacy at the individual level, the allocation can be easily 
solved at the aggregate level. See the proof of proposition 1 for detailed characterization.

**Definition 1.** Under international autarky, a market equilibrium is a set of choices of agents 
\( \{n_{j,t}, k_{j,s,t+1}\} \) and aggregate variables \( \{Y_t, Y_{s,t}, K_{s,t}, L_{s,t}, V_{s,t}, p_{s,t}, q_{s,t}, w_t, r_t, \varphi_t, \zeta_t, \delta_t, \zeta_t\} \), satisfying equations (1)-(4), (6)-(7), (10)-(13), where \( s \in \{0, 1\} \).

Under financial integration, agents are allowed to borrow and lend abroad so that the interest 
rate is aligned to the world level. Let \( \phi_t = \frac{M_t - w_t L}{w_t L} \) denote the financial inflows normalized 
by domestic saving, with negative values indicating financial outflows. Final goods are freely 
traded and serve as the vehicle for international financial flows, while sectoral goods are subject 
to prohibitively high trade costs so that the markets for sectoral goods still clear domestically. 
Use the asterisk superscript to denote the variables and parameters in the rest of the world.

**Definition 2.** Under financial integration, a market equilibrium is a set of choices of agents 
\( \{n_{j,t}, k_{j,s,t+1}\} \) and aggregate variables \( \{Y_t, Y_{s,t}, K_{s,t}, L_{s,t}, V_{s,t}, p_{s,t}, q_{s,t}, w_t, r_t, \varphi_t, \zeta_t, \delta_t, \zeta_t\} \), satisfying equations (1)-(4), (6)-(7), (10), (12)-(13), and \( r_t = r_t^* \), where \( s \in \{0, 1\} \).

\(^{15}\)In Appendix A, we analyze the case of trade integration where agents are allowed to freely import and export 
sectoral goods, while international financial flows are not allowed.

\(^{16}\)According to the Walras’ law, the market clearing conditions for sectoral goods and productive factors jointly 
imply that the market for final goods also clears, \( \int_{\varphi_t}^{\infty} \epsilon_{j,t-1}^dG(\epsilon_j) + \int_{\varphi_t}^{\infty} \epsilon_{j,t-1}^dG(\epsilon_j) + K_{1,t+1} + K_{0,t+1} = Y_t \).
Under dual integration, agents are allowed to freely borrow/lend abroad as well as import/export sectoral goods. Thus, the interest rate is aligned to the world level and so are the sectoral prices. Financial flows decouple aggregate investment from domestic saving, while trade flows decouple domestic absorption from domestic production in each sector.

Definition 3. Under dual integration, a market equilibrium is a set of choices of agents \( \{n_{j,t}, k_{j,s,t+1}\} \) and aggregate variables \( \{Y_t, Y_{s,t}, K_{s,t}, L_{s,t}, V_{s,t}, q_{s,t}, w_t, r_t, \zeta_t, \delta_t, \zeta_t\} \), satisfying equations (1)-(4), (6)-(7), (10), (12), \( r_t = r^*_t \) and \( p_{s,t} = p^*_{s,t} \), where \( s \in \{0, 1\} \).

2 Upstream Financial Flows and Allocative Efficiency

In this section, we first explore the mechanism through which the interest rate and the sectoral prices are determined under autarky. Then, we discuss the patterns of financial flows as well as the dynamics of aggregate income and allocative efficiency when country N moves from the autarkic to the new steady state under financial integration. It sets the stage for analyzing the impacts of free trade on the pattern of financial flows in section 3.

Under autarky and under financial integration, trade in sectoral goods is not allowed and hence, the sectoral absorption is equal to the sectoral output, \( V_{s,t} = Y_{s,t} \). It has two implications. First, sector 0 is active and hence, its rate of return is equal to the interest rate, \( q_{0,t+1} = r_t \). Second, given the frictionless labor market, \( V_{s,t} = Y_{s,t} \) ensures that the sectoral fraction of aggregate labor input is equal to the sector share in the final goods production,\(^{17} \zeta_{t+1} = \eta \).

Let \( \rho = \frac{\alpha}{1-\alpha} \). The law of motion for wage\(^{18} \) describes the dynamics of aggregate income.

\[
w_{t+1} = \left( \frac{\Gamma_t M_t}{\rho L} \right)^\alpha, \quad \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \left[ 1 - \left( 1 - \alpha \right) \right] \left( \frac{\partial \ln M_t}{\partial \ln w_t} - \frac{\partial \ln \Gamma_t}{\partial \ln \delta_t} + \frac{\partial \ln \delta_t}{\partial \ln \Gamma_t} \right), \tag{14}
\]

where \( \Gamma_t = \left( \frac{\delta_t}{\eta} \right)^{\eta} \left( 1 - \frac{\delta_t}{1 - \eta} \right)^{1-\eta} \) measures allocative efficiency. If the borrowing constraints are slack, the sectoral investment is efficient, \( \delta_t = \zeta_{t+1} = \eta \), and \( \Gamma_t = 1 \). If the borrowing constraints are binding, the investment in sector 1 is inefficiently low \( \delta_t < \zeta_{t+1} = \eta \) and the opposite applies to sector 0, implying that \( \Gamma_t < 1 \). Given \( \frac{\partial \ln \Gamma_t}{\partial \ln \delta_t} = \frac{\eta - \delta_t}{1 - \delta_t} > 0 \), the larger the \( \delta_t \), the less the sectoral investment distortion, the more efficient the cross-sector allocation.

Consider the dynamics of aggregate income. A rise in current income affects future income in three ways. First, it may affect the size of domestic investment. Second, if the borrowing constraints are binding, the income rise allows more agents to become entrepreneurs, which shifts domestic investment towards sector 1 and improves allocative efficiency; if the borrowing constraints are slack, \( \Gamma_t = 1 \) and the income rise does not affect allocative efficiency. Third, due to the decreasing marginal product of capital (DMPK, hereafter), capital formation raises future income less than proportionately. The three effects are summarized in equations (14) which we use to analyze the model stability and the steady-state property under two scenarios.

\(^{17}\)See the proof of proposition 1 for technical derivation and intuitive explanation.

\(^{18}\)As shown in the next two subsections, the size of domestic investment \( M_t \) and the indicator of allocative efficiency \( \Gamma_t \) are the functions of current income. As the wage rate can serve as a proxy for aggregate income, both \( M_t \) and \( \Gamma_t \) are also the functions of \( w_t \). Thus, equation (14) characterizes the law of motion for wage.
2.1 The Autarkic Equilibrium

Under autarky, domestic investment is financed by domestic saving $M_t = w_tL$ and hence, the investment size effect is constant at unity, $\frac{\partial \ln M_t}{\partial \ln w_t} = 1$. Let $\tilde{\delta}_t \equiv \min\{\left(\frac{w_t}{\bar{w}}\right)^\frac{1}{\eta} - 1\}$ denote the maximum share of domestic investment allocated in sector 1 when all entrepreneurs borrow and invest to the limit. For $w_t \geq \bar{w}_A \equiv \bar{w} \eta^{1-\theta}$, the efficient allocation is feasible, $\tilde{\delta}_t \geq \eta$. In equilibrium, the sectoral investment is efficient, i.e., $\delta_t = \zeta_{t+1} = \eta \leq \tilde{\delta}_t$ and $\Gamma_t = 1$, the sectoral rate of return equalizes, and the borrowing constraints are slack. As the allocative efficiency effect is mute, a rise in current income raises the sectoral investment in equal proportions, while the DMPK effect dampens the effect of capital formation on future income.

$$w_{t+1} = \left(\frac{w_t}{\rho}\right)^\alpha, \quad \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \left[1 - \left(1 - \alpha\right)\right]\left(\frac{1}{\text{DMPK effect}} + \frac{0}{\text{inv. size effect}} + \frac{0}{\text{alloc. effic. effect}}\right) < 1. \quad (15)$$

The DMPK effect is a convergence force that drives country N towards a steady state. The smaller the $\alpha$, the stronger the DMPK effect, the faster the convergence.

For $w_t < \bar{w}_A$, the efficient allocation is infeasible. In equilibrium, the investment in sector 1 is inefficiently low, $\delta_t = \xi_t < \zeta_{t+1} = \eta$ and $\Gamma_t < 1$, which gives rise to the sectoral rate-of-return wedge and the binding borrowing constraints, $q_{1,t+1} > q_{0,t+1} = r_t$. A rise in current income triggers the investment reallocation $\frac{\partial \ln \xi_t}{\partial \ln w_t} = \frac{1}{\theta} - 1$ and improves allocative efficiency.

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \left[1 - \left(1 - \alpha\right)\right]\left(\frac{1}{\text{DMPK effect}} + \frac{\eta - \delta_t}{\text{inv. size effect}} + \frac{\eta - \delta_t}{\delta_t} \left(\frac{1}{\theta} - 1\right)\right). \quad (16)$$

The allocative efficiency effect is positively related to the investment elasticity in the constrained sector $(\frac{1}{\theta})$, while the DMPK effect is negatively related to the share of capital in the sectoral production $(\alpha)$. If the former is dominated by the latter at any steady state, there is a unique steady steady. Let $X_A$ denote the steady-state value of variable $X_t$ under autarky.

**Proposition 1.** Let $\tilde{\theta} \equiv \frac{1}{1 + \left(1 - \alpha\right)\left(1 - \eta\right)\rho^\alpha} < \alpha$, $Z \equiv \frac{1}{1 + \frac{\eta - \delta_t}{\theta} - \rho^\alpha}$, and $\tilde{\lambda}_A \equiv 1 - Z^{1-\theta}$.

For $\theta \in [\tilde{\theta}, 1)$ and $\lambda \in [0, \tilde{\lambda}_A)$, there is a unique, stable steady state under autarky where the borrowing constraints are binding and the sectoral investment is inefficient.\(^{19}\)

Domestic investment in period $t$ is funded by domestic saving $M_t = w_tL$, which yields the total return $\sum_{s=0}^1 q_{s,t+1}K_{s,t+1} = \rho w_{t+1}L$ in period $t + 1$. Define the social rate of return

$$\Upsilon_t \equiv \sum_{s=0}^1 q_{s,t+1}K_{s,t+1} = \rho w_{t+1}, \quad (17)$$

The interest rate is coupled with the rate of return in sector 0,

$$r_t = q_{0,t+1} = \Upsilon_t[1 - \eta(1 - \mu_{t+1})] < \Upsilon_t. \quad (18)$$

In the autarkic steady state, $w_{t+1} = w_t$ and the social rate of return is constant at $\Upsilon_A = \rho$, while the interest rate depends on the sectoral rate-of-return ratio $\mu_A$.

\(^{19}\)Figure 21 in the proof of proposition 1 shows $\tilde{\theta}$ in the $(\alpha, \theta)$ space and $\tilde{\lambda}_A$ in the $(\lambda, Z)$ space.
Country N and the rest of the world are inherently identical, except that country N is a small country and more financially developed than the rest of the world.\footnote{The assumption that country N is a small open economy simplifies our analysis. One can embed our mechanism into a two-country model where the impacts of trade and/or financial flows on the world prices depend on the relative size of the two countries. Our findings still hold qualitatively, while the analysis become less tractable.}

**Assumption 1.** $\theta \in (\hat{\theta}, 1)$, $0 < \lambda^* < \lambda < \tilde{\lambda}_A$, and $\frac{L}{L+L_t} \to 0$.\footnote{By definition, the composite parameter $Z$ is independent of the level of financial development and the population size. Thus, $Z$ takes the same value for country N and for the rest of the world.}

Under assumption 1, there is a unique, autarkic steady state with the binding borrowing constraints in country N as well as in the rest of the world. The higher the level of financial development, the larger the fraction of entrepreneurs in the population, the smaller the cross-sector investment distortion, the closer the sectoral price ratio to its efficient level and the higher the income level. Thus, income per capita is higher in country N than in the rest of the world, while good 1 (0) is cheaper in country N (the rest of the world). Our model features the level of financial development as a determinant of comparative advantage.

**Lemma 1.** $w^*_A < w_A < \rho^{-\rho}$, $\chi^*_A < \chi_A < 1$, $\mu^*_A < \mu_A < 1$, and $r^*_A < r_A < \rho$.

### 2.2 The Equilibrium under Financial Integration

Suppose that the world economy is initially at the autarkic steady state. From period 0 on, agents are allowed to borrow and lend abroad. Given the initial interest rate differential, country N receives financial inflows and the interest rate is aligned to the world level. Financial inflows also decouple domestic investment from domestic saving and hence, the investment size effect is not constant at unity any more. In the following, we analyze the impacts of financial flows on income dynamics, allocative efficiency, and comparative advantage in country N.

In period 0, country N witnesses the fall in the interest rate, $r_0 = r^* < r_A$.\footnote{As country N is a small open economy, its financial opening does not affect the world prices and hence, we drop off the subscript, $r^* = r^*_A$.} which induces households to lend less and invest more in sector 0, $K_{0,0} > K_{0,A}$. Due to the predetermined wage rate $w_0 = w_A$ and the constant leverage multiplier, the mass of entrepreneurs and the total investment in sector 1 remain unchanged,\footnote{It seems extreme that financial inflows do not change the investment size in sector 1 in period 0. The point we want to make is that, as long as individual investment in sector 1 is subject to the borrowing constraints and labor is perfectly mobile, the cross-sector reallocation of investment must lag behind that of labor upon financial integration, which worsens allocative efficiency. One can use other forms of borrowing constraints, such as the one shown in the appendix of Antras and Caballero (2009). As the investment in the constrained sector cannot respond to financial flows as fast as labor, allocative efficiency gets worsened in period 0.} $K_{1,1} = K_{1,A}$, according to equation (8). It implies that the inflows of cheap foreign funds exactly cover the fall in household lending to sector 1. In other words, by crowding out household lending, financial inflows indirectly lead to a higher investment in sector 0. It amplifies the cross-sector investment distortion $\delta_0 = \frac{K_{1,1}}{K_{0,0}+K_{1,1}} < \delta_A = \frac{K_{1,A}}{K_{0,A}+K_{1,A}} < \eta$, and worsens allocative efficiency. As the investment size effect dominates the allocative efficiency effect, aggregate income rises in period 1.\footnote{As the interest rate falls in period 0 $r_0 = r^* < r_A$, $\frac{\partial \ln X_0}{\partial \ln r^*} < 0$ implies a rise in variable $X_t$, i.e., $X_0 > X_A$. See equations (59) for the derivation of the two effects.}
\[ \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = [1 - (1 - \alpha)] \left[ \frac{1 - \delta_t}{1 - \alpha(1 - \eta)} \right] + \frac{\hat{\theta} (\eta - \delta_t)}{\rho \eta} \]  

From period 1 on, as long as \( w_t < \bar{w}_F \), the borrowing constraints are binding. A rise in current income allows entrepreneurs to borrow more as well as allows more agents to become entrepreneurs. Given \( r_t = r^* \), the rise in the entrepreneurial credit demand stimulates financial inflows, which raises the size of domestic investment as well as shifts the investment composition towards sector 1. Thus, allocative efficiency improves over time. The investment size effect and the allocative efficiency effect\(^{25}\) jointly compete with the DMPK effect, which drives the income dynamics in country N.

\[ \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = [1 - (1 - \alpha)] \left[ \frac{\hat{\theta} (1 + \delta_t)}{1 - \alpha(1 - \eta)} \right] + \frac{\hat{\theta} (\eta - \delta_t)}{\rho \eta} \]  

For \( w_t \geq \bar{w}_F \), the mass of entrepreneurs is so high that the sectoral investment is efficient. Thus, the investment rate of return equalizes across sectors, the borrowing constraints are slack, and \( \Gamma_t \) is constant at unity. A rise in current income only leads to the fall in financial inflows, without affecting domestic investment and allocative efficiency. In this case, the law of motion for wage is flat at \( w_{t+1} = (r^*)^{-\rho} \).

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\(^{25}\)Equations (60) characterize analytically these two effects for period \( t \geq 1 \).
run and then improves over time, $\Gamma_0 < \Gamma_A$ and $\Gamma_{t+1} > \Gamma_t$. For $\theta \in (\hat{\theta}, \alpha)$, allocative efficiency eventually exceeds its initial level, $\Gamma_F > \Gamma_A$; for $\theta \in (\alpha, 1)$, the opposite applies, $\Gamma_F < \Gamma_A$.

According to proposition 2, the investment elasticity in the constrained sector and the capital share in the sectoral production function are two key factors determining the long-run effect of financial inflows on allocative efficiency. Intuitively, for a given interest rate decline, the lower the $\theta$, the stronger the extensive margin effect, the higher the investment elasticity in the constrained sector, the larger the financial inflows and the investment reallocation towards sector 1. Meanwhile, the larger the $\alpha$, the weaker the DMPK effect, the more the sectoral capital formation raises future income, the larger the rise in the mass of entrepreneurs and the investment reallocation towards sector 1 in the next period. Either way, the subsequent rises in allocative efficiency are more likely to dominate its initial fall so that $\Gamma_F > \Gamma_A$. Thus, whether $\Gamma_t$ eventually exceeds its initial level depends positively on $\alpha$ and negatively on $\theta$.

$\Gamma_t$ reflects the cross-sector investment distortion in period $t$ and so does the sectoral price ratio in period $t + 1$, $\chi_{t+1}$. Figure 3 shows the impulse responses of the sectoral price ratio in three cases, confirming the prediction of proposition 2 on the dynamics of $\Gamma_t$. This way, financial inflows may enhance or undermine the comparative advantage of country N, depending on the relative size of the extensive margin effect and the DMPK effect.

![Figure 3: Financial Integration and the Dynamics of Comparative Advantage](image)

Lemma 2. Given $\theta \in (\hat{\theta}, 1)$ and $\lambda^* \in (0, \hat{\lambda}_A)$, there exists a threshold value $\hat{\lambda}_F > \lambda^*$ such that, for $\lambda \in (\lambda^*, \hat{\lambda}_F)$, country N still has a comparative advantage in sector 1 and the borrowing constraints are binding at the steady state under financial integration, $\chi^* < \chi_F < 1$.

3 Trade Integration and the Re-Reversal of Financial Flows

Suppose that country N is initially in the steady state under financial integration. In period 0, it is announced that, besides financial capital mobility, agents are allowed to trade freely sectoral goods from period 1 on. According to lemma 2, country N initially has the comparative advantage. However, if the free trade policy is announced and implemented in period 0, the sectoral price ratio immediately falls to the world level $\chi_A^*$, which affects the investment return of those born in period $t = -1$ unexpectedly. In the two-period, OLG setting, announcing the free trade policy one period in advance avoids such an uncertainty.
advantage in good 1. From period 1 on, it exports good 1 and imports good 0. As a small open economy, it witnesses the fall in the sectoral price ratio to the world levels, $\chi_t = \chi^* < \chi_F$.\(^{27}\)

In this section, we show that trade integration may have the non-monotonic impacts on the aggregate income dynamics and the direction of financial flows.

### 3.1 Short-Run Impacts of Trade Integration

In period 0, the announcement of free trade affects the sector-0 rate of return in two ways. First, trade lowers the price of good 0 in period 1, which tends to reduce the sector-0 rate of return. Second, the fall in the sectoral price ratio induces country N to specialize towards sector 1. Given $w_0 = w_F$ and the constant leverage multiplier, the mass of entrepreneurs and the total investment in sector 1 remain unchanged, according to equation (8). If the total investment in sector 0 were also unchanged $K_{0,1} = K_{0,F}$, the specialization would take place only along the labor margin in period 1, $\zeta_1 > \zeta_F$, which would raise the capital-labor ratio $\frac{K_{0,1}}{(1-\zeta_1)L} > \frac{K_{0,F}}{(1-\zeta_F)L}$ and reduce the marginal product of capital (MPK) in sector 0. In this case, trade would lower the sector-0 rate of return by directly reducing the output price and indirectly raising the capital-labor ratio in sector 0. Equation (21) decomposes these two effects.

$$\ln q_{0,1} = \underbrace{\ln p_{0,1}}_{\text{sector-0 price effect (-)}} + \underbrace{(\alpha - 1) \ln \frac{K_{0,1}}{(1-\zeta_1)L}}_{\text{sector-0 MPK effect (-)}} + \underbrace{(1 - \alpha) \ln \rho.}_{\text{}}$$  \hspace{5cm} (21)

In the steady state of financial integration, sector 0 is still active in country N, with the investment rate of return equal to the world interest rate, $q_{0,F} = r_F = r^*$. As trade tends to lower the sector-0 rate of return, households invest less in sector 0 and lend more to the market in period 0. If sector 0 is still active in period 1, the no-arbitrage condition ensures that the sector-0 rate of return is unchanged, $q_{0,1} = r_0 = r^* = r_F = q_{0,F}$. It implies that the falls in the output price and in the labor input in sector 0 are exactly offset by the investment decline.\(^{28}\) If sector 0 vanishes in period 1, the sector-0 rate of return must fall below the world interest rate in period 0, justifying that households do not invest in sector 0 at all. In both cases, as the total entrepreneurial credit demand stays unchanged, the rise in household lending crowds out financial inflows, leading to financial outflows for country N in period 0. The reversal of financial flows is consistent with the finding of Antrás and Caballero (2009).

In the long run, country N may witness the re-reversal of financial flows, depending on the dynamics of aggregate income and the degree of specialization.

### 3.2 Aggregate Income Dynamics and the Degree of Specialization

Under dual integration, trade has two opposite effects on aggregate income in period 1. First, trade reduces the sectoral price ratio and triggers the labor reallocation towards sector 1, which creates the static gains and tends to raise aggregate income. Second, trade induces households to invest less in sector 0, which tends to reduce sectoral and aggregate output. In order to figure out the net impact on aggregate income, we first specify the law of motion for wage.

\(^{27}\)As country N is a small open economy, its trade and financial opening does not affect the world prices and hence, we drop off the subscript, $r^* = r^*_A$, $\chi^* = \chi^*_A$, $\mu^* = \mu^*_A$.

\(^{28}\)According to equation (21), $q_{0,1} = q_{0,F}$ requires $\partial \ln K_{0,1} = \frac{1}{1-\alpha} \partial \ln p_{0,1} + \partial \ln (1-\zeta_1)L$. 

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Figure 4: Sectoral Rates of Return, the Degree of Specialization, and the Borrowing Constraints

The two solid curves in figure 4 show the sectoral rates of return as the functions of aggregate income under dual integration, where \( \bar{w}_D = \frac{\rho\bar{w}^{1-\theta}}{(\chi^*)^{1-\eta}} \) and \( w_D = \bar{w}_D(\mu^*)^{\frac{\theta}{\eta}} < \bar{w}_D \) denote two threshold values.\(^{29}\) Trade implicitly aligns the sectoral rate-of-return ratio to the world level, \( \frac{q_{0,t+1}}{q_{1,t+1}} = \mu_{t+1} = \frac{1}{\chi_{t+1}} = (\chi^*)^{\frac{1}{\eta}} = \mu^* < \mu_F \). The law of motion for wage is a piecewise function over three intervals, depending on whether country N fully specializes in sector 1 and whether the borrowing constraints are binding.

- For \( w_t < w_D \), the mass of entrepreneurs and their total investment are so low that sector 1 cannot hire the entire labor input in country N at the market wage rate. Thus, sector 0 is still active and the no-arbitrage condition is \( q_{0,t+1} = r^* \).\(^{30}\) The sector-1 rate of return is constant at \( q_{1,t+1} = \frac{q_{0,t+1}}{\mu_{t+1}} = \frac{r^*}{\mu^*} > r^* \) and hence, the borrowing constraints are binding there. See figure 4. In this case, dual integration leads to the factor price equalization (FPE),\(^{31}\) i.e., \( q_s,t+1 = q_s^* \) for \( s \in \{0,1\} \), and the law of motion for wage is flat at \( w_{t+1} = w^* \).\(^{32}\)

- For \( w_t \in (w_D, \bar{w}_D) \), the mass of entrepreneurs and their total investment are so high that

\(^{29}\) See the proof of proposition 3 for the derivation of two threshold values.

\(^{30}\) If \( q_{0,t+1} > r^* \), households would borrow from abroad and invest in sector 0. By assumption, the investment in sector 0 is not subject to financial frictions. Then, financial inflows ensure that \( q_{0,t+1} = r^* \) holds in equilibrium. Alternatively, one can assume that households who invest in sector 0 are subject to the same borrowing constraints as entrepreneurs. In that case, there is another threshold value \( w_D \). For \( w_t \in (w_D, \bar{w}_D) \), \( q_{1,t+1} = \frac{q_{0,t+1}}{\mu_{t+1}} > r^* \) holds so that the borrowing constraints are slack (binding) in sector 0 (1) and the law of motion for wage is flat at \( w_{t+1} = w^* \). For \( w_t < w_D \), \( q_{1,t+1} = \frac{q_{0,t+1}}{\mu_{t+1}} > q_{0,t+1} > r^* \) holds so that both households and entrepreneurs borrow from abroad, the borrowing constraints are binding in both sectors, the sectoral rates of return decrease in the income level, and the law of motion for wage is upward-sloping. See figure 24 in the proof of proposition 3. As the re-reversal of financial flows occurs when country N fully specializes in the high-return sector (sector 1), we assume away the borrowing constraints in sector 0 and focus on the threshold values \( w_D \) and \( \bar{w}_D \).

\(^{31}\) In the case of partial specialization, sector 0 is active and the no-arbitrage condition is \( q_{0,t+1} = r_t \). By aligning the interest rate to the world level, financial flows lead to the international equalization of the sector-0 rate of return, \( q_{0,t+1} = r_t = r^* = q_0^* \). By aligning the sectoral price ratio to the world level, trade flows lead to the international equalization of the sectoral rate-of-return ratio, \( \mu_{t+1} = \mu^* \). The two conditions jointly imply that the sector-1 rate of return is also aligned to the world level, \( q_{1,t+1} = \frac{q_{0,t+1}}{\mu_{t+1}} = \frac{q_0^*}{\mu^*} = q_1^* \). Combine equations (1)-(2) to get \( w_{t+1} = \left(q_{0,t+1}q_{1,t+1}\right)^{-\rho} \). Thus, \( q_s,t+1 = q_s^* \) gives \( w_{t+1} = w^* \).

\(^{32}\) Under dual integration, a rise in current income allows more agents to become entrepreneurs and induces country N to specialize further towards sector 1 along the labor margin, which enhances the static gains from trade. Meanwhile, a rise in current income tends to reduce the sector-0 rate of return, which induces households to invest even less in sector 0. Given the FPE, the law of motion for wage is flat at \( w_{t+1} = w^* \), implying that the static gains from trade exactly offset the investment decline in sector 0.

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\( q_{0,t+1} \) and \( q_{1,t+1} \) represent the sectoral rates of return in sectors 0 and 1, respectively. \( \bar{w}_D \) and \( w_D \) denote the upper and lower thresholds for wage, respectively.
sector 1 hires the entire labor input in country N at the market wage rate and hence, sector 0 vanishes, \( \delta_t = \zeta_{t+1} = 1 \). A rise in current income raises the mass of entrepreneurs and stimulates the total investment in sector 1, which raises the capital-labor ratio and reduces the MPK in sector 1, given the total labor input constant at \( L = 1 \). Thus, the sector-1 rate of return declines in current income, as shown in figure 4. As \( q_{1,t+1} > \rho^* \) holds in this case, the borrowing constraints are still binding\(^{33}\) and the law of motion for wage is characterized by equations (8) and (22).

\[
\begin{align*}
  w_{t+1} &= p_1^* \left( \frac{K_{1,t+1}}{\rho L} \right)^\alpha = \frac{w_t^{\alpha}}{\left[ (\mu^*)^{1-\eta} \rho w^{1-\theta} \right]^\alpha}, \\
  \frac{\partial \ln w_{t+1}}{\partial \ln w_t} &= \left[ 1 - (1 - \alpha) \right] \left[ \frac{\partial \ln w_t L}{\partial \ln w_t} + (1 - \theta) \frac{\partial \ln \tau_t}{\partial \ln w_t} \right] = \frac{\alpha}{\theta}.
\end{align*}
\]

Due to the extensive margin effect, a rise in current income raises the investment in sector 1 more than proportionately. Due to the DMPK effect, capital formation raises future income less than proportionately. For \( \theta \in (\hat{\theta}, \alpha) \), the extensive margin effect dominates the DMPK effect so that a change in current income gets amplified over time. For \( \theta \in (\alpha, 1) \), the opposite applies so that a change in current income gets dampened.

- For \( w_t \geq \bar{w}_D \), the mass of entrepreneurs and the total investment in sector 1 are sufficiently high so that the sector-1 rate of return falls to the level of the world interest rate, \( q_{1,t+1} = \rho^* \) and the borrowing constraints become slack, as shown in figure 4. In this case, entrepreneurs do not have strong incentive to borrow and invest to the limit and hence, a rise in current income only reduces financial inflows, without affecting the size of domestic investment. The law of motion for wage is flat at \( w_{t+1} = \frac{w^*}{(\mu^*)^\rho} > w_F \).

**Proposition 3.** Under dual integration, for \( \theta \in (\alpha, 1) \), there is a unique steady state; for \( \theta \in (\hat{\theta}, \alpha) \), multiple steady states may arise. Start from the steady state of financial integration. Trade integration reverses financial flows in the short run. If it induces country N to specialize fully in sector 1, financial flows may be re-reversed in the long run.

Whether country N witnesses the re-reversal of financial flows depends on the cross-country difference in financial development and the investment elasticity in the constrained sector.

### 3.3 The Role of the Cross-Country Difference in Financial Development

For \( \{\alpha, \theta\} \) in region M of the left panel of figure 5, \( \theta \in (\hat{\theta}, \alpha) \) and multiple steady states may arise under dual integration. We first analyze the dynamics of financial flows in this case, while keeping the case of \( \theta \in (\alpha, 1) \) for subsection 3.4. The flat curve in the middle panel of figure 5 shows the threshold value \( \hat{\lambda}_F \) specified in lemma 2. We focus on the region below this curve and above the 45\(^o\) line, i.e., \( \lambda^* \in (0, \hat{\lambda}_A) \) and \( \lambda \in (\lambda^*, \hat{\lambda}_F) \).

For \( \{\lambda^*, \lambda\} \) in region IR of the middle panel of figure 5, the solid (dashed) curve in the left panel of figure 6 shows the law of motion for wage under dual (financial) integration, while

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33Strictly speaking, if no one invests in sector 0 in equilibrium, the sector-0 rate of return cannot be measured. In figure 4, the dashed curve shows the hypothetical rate of return if an household had invested in sector 0.
Figure 5: Impacts of Trade on Income Dynamics and Financial Flows: Threshold Values

The middle and the right panels of figure 6 show the impulse responses of the wage rate and the normalized financial flow, respectively. Start from the steady state $F$. The announcement of free trade leads to the reversal of financial flow in period 0, $\phi_0 < 0 < \phi_F$, as explained in subsection 3.1. In this case, the cross-country difference in financial development is small and so are the international sectoral price differentials and the static gains from trade. In period 1, the static gains from trade are dominated by the investment decline in sector 0, which reduces aggregate income in period 1. Then, country $N$ converges to the new steady state $L$, with both sectors active and a lower income level, $w_L = w^* < w_F$. According to lemma 1 and proposition 2, country $N$ initially has a higher income per capita than the rest of world, $w_F > w_A > w^*$. Allowing additionally free trade leads to the reversals of both financial flows and income per capita for country $N$, consistent with the findings of Antras and Caballero (2009).

Figure 6: Free Trade and Immediate Reversal of Financial Flow: Case IR with $\theta \in (\hat{\theta}, \alpha)$

For $\{\lambda^*, \lambda\}$ in region RR of the middle panel of figure 5, figure 7 shows the relevant laws of motion for wage and the impulse responses. Start from the steady state $F$. The announcement of free trade leads to the reversal of financial flow, the same as in case IR. However, given the level of financial development in country $N$ ($\lambda$), the cross-country difference in financial development ($\lambda - \lambda^*$) is much larger in case RR than in case IR and so are the international sectoral price differentials.

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34 Given other model parameters, the border between region IR and GR in the middle panel of figure 5 is characterized by $(\lambda^*, \lambda)$ satisfying $w_F = w_D$. 

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differentials and the static gains from trade. Thus, trade substantially reduces the price of good 0 and raises the price of good 1 in country N, which drives the MRL in sector 0 below that in sector 1. In equilibrium, sector 1 hires the entire labor input in country N at the market wage rate, while sector 0 vanishes. Meanwhile, the static gains from trade dominate the investment decline in sector 0 so that aggregate income rises in period 1. According to equation (23), for \( \theta \in (\hat{\theta}, \alpha) \), dual integration amplifies this initial income rise over time. The gradual expansions in aggregate income raise the mass of entrepreneurs and stimulate domestic investment along the extensive margin. In the long run, country N converges to a new steady state \( H \) where it specializes fully in sector 1 and the borrowing constraints are slack. The steady-state income level is so high \( w_H > \bar{w}_D > \bar{w} \) that domestic investment exceeds domestic saving and hence, country N witnesses the **re-reversal of financial flow**, \( \phi_H = \frac{K_{1,H} - w_H L}{w_H L} > 0 > \phi_0 \).

***Figure 7: Trade Integration and the Re-Reversal of Financial Flows: Case RR with \( \theta \in (\hat{\theta}, \alpha) \)***

It is worth to mention that full specialization in sector 1 is a necessary but not sufficient condition for the re-reversal of financial flows. For \( \{\lambda^*, \lambda\} \) in region GR of the middle panel of figure 5, figure 8 shows the relevant laws of motion for wage and the impulse responses. Given the level of financial development in country N, the cross-country difference in financial development is moderate, compared to case IR and case RR. Start from the steady state \( F \). The announcement of free trade induces country N to specialize fully in sector 1. However, the static gains from trade are dominated by the investment decline in sector 0, which reduces aggregate income in period 1. Given \( \theta \in (\hat{\theta}, \alpha) \), the fall in aggregate income gets amplified over time. In the long run, country N converges to a new steady state \( L \), with the partial specialization and a lower income level, \( w_L = w^* \).

Intuitively, the cross-country difference in financial development \( \lambda - \lambda^* \) determines the international sectoral price differentials and the static gains from trade. The former determines the degree of specialization, while the latter competes with the investment decline in sector 0, which determines the income change in period 1. For \( \theta \in (\hat{\theta}, \alpha) \), the financial flow re-reversal occurs, as long as trade induces country N to specialize fully in sector 1 and aggregate income rises in period 1. This way, the cross-country difference in financial development matters for the possibility of the financial flow re-reversal.

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35 Given other model parameters, the border between region RR and GR in the middle panel of figure 5 is characterized by \( (\lambda^*, \lambda) \) such that there is an unstable steady state \( U \) under dual integration with \( w_U = w_F \).
3.4 The Role of the Investment Elasticity in the Constrained Sector

For \( \theta \in (\alpha, 1) \), full specialization in sector 1 and the income rise in period 1 are not sufficient for the re-reversal of financial flows in country N. This subsection shows that the investment elasticity of the constrained sector is another key factor for the financial flow re-reversal.

For \( \{\alpha, \theta\} \) in region U of the left panel of figure 5, \( \theta \in (\alpha, 1) \) and there exists a unique steady state under dual integration. Let \( X_D \) denote the steady-state value of variable \( X_t \). For \( \{\lambda^*, \lambda\} \) in region GR of the right panel of figure 5, figure 9 shows the relevant laws of motion for wage and the impulse responses. Start from the steady state F. Given \( w_F > \bar{w}_D \), the announcement of free trade induces country N to specialize fully in sector 1. Besides, as the cross-country difference in financial development is moderately large, the static gains from trade dominate the investment decline in sector 0 so that aggregate income rises in period 1. Given \( \theta \in (\alpha, 1) \), the initial income rise gets dampened, according to equation (23). Although country N converges to the steady state D with a higher income level \( w_D > w_F \), the income rise is not large enough \( w_D < \bar{w} \) and domestic investment is still less than domestic saving, \( \frac{K_{1,D}}{w_D} = \left( \frac{w_D}{\bar{w}} \right)^{\frac{1-\theta}{\sigma}} < 1 \), according to equation (8). In this case, country N witnesses financial outflows in the short run as well as in the long run, \( \phi_0 < \phi_D = \left( \frac{w_D}{\bar{w}} \right)^{\frac{1-\theta}{\sigma}} - 1 < 0 \).

For \( \{\lambda^*, \lambda\} \) in region RR of the right panel of figure 5, figure 10 shows the relevant laws of motion for wage and the impulse responses. Compared to case GR, the larger cross-country
difference in financial development implies the larger international sectoral price differentials. In this case, the static gains from trade not only dominate the investment decline in sector 0 but also induce country N to reach a steady state where domestic investment exceeds domestic saving. Thus, the financial flow re-reversal occurs in the long run, \( w_D > \bar{w} \) and \( \phi_D = \left( \frac{w_D}{\bar{w}} \right)^{\frac{1-\theta}{\theta}} - 1 > 0 \).

Given other model parameters, the border between region RR and GR in the right panel of figure 5 is characterized by \( (\lambda^*, \lambda) \) such that \( w_D = \bar{w} \).

As shown in subsection 2.1, if \( \delta_A = \frac{1-\theta}{1-\lambda} < \eta \) holds, the borrowing constraints are binding in the autarkic steady state. Under dual integration, country N witnesses financial inflows in the steady state if it specializes fully in sector 1 and domestic investment exceeds domestic saving, i.e., \( M_D = K_{1,D} > w_D L \) or equivalently, \( \frac{\tau_1 - \theta}{1-\lambda} > 1 \), according to equation (9). These two conditions can hold at the same time, \( \frac{\tau_1 - \theta}{1-\lambda} < \eta < < \frac{\tau^* \theta}{1-\lambda} \) iff the mass of entrepreneurs \( \tau_1 \) is endogenous.

**Remark 1: Why do we endogenize the mass of entrepreneurs?**

In our model, the financial flow re-reversal occurs, if trade induces country N to specialize fully in the constrained sector and domestic investment eventually exceeds domestic saving. The endogenous mass of entrepreneurs is crucial for these two conditions to be met. In the static model of Antras and Caballero (2009), only a fixed mass \( \tau \) of agents can invest in sector 1, subject to the borrowing constraints. Besides, agents are homogenous in labor endowment and there is no MIR. One can embed their static model into a two-period OLG setting.

![Figure 10: Impacts of Trade on Income Dynamics and Financial Flows: the Case of RR](image-url)
Given assumption 1 of Antras and Caballero (2009), the fraction of domestic investment in the constrained sector is constant and inefficiently low under autarky, $\delta_t = \frac{K_{1,t+1}}{M_t} = \frac{\tau w_t L}{w_t L} = \frac{\tau}{1-\lambda} < \eta$, which ensures the binding borrowing constraints. If the cross-country difference in financial development is sufficiently large, country N may specialize fully in sector 1 under dual integration. However, due to the exogenous mass of entrepreneurs, domestic investment is strictly lower than domestic saving, $\frac{M_t}{w_t L} = \frac{K_{1,t+1}}{w_t L} = \frac{\tau}{1-\lambda} < \eta < 1$ and hence, the re-reversal of financial flows cannot occur. It confirms that the endogenous mass of entrepreneurs is crucial for the re-reversal of financial flows in our model.

Antras and Caballero (2009) embed their static model into a continuous-time setting with two key assumptions: (1) agents are born at a constant rate per unit of time and die at the same rate, (2) agents save all their (labor and investment) income and consume only when they die. Due to the law of large numbers, the mass of entrepreneurs is constant and hence, the extensive margin of domestic investment is mute. Different from the two-period OLG setting, agents can accumulate wealth over their lifetime. Thanks to the privilege of investing in the constrained sector at a higher rate of return, entrepreneurs accumulate wealth at a faster speed than others so that their wealth share in national wealth is endogenous along the intensive margin. By aligning the sectoral price ratio at the world level, trade integration raises the sectoral rate-of-return differential in country N, which allows entrepreneurs to accumulate wealth at a faster speed than in the autarkic steady state. This way, trade affects the investment fraction of the constrained sector along the intensive margin. Whether trade can generate the re-reversal of financial flows in that setting is beyond the scope of our paper.

Remark 2: Dual Integration and the FPE

Antras and Caballero (2009) show that, in the presence of sector-specific financial frictions, trade does not equalize the prices of productive factors (i.e., the wage rate, the interest rate, etc) across countries, while trade and financial integration jointly leads to the FPE. In their model, financial frictions distort domestic allocation along the intra- and the intertemporal dimensions. The cross-sector investment distortion keeps the sectoral price ratio below its efficient level, $1 - \chi_t$, while the constraints on the aggregate credit demand keep the interest rate below the social rate of return, $\Upsilon_t - r_t$. Trade (financial) integration only equalizes across countries the impacts of distortion along the intratemporal (intertemporal) dimension, while dual integration equalizes the impacts of distortions across countries along both dimensions and the FPE holds.

In our model, for a small cross-country difference in financial development, dual integration leads to partial specialization in country N. In this case, as country N is still in the same cone of diversification as the rest of the world, the FPE holds and the income convergence occurs. However, for a large cross-country difference in financial development, dual integration leads to full specialization in country N. In this case, as country N is not in the same cone of diversification as the rest of the world, the FPE does not hold and income divergence occurs.

Remark 3: Declining Trade Costs and the Patterns of Financial Flows

We analyze the impacts of trade on income dynamics and financial flows by comparing two extreme cases. In the first case, country N is in the steady state of financial integration where
sectoral goods are subject to prohibitively high trade costs and hence, trade in sectoral goods does not occur. In the second case, there is no trade cost and free trade equalizes the sectoral price ratio across countries. Alternatively, one can explicitly introduce trade costs into our model and analyze how the gradual decline in trade costs affects income dynamics and financial flows. Start from the case of prohibitively high trade costs. A small decline in trade costs only slightly reduces the sectoral price ratio in country N, which leads to small trade and financial flows under dual integration. In this case, country N specializes partially towards sector 1 and the reversal of financial flows occurs. If trade costs continue to fall over time, country N specializes further towards sector 1, which amplifies financial outflows. However, if trade costs fall below a threshold value, country N may specialize fully in sector 1 and aggregate income may rise over time, which may trigger the re-reversal of financial flows. Thus, the decline in trade costs may have non-monotonic impacts on income dynamics and financial flows.

**Remark 4: Interest Rate Dynamics and the Patterns of Financial Flows**

In order to highlight the core mechanism behind the reversal of financial flows, Antras and Caballero (2009) show that trade integration leads to the interest rate reversal, given that the model economy is initially in the autarkic steady state. For comparison purpose, we analyze in appendix A the interest rate responses to trade integration, given that country N is initially in the autarkic steady state. According to lemma 1, the interest rate is initially higher in country N. In period 0, the announcement of free trade induces country N to specialize toward sector 1 along the labor margin, which raises the capital-labor ratio and reduces the rate of return in sector 0. Coupled with the sector-0 rate of return, the interest rate in country N falls below the world level, the result Antras and Caballero (2009) call the interest rate reversal. If the cross-country difference in financial development and the investment elasticity of the constrained sector are sufficiently large, trade may induce country N to fully specialize in sector 1. In this case, the interest rate is decoupled from (coupled with) the rate of return in sector 0 (1). In the new steady state, the interest rate is higher in country N again, a result we call the interest rate re-reversal. The dynamic pattern of the interest rate helps predict the patterns of financial flows.

4 Recurrent Reversal and Re-Reversal of Financial Flows

In this section, we embed our model in a three-sector setting and show that the financial flow re-reversal may arise recurrently if trade induces country N to sequentially offshore low-return sectors and upgrade to high-return sectors. See appendix B for a multi-sector version.

There are three sectors in country N, indexed by \( s \in \{0, 1, 2\} \) and ranked in ascending order with respect to the MIR, i.e., \( m_0 = 0 < m_1 < m_2 \). Let \( \delta_{s,t} \) and \( \zeta_{s,t+1} \) denote the respective shares of domestic investment and labor input for the production of good \( s \), \( Y_{s,t+1} = \left( \frac{\delta_{s,t}M_t}{\alpha} \right)^{\alpha} \left( \frac{\zeta_{s,t+1}L_t}{1-\alpha} \right)^{1-\alpha} \). Sectoral outputs are combined for the production of final goods, \( Y_{t+1} = \prod_{s=0}^{2} \left( \frac{V_{s,t+1}}{\eta_s} \right)^{\eta_s} \), where \( \eta_s \) denotes the sector share and \( \sum_{s=0}^{2} \eta_s = 1 \). Investors in sector 1 and 2 are subject to the borrowing constraints. The agents with \( \epsilon_j \geq \epsilon_{s,t} \) can meet the MIR and invest in sector \( \nu \geq s \), where \( \epsilon_{s,t} = \frac{m_0}{u_t} \frac{1-\lambda}{1-\theta} \).
4.1 From International Autarky to Financial Integration

In the absence of trade flows, all sectors are active and the sectoral labor input is constant at $\bar{\zeta}_{s,t+1} = \eta_s$. If the borrowing constraints are binding in sector $s \in \{1, 2\}$, the mass of investors in sector $s$ is inefficiently low and so is the sectoral investment.

Assumption 2. $\gamma \equiv \frac{m_1}{m_2} < \left(\frac{\eta_2}{\eta_1 + \eta_2}\right)^{\frac{1}{1-\theta}}$.

Let $\bar{w}_s \equiv \frac{m_s(1-\lambda)}{1-\theta} \bar{w}$, $\bar{w}_{1,A} \equiv \bar{w}_1 \left[1 + \frac{w_2}{\eta_1} \left(1 - \gamma \frac{1-\theta}{\theta} \right)\right]^{-\frac{1-\theta}{\theta}} < \bar{w}_1$, and $\bar{w}_{2,A} \equiv \bar{w}_2 \bar{w}^\theta_2 < \bar{w}_2$.

Under autarky, for $w_t < \bar{w}_{1,A}$, the mass of investors in sector $s \in \{1, 2\}$ is inefficiently low and so is the aggregate credit demand. The rate-of-return spread, $r_t < q_{s,t+1}$, arises and the borrowing constraints are binding in sector 1 and 2. Under autarky, domestic investment is financed by domestic saving $M_t = w_t L$ and the sectoral fractions of domestic investment are

$$\delta_{2,t} = \frac{\int_{\bar{w}_{2,t}}^{\infty} \frac{(1-\theta) \bar{w} \eta_0}{1-\lambda} dG(\epsilon_j)}{M_t} = \left(\frac{w_t}{\bar{w}_2}\right)^{\frac{1-\theta}{\theta}},$$

$$\delta_{1,t} = \frac{\int_{\bar{w}_{1,t}}^{\infty} \frac{(1-\theta) \bar{w} \eta_0}{1-\lambda} dG(\epsilon_j)}{M_t} = \left(\frac{w_t}{\bar{w}_1}\right)^{\frac{1-\theta}{\theta}} - \left(\frac{w_t}{\bar{w}_2}\right)^{\frac{1-\theta}{\theta}} = \delta_{2,t} \left(\gamma \frac{1-\theta}{\theta} - 1\right),$$

$$\delta_{0,t} = 1 - (\delta_{1,t} + \delta_{2,t}) = 1 - \left(\frac{w_t}{w_1}\right)^{\frac{1-\theta}{\theta}}.$$  

For $w_t < \bar{w}_{1,A}$, the sectoral capital-labor ratio is descending, implying the ascending sectoral rate of return. The interest rate is determined by the lowest-return sector (sector 0).

$$\frac{\delta_{0,t}}{\eta_0} > \frac{\delta_{1,t}}{\eta_1} > \frac{\delta_{2,t}}{\eta_2}, \quad \Rightarrow \quad r_t = q_{0,t+1} < q_{1,t+1} < q_{2,t+1}. \quad (27)$$

A rise in current income raises the fractions of domestic investment in sector 1 and 2 along the extensive margin, while it reduces the fraction of domestic investment in sector 0.  

Let $\bar{\theta}_3 \equiv \frac{1}{1+11(1-\rho_0)}$, $F \equiv \left(\frac{\eta_1}{1-\gamma} \frac{1-\theta}{\theta} + \eta_0\right)^{\rho-\theta} \left[\eta_2 \left(\gamma \frac{1-\theta}{\theta} - 1\right)\right]^{\rho_0} \bar{w}^{\theta-\theta}$, and $Z_3 \equiv \frac{1}{\rho_0} \eta_2^{\frac{1-\theta}{\theta}}$.  

Proposition 4. For $\theta \in (\bar{\theta}_3, 1)$, $Z_3 < F$, and $\lambda < \bar{\lambda}_{A,3} \equiv 1 - \left(\frac{Z_3}{F}\right)^{1-\theta}$, there is a unique, autarkic steady state where the borrowing constraints are binding in sector 1 and 2.

Assumption 3. $\theta \in (\bar{\theta}_3, 1)$, $Z_3 < F$, $0 < \lambda^* < \lambda < \bar{\lambda}_{A,3}$, and $\frac{1}{L+L^*} \to 0$.

38For $w_t \in (\bar{w}_{1,A}, \bar{w}_{2,A})$, the mass of investors and the total investment in sector 2 are so low that the rate-of-return spread $r_t < q_{2,t+1}$ ensures the binding borrowing constraints there. The investment share of sector 2 is still specified by equation (24). However, the mass of investors and the total investment in sector 1 are so large that the rate-of-return spread vanishes $q_{1,t+1} = r_t = q_{0,t+1}$ and the borrowing constraints are slack there. The capital-labor ratio equalizes in sector 0 and 1, $\frac{\delta_{1,t}}{\eta_1} = \frac{\delta_{2,t}}{\eta_2} = 1 - \frac{\delta_{0,t}}{\eta_0}$. A rise in current income raises the investment share of sector 2 along the extensive margin, while that of sector 0 and 1 falls in equal proportions. In this case, the three-sector model behaves like the two-sector model that we have analyzed before.

For $w_t > \bar{w}_{2,A}$, the mass of investors in sector 1 and 2 are so large that the rate of return equalizes across the three sectors and the borrowing constraints are slack $q_{s,t+1} = r_t$ for $s \in \{0, 1, 2\}$. Thus, the sectoral investment shares are constant at $\delta_{s,t} = \eta_s$, independent of the change in current income.

39See figure 25 in the proof of Proposition 4 for the graphic illustration of the threshold values.
Let $X_{s,A}$ denote the autarkic steady-state value of $X_{s,t}$. Let $\chi_{s,t} \equiv \frac{p_{s,t}}{p_{2,t}}$ and $\mu_{s,t} \equiv \frac{q_{s,t}}{q_{2,t}}$ denote the normalized output price and the normalized rental price of capital in sector $s \in \{0, 1\}$.

**Lemma 3.** Under assumption 3, $\chi^*_0 < \chi^*_A < \chi^*_1$, $\mu^*_0 < \mu^*_A < \mu^*_1$, and $r^*_A < r_A < \rho$.

Proposition 4, assumption 3, lemma 3 closely resemble proposition 1, assumption 1, and lemma 1 in the two-sector setting, respectively. It is worth to mention that country N has the comparative advantage in the constrained sectors (sector 1 and 2) at the autarkic steady state.

Under financial integration, given assumption 3, the claims in proposition 2 also hold in the three-sector setting. Besides, there exists a threshold value $\tilde{\lambda}_{F,3}$ such that, for $\lambda \in (\lambda^*, \tilde{\lambda}_{F,3})$, country N still has a comparative advantage in sector 1 and sector 2 in the steady state of financial integration, with the binding borrowing constraints. It resembles lemma 2.

### 4.2 Sequential Upgrading and Recurrent Re-Reversal of Financial Flows

In the two-sector setting, $w_{D}$ and $\bar{w}_{D}$ denotes the threshold values of the income level at which country N fully offshores sector 0 and the borrowing constraints slack under dual integration, respectively. The law of motion for wage is a piecewise function over three intervals.

In the three-sector setting, let $w_{D1}$ and $w_{D2}$ denote the threshold values of the income level at which country N fully offshores sector 0 and 1, respectively; let $\bar{w}_{D1}$ and $\bar{w}_{D2}$ denote the threshold values of the income level at which the borrowing constraints become slack in sector 1 and 2, respectively. Thus, the law of motion for wage is a piecewise function over five intervals specified by the four threshold values. Similar as in the two-sector setting, if the borrowing constraints are slack in any active sector, the law of motion for wage is flat; if the borrowing constraints are binding in all active sector(s), the law of motion for wage in logarithm is upward-sloping with the slope $\alpha$ as shown in equation (23).

**Proposition 5.** Under dual integration, for $\theta \in (\alpha, 1)$, there is a unique steady state; for $\theta \in (\hat{\theta}_{3}, \alpha)$, multiple steady states may arise.

Start from the steady state of financial integration. Trade integration reverses financial flows in the short run. If the cross-country difference in financial development and the investment elasticity of the constrained sector are sufficiently large, trade may induce country N to offshore the low-return sectors and upgrade to the high-return sectors sequentially. In this process, country N may witness the re-reversal of financial flows recurrently.

Proposition 5 closely resembles proposition 3 in the two-sector setting. For $\{\alpha, \theta\}$ in region M of the left panel of figure 11, $\theta \in (\hat{\theta}_{3}, \alpha)$ and multiple steady states may arise under dual integration. In this subsection, we analyze this case. The flat curve in the middle panel of figure 5 shows the threshold value $\tilde{\lambda}_{F,3}$ specified above. We focus on the region below this curve and above the $45^\circ$ line, i.e., $\lambda^* \in (0, \tilde{\lambda}_{A,3})$ and $\lambda \in (\lambda^*, \tilde{\lambda}_{F,3})$.

For $\{\lambda^*, \lambda\}$ in region RR1 of the right panel of figure 11, figure 12 shows the relevant laws of motion for wage and the impulse responses in this case. In the steady state of financial integration, country N has the comparative advantage in sector 1 and 2. Trade induces country N to import good 0 and export goods 1 and 2. In this case, the cross-country difference in financial
The Case of $\theta \in (\hat{\theta}_3, \alpha)$

For $\{\lambda^*, \lambda\}$ in region RR2 of the right panel of figure 11, figure 13 shows the relevant laws of motion for wage and the impulse responses in this case. Start from the steady state F. Free trade induces country N to offshore sector 0 and aggregate income rises over time, leading to the reversal and re-reversal of financial flows, similar as in case RR1. Given the level of financial development in country N, the cross-country difference in financial development is larger than in case RR1 and so are the international sectoral price differentials and the static gains from trade. Thus, country N offshores not only sector 0 but also sector 1, which raises aggregate income further. Then, the reversal and re-reversal of financial flows occur again. Eventually, country N reaches the new steady state H where it fully specializes in sector 2.
5 Final Remarks

This paper revisits the predictions of Antras and Caballero (2009) and finds that trade integration may not dampen or reverse upstream financial flows, if it induces North to specialize fully in high-return sectors. Grossman and Rossi-Hansberg (2006) and Baldwin (2016) argue that task trade and global value chains have transformed the industrial structures in advanced and emerging economies. By reinterpreting “sectors” as production stages/tasks, one can use our model to analyze how the rising global value chains may affect the direction of financial flows.40

For example, fabrication & assembly in the manufacturing industries are involved with standardized, routine tasks that require intensively the input of tangible investment, while upstream activities (such as R&D, product design, or the manufacturing of key parts and components) and downstream activities (such as marketing, brand building, and customer service) are more involved with knowledge-intensive, non-routine tasks that require mostly the input of intangibles. Compared to tangible investment, intangibles are subject to higher MIR and more severe financial frictions. Our model predicts that the output price and the investment return are higher in upstream & downstream activities than in fabrication & assembly, which is consistent with the “U-shaped” value-added pattern along the supply chain (Baldwin, Ito, and Sato, 2014; Kimura, 2003). By allowing North to offshore low-return fabrication & assembly activities and specialize fully in high-return upstream & downstream activities, supply-chain trade may contribute to upstream financial flows. See appendix C for the detailed discussion.

References


40As our model does not feature sequential production nor intermediates, it is difficult to map our results to actual value-chains. Nevertheless, the logic of our findings should still apply.


Online Appendices

A Trade Integration in the Two-Sector Model

The world economy is initially at the autarkic steady state. In period 0, it is announced that agents are allowed to trade freely sectoral goods from period 1 on. According to lemma 1, country N initially has the comparative advantage in good 1. From period 1 on, it exports good 1 and imports good 0. As a small open economy, country N witnesses the fall in the sectoral price ratio to the world levels, \( \chi_t = \chi^* < \chi_F \). In this section, we show that trade integration may have the non-monotonic impacts on the interest rate, which helps predict the direction of financial flows (if allowed).

A.1 Specialization as An Amplification Mechanism

As long as \( \delta_t < 1 \), sector 0 is active and, according to equations (4) and (6), the borrowing constraints are binding so that \( \delta_t \) is determined by equation (10).

**Lemma 4.** For \( w_t < \bar{w} \ (w_t \geq \bar{w}) \), country N specializes partially (fully) in sector 1.

The law of motion for wage is piecewise and characterized by equation (28).

\[
w_{t+1} = \left( \frac{w_t \Gamma_t}{\rho} \right)^{\alpha}, \text{ where } \Gamma_t = (\mu^*)^\eta \left[ 1 + \frac{(1 - \mu^*) \delta_t}{\mu^*} \right] \text{ and } \delta_t = \min \left\{ \left( \frac{w_t}{\bar{w}} \right)^{\frac{1-\theta}{\gamma}}, 1 \right\}.
\]

In period 0, country N is in the autarkic steady state, \( w_0 = w_A < \bar{w}_A < \bar{w} \). The advanced announcement of free trade does not change the sectoral investment, \( \delta_0 = (\frac{w_0}{\bar{w}})^{\frac{1-\theta}{\gamma}} = \delta_A \). In period 1, trade induces country N to specialize towards sector 1 along the labor margin. The larger the cross-country difference in financial development, the larger the international price differentials, the larger the static gains.

\[
\frac{\partial \ln w_t}{\partial \ln \chi_1} = -\frac{\chi^*_A - (\chi^*)^\frac{1}{\gamma}}{\chi^*_A + (\chi^*)^\frac{1}{\gamma}} < 0.
\]

The static gains from trade \( w_1 > w_0 \) allow more agents to overcome the MIR and invest in sector 1, \( \delta_1 = (\frac{w_1}{\bar{w}})^{\frac{1-\theta}{\gamma}} > \delta_0 \), which enhances country N's comparative advantage and induces it to specialize towards sector 1 along the labor margin in period 2. Thus, trade triggers a dynamic, virtuous cycle through which the rise in aggregate income and the cross-sector resource reallocation reinforce each other.

\[
\frac{\partial \ln \Gamma_t}{\partial \ln \delta_t} = \frac{1}{\mu^*_A}, \text{ and } \frac{\partial \ln \delta_t}{\partial \ln w_t} = \begin{cases} \frac{1}{\gamma} - 1, & \text{if } w_t < \bar{w}; \\ 0, & \text{if } w_t \geq \bar{w}. \end{cases}
\]

The allocative efficiency effect depends on two factors. First, the lower the \( \theta \), the smaller the wealth inequality, the larger the mass of entrepreneurs responds to the gains from trade, the larger the cross-sector investment reallocation \( \frac{\partial \ln \Gamma_t}{\partial \ln w_t} \). Second, the lower the \( \lambda^* \), the lower the \( \mu^*_A \), the larger the sectoral rate-of-return differential in the rest of the world, the larger country N gains from specializing in the high-return sector \( \frac{\partial \ln \Gamma_t}{\partial \ln \delta_t} \). As long as the allocative efficiency effect dominates the DMRK effect, the virtuous cycle goes on over time until the mass of entrepreneurs becomes so large that sector 0 vanishes in country N, i.e., \( w_t \geq \bar{w} \) and \( \delta_t = 1 \). Then, the income dynamics are driven by the DMRK effect. Let \( X_T \) denote the steady-state value of variable \( X_t \) under trade integration.
Proposition 6. If the cross-country difference in financial development and the investment elasticity of the constrained sector in country N are sufficiently large and the level of financial development in the rest of the world is sufficiently low, trade integration induces country N to converge towards a unique steady state where country N specializes fully in sector 1.

Let case UF1 denote the case described in proposition 6. In figure 14, the left panel shows the threshold value \( \tilde{\theta} \) in the \( \{\theta, Z\} \) space, while the right panel shows the threshold values \( \tilde{\lambda}_T \) and \( \tilde{\lambda}^*_T \) in the \( \{\lambda^*, \lambda\} \) space. Case UF1 arises for \( \{\theta, Z, \lambda^*, \lambda\} \) in region UF1, i.e., \( \theta \in [\tilde{\theta}, \tilde{\theta}) \), \( \lambda^* < \tilde{\lambda}_T \), and \( \lambda > \tilde{\lambda}_T \).

Next, we focus on case UF1 and analyze the dynamic responses of aggregate income and the interest rate under trade integration.\(^{41}\)

![Figure 14: Threshold Values for the Free-Trade Equilibrium](image)

![Figure 15: Income Dynamics: From Autarkic Steady State to Free Trade](image)

In figure 15, the solid (dashed) curve in the left panel shows the law of motion for wage under trade (autarky), while the solid curve in the right panel shows the impulse responses of wage under trade.\(^{42}\) In

\(^{41}\)The proof of proposition 6 offers a detailed description of the dynamic and steady-state properties under trade integration.

\(^{42}\)For illustration clarity, the axes in the left panel are scaled in logarithm and so is the vertical axis in the right panel, while the horizontal axis in the right panel shows the period index. This scaling also applies to figures 16.
a numerical example, for period \( t \leq 6 \), \( w_t < \bar{w} \) and \( \delta_t < 1 \) hold, implying that sector 0 is active and the income dynamics are driven by the allocative efficiency effect and the DMRK effect. For period \( t \geq 7 \), \( w_t > \bar{w} \) and \( \delta_t = 1 \) hold, implying that country N fully offshores sector 0 and the income dynamics are driven purely by the DMRK effect.

### A.2 Interest Rate Reversal and Re-reversal under Trade Integration

**Lemma 5.** Under trade integration, the interest rate in country \( N \) is a piecewise function of the income level, depending on whether the low-return sector is fully offshored.

1.) For \( w_t \in (0, \bar{w}) \), the mass of entrepreneurs is so small that their total borrowing capacity is less than the entire household saving. In equilibrium, sector 0 is still active, the interest rate is aligned with the rate of return in sector 0, and the borrowing constraints are binding.

\[
\rho_t = q_{0,t-1} = \frac{\Upsilon_t}{1 + (1 - \mu^*) \delta_t} < \Upsilon_t, \quad \frac{\partial \ln \rho_t}{\partial \ln w_t} = - \left( 1 - \alpha \right) \frac{1}{1 + \frac{1}{1 - \mu^*} \delta_t} < 0. \tag{32}
\]

2.) For \( w_t \geq \bar{w} \), the mass of entrepreneurs is so large that their total borrowing capacity exceeds the entire household saving. In equilibrium, sector 0 vanishes, the interest rate is aligned with the rate of return in sector 1, and the borrowing constraints are slack.

\[
\rho_t = q_{1,t-1} = \Upsilon_t = \frac{\rho w_{t+1}}{w_t}, \quad \frac{\partial \ln \rho_t}{\partial \ln w_t} = - \left( 1 - \alpha \right) < 0. \tag{33}
\]

In figure 16, the left panel shows the interest rate as a piecewise function of the income level, while the right panel shows the impulse responses of the interest rate under trade integration.\(^{43}\)

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\(^{43}\)As noted in footnote 42, the axes of the left panel are in logarithm and so is the vertical axis in the right panel.
Besides, the price of good 0 also falls in period 1. Overall, the MRK in sector 0 falls in period 1 and so does the interest rate in period 0.

\[ q_{0,1} = p_{0,1} \left( \frac{K_{0,1}}{\rho L_{0,1}} \right)^{\alpha-1} < q_{0,0} = p_{0,0} \left( \frac{K_{0,0}}{\rho L_{0,0}} \right)^{\alpha-1} \Rightarrow r_0 < r_A. \]

Furthermore, the allocative efficiency effect dominates the financial development effect so that the interest rate in country N is even lower than the world interest rate in period 0, \( r_0 < r^*_A < r_A \), a result Antras and Caballero (2009) calls the interest rate reversal.

In period \( t \in \{1, ..., 6\}, w_t < \bar{w} \) and sector 0 is active. As the sectoral input of labor is frictionless and that of investment is frictional, trade triggers an asymmetric resource reallocation, which raises the capital-labor share and reduces the MPK in sector 0. Besides, the gains from trade raise aggregate income and the size of domestic investment. Overall, the allocative efficiency effect and the investment size effect jointly reduce the MRK in sector 0. Thus, the interest rate also falls over time.

\[
\frac{K_{0,t+1}}{L_{0,t+1}} = \frac{1 - \delta_t w_t L_{t+1}}{1 - \zeta_{t+1}} = [1 + \delta_t \left( 1 - \mu^* \right)] \frac{w_t}{\text{investment size effect}}.
\]

In period \( t = 7, w_t > \bar{w} \) and country N fully offshores sector 0. Thus, the interest rate is decoupled from (coupled with) the rate of return to sector 0 (1), shown by an interest rate jump in the right panel of figure 16. We call it the interest rate re-reversal. From then on, the interest rate is driven by the DMRK effect and falls over time until country N reaches the new steady state T where the interest rate is even higher than its autarkic value, \( r_T = \bar{Y}_T = \rho > r_A > r^*_A \).

To sum up, the world econour witnesses sequentially the interest rate reversal and re-reversal when country N converges from the autarkic to the new steady state.

**Proposition 7.** In case UF1, the interest rate in country N has a non-monotonic pattern along the convergence path under trade integration. In the new steady state, \( r_T > r_A > r^*_A \).

For the parameter configurations outside region UF1 of figure 14, the static and dynamic gains from trade are too weak to ensure that country N fully offshores sector 0 in the long run. Hence, trade integration just leads to the interest rate reversal, as predicted by Antras and Caballero (2009).

In our model, trade integration may have non-monotonic impacts on industrial composition in country N, which then affects the patterns of financial flows. Undoubtedly, various factors, e.g., globalization, technology progress, industrial policies, and etc., may induce country N to upgrade along the value chain. No matter what the causes are, if the shifts in industrial composition change fundamentally the way the interest rate is determined, upstream financial flows may arise.

## B The Multi-Sector Model

This section analyzes the S-sector model with \( S \geq 2 \). In period \( t \), a fraction \( \delta_{s,t} \) of domestic saving \( w_t L \) is allocated in sector \( s \in \{0, 1, ..., S-1\} \), which yields \( K_{s,t+1} = \delta_{s,t} w_t L \) units of capital in period \( t+1 \). They are hired together with \( \zeta_{s,t+1} L \) units of labor to produce \( Y_{s,t+1} \) units of good \( s \). \( V_{s,t+1} \) units of good \( s \) are combined with the goods from other sectors to produce \( Y_{t+1} \) units of final goods. The final goods serve as the numeraire and are used for consumption and investment. Let \( \eta_s \) denote the sector share in the production of final goods and \( \sum_{s=0}^{S-1} \eta_s = 1 \). Let \( \bar{Y}_{t+1} \equiv \left( \frac{w_t L}{\alpha} \right) \left( \frac{L}{1-\alpha} \right)^{1-\alpha} \) denote the maximum
possible output in a particular sector if domestic saving and labor are fully hired in that sector.

\[
Y_{s,t+1} = \left( \frac{\delta_{s,t} L_w}{\alpha} \right)^\alpha \left( \frac{\zeta_{s,t+1} L}{1 - \alpha} \right)^{1-\alpha} = \delta_{s,t}^{\alpha} \zeta_{s,t+1}^{1-\alpha} Y_{t+1}, \quad Y_{t+1} = \Pi_{s=0}^{s=1} \left( \frac{V_{s,t+1}}{\eta_s} \right). \tag{34}
\]

\[
q_{s,t+1} \delta_{s,t} L_w = p_{s,t+1} Y_{s,t+1} = \frac{w_{s+1} \zeta_{s,t+1} L}{1 - \alpha}, \quad \frac{p_{s,t+1} V_{s,t+1}}{\eta_s} = Y_{t+1}. \tag{35}
\]

In the absence of international mobility of labor and capital, use the domestic labor market clearing condition to derive the production possibility frontier (PPF, hereafter),

\[
\zeta_{s,t+1} = \left( \frac{Y_{s,t+1}}{\bar{V}_{t+1} \delta_{s,t}} \right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad \sum_{s=0}^{S-1} \zeta_{s,t+1} = 1, \quad \Rightarrow \sum_{s=0}^{S-1} \left( \frac{Y_{s,t+1}}{\delta_{s,t}^{\alpha}} \right)^{\frac{1}{1-\alpha}} = \bar{V}_{t+1}^{\frac{1}{1-\alpha}},
\]

which holds under autarky as well as under free trade.

Sectors are ranked in terms of the sector-specific MIR, \( m_0 \equiv 0, m_{s-1} < m_s \) for \( s \in \{1, ..., S - 1\} \). For notational simplicity, define \( m_S \equiv \infty \). For \( z \in \{0, ..., S - 1\} \), define two series of threshold values,

\[
w_z \equiv \frac{m_z}{1 - \theta} (1 - \lambda)^{-\frac{1}{1-\theta}}, \quad \text{and} \quad w_{z,A} = \frac{w_z}{\left( 1 + \frac{\sum_{v=0}^{z-1} \eta_v}{\eta_z} \left[ 1 - \left( 1 \frac{m_z}{m_{z+1}} \right)^{\frac{1-\theta}{\theta}} \right] \right)^{\frac{1}{1-\theta}}}. \tag{36}
\]

Let \( a \equiv \min\{v : \delta_v > 0\} \) denote the lowest sector index among all active sectors. As all goods are essential for the production of final goods, \( a = 0 \) must hold under autarky. Under trade, country N may fully offshore low-index sectors so that \( 0 \leq a \leq S - 1 \) holds.

### B.1 The Autarkic Equilibrium: \( a = 0 \)

**Assumption 4.** \( \frac{m_z}{1 - \theta} \frac{m_z^{1-\theta}}{m_{z+1}^{1-\theta}} < \frac{\eta_{z+1}}{\eta_z} \) holds for \( s \in \{1, ..., S - 2\} \).

**Lemma 6.** Under autarky, for \( w_t \in [\bar{w}_{z,A}, \bar{w}_{z+1,A}] \), the borrowing constraints are slack and the rate of return equalizes in sector \( s \in \{0, ..., z\} \), while the borrowing constraints are binding and the sectoral rate of return is ascending in sector \( s \in \{z + 1, ..., S - 1\} \).

Under autarky, the law of motion for wage is a piecewise function.\(^{44}\) For \( w_t \in [\bar{w}_{z,A}, \bar{w}_{z+1,A}] \),

\[
w_{t+1} = \left( \frac{w_t}{\rho} \Gamma_t \right)^\alpha, \quad \Gamma_t = \left( \frac{\delta_{a,t}}{\eta_a} \right) \Pi_{v=z+1}^{S-1} \left( \frac{\delta_{v,t}}{\eta_v} \right),
\]

where \( \delta_{v,t} = \frac{1-\theta}{\theta} \kappa_v \), \( \kappa_v \equiv \bar{w}_v - \bar{w}_{v+1} \) for \( v \in \{z + 1, ..., S - 1\} \),

\[
\delta_{a,t} \equiv \frac{1-\theta}{\theta} \kappa_{a,t}, \quad \kappa_{a,t} \equiv \bar{w}_t - \bar{w}_{z+1}, \quad \eta_v \equiv \sum_{v=0}^{z} \eta_v.
\]

Let us analyze the sectoral rate-of-return pattern. Define \( \mu_{s,t+1} = \frac{q_{s,t+1}}{q_{s-1,t+1}} \) as the rate of return in sector \( S - 1 \), and thus consider the case of \( w_t \in [\bar{w}_{z,A}, \bar{w}_{z+1,A}] \).

- In sector \( s \in \{0, ..., z\} \), the borrowing constraints are slack. The rate of return equalizes among these sectors \( q_{s,t+1} = q_{z,t+1} = r_t \) and so does the capital-labor ratio. Combine them for \( \mu_{s,t+1} \)\(^{45}\)

\[
\mu_{s,t+1} = \mu_{z,t+1} = \frac{\delta_{s-1,t}}{\delta_{s,t}} \delta_{z,t} \zeta_{s,t+1} = \frac{\delta_{s-1,t}}{\delta_{s,t}} \zeta_{s,t+1} = \frac{\delta_{s-1,t}}{\delta_{s,t}} \zeta_{s,t+1}.
\]

\(^{44}\)See the proof of lemma 6 for derivation.

\(^{45}\)As shown in the proof of Lemma 6, \( \delta_{a,t} = \sum_{v=0}^{z} \delta_{v,t} \) and \( \delta_{s,t} = \sum_{s=0}^{S-1} \zeta_{s,t+1} \). The sectoral capital-labor ratio equalizes among sector \( s \in \{0, ..., z\} \), \( \delta_{a,t} = \sum_{s=0}^{S-1} \zeta_{s,t+1} \).
\[ \mu_{s,t+1} = \frac{\delta_{S-1,t}}{\zeta_{S-1,t+1}} = \frac{m_{S-1}}{m_z - \frac{1}{\eta} - \frac{1}{\eta} \eta_{S-1}}. \]

(39)

Assumption 4 ensures the ascending relative sectoral rate of return, \( \mu_{s,t+1} < \mu_{s+1,t+1} \).

![Figure 17: Patterns of Relative Sectoral Rate-of-Return](image)

Given \( w_t = (\bar{w}_{zL,A}, \bar{w}_{zL+1,A}) \), the left panel of figure 17 shows the relative sectoral rate-of-return, with the sector index on the horizontal axis.\(^{46}\) According to equations (38) and (39), \( \mu_{s,t+1} \) is constant at \( \mu_{zL,t+1} \) in sector \( s \in \{0, \ldots, z_L\} \) and ascending in sector \( s \in \{z_L + 1, \ldots, S - 1\} \).

- Given \( \lambda \), if the income level rises to be in the interval of \( w_t = (\bar{w}_{zH,A}, \bar{w}_{zH+1,A}) \), the relative rate of return in sector \( s \in \{z_H + 1, \ldots, S - 1\} \) does not change, while that in sector \( s \in \{0, \ldots, z_H\} \) shifts upwards and becomes flat, with \( \mu_{s,t+1} = \mu_{zH,t+1} \). See the right panel of figure 17.

- Given \( w_t \), a rise in the level of financial development \( \lambda \) reduces the threshold values \( \bar{w}_{z,A} \), which affects the relative sectoral rate of return in the same way as the rise in the income level.

The rise in \( w_t \) and/or \( \lambda \) allows each agent to raise the investment. In sector \( s \in \{z_H + 1, \ldots, S - 1\} \), some agents who previously invest in sector \( s - 1 \) can overcome the MIR and invest in sector \( s \), while some agents who previously invest in sector \( s \) can overcome the MIR and invest in sector \( s + 1 \). Given the Pareto distribution of labor endowment, the sectoral investment share rises in equal proportions in these sectors and hence, the relative sectoral rate of return stays constant. Meanwhile, as domestic investment shifts towards sector \( s \in \{z_H + 1, \ldots, S - 1\} \), the overinvestment problem in sector \( s \in \{0, \ldots, z_L\} \) is ameliorated so that the relative rate of return in these sectors gets closer to the efficient level.

Let \( \tilde{\lambda} \) denote a series of threshold values,\(^{47}\) where \( z \in \{0, \ldots, S - 1\} \).

**Proposition 8.** For \( 1 - \eta_0 > \frac{\bar{z}}{\bar{S} - 1} \) and \( \lambda \in (\tilde{\lambda}_z, \tilde{\lambda}_{z+1}) \), there exists a unique steady state under autarky with the steady-state wage rate \( w_A = (\bar{w}_{z,A}, \bar{w}_{z+1,A}) \).

Country N is a small open economy, which is more financially developed than the rest of the world, \( \lambda^* < \lambda < \tilde{\lambda}_{S-1} \). The world economy is initially at the autarkic steady state.

Let \( \chi_{s,t} = \frac{\mu_{s,t}}{\mu_{S-1,t}} \) denote the relative output price in sector \( s \) with respect to sector \( S - 1 \). Since \( \chi_{s,t} = \mu_{s,t}^* \), the relative sectoral price has the same pattern as the relative sectoral rate of return. The solid

---

\(^{46}\) The exact shape of the curve depends on the parameter values of \( m_z \) and \( \eta_z \).

\(^{47}\) \( \tilde{\lambda}_z \) is specified by equation (118) in the proof of proposition 8.
Figure 18: Steady-State Patterns of the Relative Sectoral Output Prices under Autarky

(dashed) curve in figure 18 shows the steady-state pattern of the relative sectoral price in country N (the rest of the world). In country N, with \( w_A \in (\bar{w}_z,A, \bar{w}_{z+1},A) \), the borrowing constraints are slack in sector \( s \in \{0, ..., z\} \) so that the relative sectoral price is equalized at \( \chi_{s,A} = \chi_z,A \); the borrowing constraints are binding in sector \( s \in \{z + 1, ..., S - 1\} \) and \( \chi_{s,A} \) is ascending in the sector index. The rest of the world is less financially developed and the autarkic steady-state wage is \( w^*_A \in (\bar{w}^*_z,A, \bar{w}^*_{z+1},A) \), where \( z^* < z \). The relative sectoral price in the rest of the world \( \chi^*_s,A \) is identical as that in country N for sector \( s \in \{z + 1, ..., S - 1\} \) and lower than that in country N for sector \( s \in \{0, ..., z\} \). Thus, country N has the comparative advantage in the sector with the binding borrowing constraints \( s \in \{z + 1, ..., S - 1\} \), while the rest of the world has the comparative advantage in other sectors, \( s \in \{0, ..., z\} \).

B.2 The Free-Trade Equilibrium: \( a \geq 0 \)

Under free trade, the relative sectoral prices in country N are aligned with the world levels \( \chi_{s,t+1} = \chi^*_s,A \) and so are the relative sectoral rates of return, \( \mu_{s,t+1} = \mu^*_s,A \). For \( w_t \in (\bar{w}_z, \bar{w}_{z+1}) \) and \( z \in \{z^* + 1, ..., S - 1\} \), the mass of investors in sector \( s \in \{z, ..., S - 1\} \) is so large that domestic saving is fully invested there, while sector \( s \in \{0, ..., z - 1\} \) vanishes, \( \delta_{s,t} = 0 \). The borrowing constraints are slack in sector \( z \) and binding in sector \( s \in \{z + 1, ..., S - 1\} \). The law of motion for wage is a piecewise function.

\[
\begin{align*}
    w_{t+1} &= \left(\frac{w_t}{\rho} \Gamma_t\right)^{\alpha}, \quad \text{where} \quad \Gamma_t \equiv \frac{\mu^*_s,A \delta_{z,t} + \sum_{v=z+1}^{S-1} \mu^*_{v,A} \delta_{v,t}}{\Pi_{v=0}^{S-1}(\mu^v_{v,A})^{\eta_v}}, \\
    \delta_{z,t} &= \left(\frac{1}{\theta}\right)^{1-\theta} \kappa_{z,t}, \quad \kappa_{z,t} \equiv w_t \left(\frac{1-\theta}{\rho}\right) - \bar{w}_{z+1}^{1-\theta}, \\
    \delta_{v,t} &= \left(\frac{1}{\theta}\right)^{1-\theta} \kappa_v, \quad \kappa_v \equiv \bar{w}_v \left(\frac{1-\theta}{\rho}\right) - \bar{w}_{v+1}^{1-\theta} \quad \text{for} \quad v \in \{z + 1, ..., S - 1\}.
\end{align*}
\]

Corollary 1. Trade integration may allow country N to sequentially offshore the low-index sectors and upgrade to the high-index sectors.

The conditions for sequential production upgrading are qualitatively similar as in proposition 6.

C Value-Added Trade and Upstream Financial Flows

In this subsection, we use the mechanism featured above to explain intuitively how the rise of value-added trade may affect the direction of financial flows in advanced economies.

In manufacturing industries, upstream activities (such as R&D, product design, or the manufacturing of key parts and components) and downstream activities (such as marketing, brand building, and customer

36
service) constitute a large share of value-added, while the intermediate production stages (such as component fabrication and final assembly) account for a small value-added share (Kimura, 2003). Stan Shih, the founder of Acer, introduced the smile curve to feature such an “U-shaped” value-added pattern along the production chain (Baldwin, Ito, and Sato, 2014). Compared with those of upstream and downstream activities, the value-added shares of fabrication and assembly activities have declined substantially in the OECD countries since the 1970s (Baldwin and Lopez-Gonzalez, 2015; Gereffi, 1999; Koopman, Wang, and Wei, 2014). These facts can be justified in our model as follows.

Fabrication and assembly are involved intensively with standardized, routine tasks that require mainly the input of tangible investment, while upstream and downstream activities are more involved with knowledge-intensive, non-routine tasks that require mostly the input of intangibles. Compared to tangible investment, intangibles are subject to higher MIR and more severe financial frictions. In our model, investment can be tangible or intangible, while “sectors” can be interpreted broadly as production stages or tasks. Our model predicts that the output price and the investment return are higher in upstream and downstream activities than in fabrication and assembly. One may use the smile curve to feature the rate-of-return pattern across production stages.48

Before moving on to the smile curve analysis, we first distinguish two ways of ranking tasks in the production chain. Suppose that capital $K_s$ and labor $L_s$ are hired to complete various tasks, where subscript $s$ denotes the task index. In the left panel of figure 19, the rightward pointing arrow shows the direction of the production process and tasks are ranked in sequential order, which is common in the literature of global value chain (Antras and Chor, 2013). As our model features cross-task heterogeneity in the MIR and the rate of return, we rank tasks in ascending order in terms of the MIR, as shown by the upward pointing arrow in the right panel.49 A task may show up in different positions in the two panels. For example, if the production process starts with R&D which require the largest MIR, the R&D shows up as “Task a” in the left panel and “Task 4” in the right panel; if assembly activities take place in the middle of the production process and do not require the MIR, the assembly activities show up as “Task c” in the left panel and “Task 0” in the right panel.

Without loss of generality, we assume that fabrication and assembly activities do not require the MIR, while upstream and downstream activities require roughly the same MIR. Thus, the former are regarded as “task 0”, while the latter as “task 1” in our two-sector setting.50

48 The original smile curve shows the value-added pattern along the production chain. Since the production stages/tasks that account for the high value-added shares usually have the high MRK, we use the smile curve to show the MRK pattern along the value chain.

49 Although tasks may differ in many other dimensions (e.g., capital intensity, riskiness), we focus only on the task-specific MIR in this paper.

50 One can further disaggregate upstream and downstream activities into multiple stages, according to their re-
Let us use the smile curve to feature the sectoral rate-of-return pattern. As advanced economies (North) are more developed than emerging economies (South) in financing intangibles (Corrado et al., 2013), the sectoral rate-of-return differential is smaller and the smile curve is flatter in North than in South. In the upper panel of figure 20, variables in North (South) are denoted without (with) the asterisk superscript. The interest rate $r$ is coupled with the MRK in the lowest-return production stages/tasks that are active, $r = \min \{ MRK_s \| K_s > 0 \}$. At the autarkic steady state, the interest rate is higher in North than in South, $r_A > r^*_A$. If allowed, financial flows are upstream from South to North. Besides, North (South) has a comparative advantage in upstream and downstream (fabrication and assembly) activities.

Consider North as a small open economy. In the presence of high trade costs, the incentive of offshoring is small and a marginal decline in trade costs only allows North to offshore part of fabrication and assembly activities, leading to a “falling jaw” of its smile curve, as shown in the middle panel of figure 20. It is consistent with the dynamics of value-added shares in OECD countries since the 1970s (OECD, 2013). As the interest rate in North is still coupled with the MRK in fabrication and assembly, respective MIR. Then, one just follows the analysis in the multi-sector setting as shown in appendix B.

Figure 20: The “Falling vs. Missing Jaw” of the Smile Curve in North
the interest rate reversal occurs, i.e., \( r_T = MRK_0 < r_A^* < r_A \), consistent with the prediction of Antras and Caballero (2009).

Since the 1990s, technological progress and world-wide economic liberalization have substantially reduced the costs of transportation, communication, and coordination, leading to the boom of supply-chain trade. Vertically and horizontally linked production stages/tasks have been increasingly conducted in different countries. In our model, a substantial decline in trade costs may allow North to offshore all fabrication/assembly and specialize fully in upstream/downstream activities. If so, the smile curve in North witnesses a “missing jaw” and the interest rate is coupled with the MRK in upstream and downstream activities. As shown in the lower panel of figure 20, supply-chain trade leads to the interest rate re-reversal, which amplifies the initial interest rate differential, \( r_T = MRK_1 > r_A > r_A^* \).

\[ (PPF, \text{hereafter}) \]

\[ \text{Proof of Proposition 1} \]

**Proof.** The proof consists of two steps.

**Step 1: Use the PPF and the Isoquant to determine the cross-sector allocation**

Combine equations (1) and (3) to get (4), which implicitly describes the production possibility frontier (PPF, hereafter) in period \( t + 1 \) and reflects the domestic supply of sectoral outputs.

\[
\frac{q_{1,t+1} \delta_1 M_t}{\alpha} \equiv p_{1,t+1} Y_{1,t+1} = \frac{w_{t+1} \zeta_{t+1} L}{1 - \alpha}, \quad \frac{w_{t+1}}{q_{1,t+1}} = \frac{\delta_1 M_t}{\rho \zeta_{t+1} L},
\]

\[
\frac{q_{0,t+1} (1 - \delta_t) M_t}{\alpha} \equiv p_{0,t+1} Y_{0,t+1} = \frac{w_{t+1} (1 - \zeta_{t+1}) L}{1 - \alpha}, \quad \frac{w_{t+1}}{q_{0,t+1}} = \frac{(1 - \delta_t) M_t}{\rho (1 - \zeta_{t+1}) L},
\]

\[
\mu_{t+1} \equiv \frac{q_{0,t+1}}{q_{1,t+1}} = \left( \frac{\delta_t}{1 - \zeta_{t+1}} \right), \quad \zeta_{t+1} = \frac{(1 - \delta_t) \mu_{t+1}}{\delta_t} + \delta_t
\]

\[
Y_{1,t+1} = \left( \frac{\delta_t M_t}{\rho \zeta_{t+1} L} \right)^{\alpha} \zeta_{t+1} L \equiv p_{1,t+1} Y_{1,t+1} \frac{w_{t+1}^{1-\alpha}}{(1 - \alpha)},
\]

\[
Y_{0,t+1} = \left( \frac{(1 - \delta_t) M_t}{\rho (1 - \zeta_{t+1}) L} \right)^{\alpha} (1 - \zeta_{t+1}) L \equiv p_{0,t+1} Y_{0,t+1} \frac{w_{t+1}^{1-\alpha}}{(1 - \alpha)},
\]

\[
p_{1,t+1} = \left( q_{1,t+1} \right)^{\alpha} w_{t+1}^{1-\alpha}, \quad p_{0,t+1} = \left( q_{0,t+1} \right)^{\alpha} w_{t+1}^{1-\alpha}, \quad \Rightarrow \chi_{t+1} = \mu_{t+1}^\alpha.
\]

\[
Y_{t+1} = \left( \frac{V_{1,t+1}}{\eta} \right) \left( \frac{V_{0,t+1}}{\eta} \right)^{1-\eta} = \left( \frac{Y_{1,t+1}}{p_{1,t+1}} \right) \left( \frac{Y_{0,t+1}}{p_{0,t+1}} \right)^{1-\eta}, \quad \Rightarrow p_{1,t+1} p_{0,t+1}^{1-\eta} = 1.
\]

Combine equations (1) and (3) to derive the marginal rate of transformation (MRT, hereafter),

\[
\ln Y_{1,t+1} = \alpha \ln \delta_t + (1 - \alpha) \ln \zeta_{t+1} + \ln Y_{t+1}, \quad \text{where} \quad Y_{t+1} \equiv \left( \frac{M_t}{\alpha} \right)^{\alpha} \left( \frac{L}{1 - \alpha} \right)^{1-\alpha}
\]

\[
\ln Y_{0,t+1} = \alpha \ln (1 - \delta_t) + (1 - \alpha) \ln (1 - \zeta_{t+1}) + \ln Y_{t+1}
\]

\[
\frac{\partial \ln Y_{1,t+1}}{\partial \ln \zeta_{t+1}} = \alpha \frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1 - \alpha), \quad \frac{\partial \ln Y_{0,t+1}}{\partial \ln \zeta_{t+1}} = \frac{-\delta_t \alpha}{1 - \delta_t} \frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + \frac{-\zeta_{t+1} (1 - \alpha)}{1 - \zeta_{t+1}}
\]

\[
MRT_{0,t+1} = \frac{\partial Y_{1,t+1}}{\partial Y_{0,t+1}} = -\frac{\partial \ln Y_{1,t+1} Y_{t+1} Y_{0,t+1}}{\partial \ln Y_{0,t+1} Y_{t+1}} = \frac{\alpha}{\alpha - \delta_t} \frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + \frac{(1 - \alpha) \zeta_{t+1}}{1 - \zeta_{t+1}} Y_{1,t+1}
\]

If the sectoral investment is efficient, \( \mu_{t+1} = 1 \) holds; combine it with equation (44) to get \( \zeta_{t+1} = \delta_t \) and \( \frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} = 1 \). If the sectoral investment is inefficient, \( \mu_{t+1} < 1 \) holds; given the predetermined \( \delta_t \),

\[ 39 \]
\[ \frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} = 0. \] Thus, the (absolute) gradient of the PPF is
\[ \text{MRT}_{0,1} = 1 - \frac{\zeta_{t+1} Y_{1,t+1}}{Y_{0,t+1}}. \] (49)

Equation (2) specifies the isoquant with the (absolute) gradient of
\[ \text{MRS}_{0,1} \equiv -\frac{\partial V_{1,t+1}}{\partial V_{0,t+1}} = 1 - \frac{1 - \eta}{\eta \frac{V_{1,t+1}}{V_{0,t+1}}}. \] (50)

In the absence of trade in sectoral goods, the markets of sectoral goods clear domestically \( V_{s,t+1} = Y_{s,t+1} \) so that the PPF and the isoquant are tangent. Combine it with equations (49) and (50) to get
\[ \text{MRS}_{0,1} = \text{MRT}_{0,1}, \quad \Rightarrow \quad \frac{1 - \eta}{\eta} = 1 - \frac{1 - \zeta_{t+1}}{\zeta_{t+1}}, \quad \Rightarrow \quad \zeta_{t+1} = \eta. \] (51)

**Step 2: the Condition for the Unique Steady State under Autarky**

Under autarky, the sectoral fraction of labor input is equal to the sector share in the final goods production, \( \zeta_{t+1} = \eta \). Whether it also applies to the investment share depends on how far financial frictions distort the maximum share of domestic investment in sector 1.

\[ \delta_t = \begin{cases} \tilde{\delta}_t < \zeta_{t+1} = \eta, & \text{if } \tilde{\delta}_t < \eta; \\ \zeta_{t+1} = \eta, & \text{if } \tilde{\delta}_t \geq \eta. \end{cases} \] (52)

Combine equations (51)-(52) with (1)-(3) and (4) to get (14) specifying the law of motion for wage.

- If \( w_t \geq \bar{w}_A \equiv \eta^{\frac{\theta}{\alpha}} \bar{w} \), \( \tilde{\delta}_t \geq \eta \) so that the sectoral investment is efficient, i.e., \( \delta_t = \eta \) and \( \Gamma_t = 1 \), and the borrowing constraints are slack. The law of motion for wage degenerates into (15).

- If \( w_t < \bar{w}_A \), \( \tilde{\delta}_t < \eta \) so that the sectoral investment is inefficient, i.e., \( \delta_t = \tilde{\delta}_t < \eta \) and \( \Gamma_t < 1 \), and the borrowing constraints are binding. Combine equations (10) and (14) to get (16). The condition for the stable steady state is
\[ \frac{\partial w_{t+1}}{\partial w_t}_{w_{t+1}=w_t} < 1, \quad \Rightarrow \quad \mu_A > 1 - \frac{g}{\eta}, \quad \text{where } g = \frac{1}{\alpha} - \frac{1}{\eta}. \] (53)

Given \( \mu_{t+1} \in (0, 1] \), a sufficient condition for inequality (53) to hold is \( \theta \geq \hat{\theta} \equiv \frac{\eta}{\eta \alpha + (1-\alpha)}. \)

\[ \begin{align*}
\begin{array}{c}
\text{Figure 21: Threshold Values for the Autarkic Equilibrium} \\
\end{array}
\end{align*} \]

The left pane of figure 21 shows \( \hat{\theta} \) in the \( \{\alpha, \theta\} \) space. Consider first the case of \( \theta \geq \hat{\theta} \), i.e., \( \{\alpha, \theta\} \) in region U of the left panel of figure 21. Let us derive the condition under which the borrowing constraints
are binding at the autarkic steady state, i.e., \( \mu_A < 1 \) or \( \delta_A < \eta \). Combine \( w_{t+1} = w_t = w_A \) with equations (10) and (14) to get \( \mu_A < 1 \) as a function of \( \lambda \),

\[
Z \equiv \frac{1}{\frac{m}{1-\eta} \frac{s}{1-\eta} \rho^\rho} = \frac{(1-\lambda)^{\frac{1}{\eta} \eta} (\mu_A)^{\frac{\rho}{s}} (1-\eta + \eta \mu_A)^{\frac{\rho}{s}}}{(1-\eta + \eta \mu_A)^{\frac{\rho}{s}}}, \quad \frac{\partial \ln \mu_A}{\partial \ln \lambda} = \frac{1}{(1-\delta)(1-(1-\mu_A)^{\eta})}. \tag{54}
\]

Given \( \theta > \bar{\theta}, \varrho > \eta \) so that \( \frac{(1-\mu_A)\eta}{\rho} < 1 \) and \( \frac{\partial \ln \mu_A}{\partial \ln \lambda} > 0 \). Define \( \lambda_A \) as a threshold value such that, for \( \lambda = \lambda_A, \delta_A = \eta \) and \( \mu_A = 1 \) hold. Combine them with equation (54) to get \( \lambda_A \equiv 1 - Z^{1-\theta} \). The right panel of figure 21 shows \( \lambda_A \) in the \( \{\lambda, Z\} \) space.

Thus, for \( \{\alpha, \theta\} \) in region U of the left panel and \( \{\lambda, Z\} \) in region UB (US) of the right panel, there exists a unique steady state with \( w_A < \bar{w}_A \) (\( w_A > \bar{w}_A \)) and the binding (slack) borrowing constraints.\(^{51}\)

![Figure 22: Laws of Motion for Wage under Autarky: \( \theta \geq \bar{\theta} \)](image)

In figure 22, the dashed curves show the law of motion for wage under autarky in the case of perfect credit markets \( \lambda = 1 \), while the solid curves show those in the case where \( \{\alpha, \theta\} \) are in region U and \( \{\lambda, Z\} \) in region UB and US of figure 21, respectively. For \( w_t \in (0, \bar{w}_A) \), the solid curve lies below the dashed curve and the gap reflects the efficiency losses, \( (1 - \Gamma_t) \left( \frac{w_t}{\rho} \right)^\alpha \).

**Proof of Lemma 1**

**Proof.** Under autarky, the markets for sectoral outputs clear domestically, \( V_{s,t} = Y_{s,t} \), implying \( K_{0,t+1} > 0 \) and \( r_t = q_{0,t+1} \). According to (45)-(48), \( w_{t+1}^{1-\alpha} (q_{0,t+1})^{\alpha(1-\eta)} (q_{1,t+1})^{\eta} = 1 \). Combine them to get \( w_{t+1} = \left( \frac{1}{\rho} \right)^\rho \). Combine it with (14) to get (18).

For \( \theta \in (\bar{\theta}, 1) \) and \( \lambda \in [0, \lambda_A] \), there exists a unique steady state with

\[
w_A = \left( \frac{\Gamma_A}{\rho} \right)^\rho, \quad \text{where} \quad \Gamma_A = \frac{\mu_A^\eta}{1 - \eta + \eta \mu_A} \quad \text{and} \quad \frac{\partial \ln \Gamma_A}{\partial \ln \mu_A} = \eta(1 - \delta_A) > 0.
\tag{55}
\]

In the autarkic steady state, \( w_{t+1} = w_t \) implies that \( Y_A = \rho \) and \( r_A = \rho[1 - \eta + \eta \mu_A] \). According to equation (54), \( \frac{\partial \ln \mu_A}{\partial \lambda} > 0 \) for \( \theta \in (\bar{\theta}, 1) \). Thus, \( \frac{\partial r_A}{\partial \lambda} > 0, \frac{\partial w_A}{\partial \lambda} > 0, \frac{\partial \ln \lambda_A}{\partial \lambda} > 0 \). Given \( \lambda^* < \lambda < \lambda_A \), it holds that \( Y_A^* < Y_A, \chi_A^* < \chi_A < 1 \), and \( r_A^* < r_A < \rho \).

**Proof of Proposition 2**

\(^{51}\)See the proof of Proposition 1 for the laws of motion for wage in case UB and US, respectively. For \( \{\alpha, \theta\} \) in region M, multiple steady states arise. See the proof of proposition 1 for the analysis of that case.
The law of motion for wage under financial integration is specified by equation (56), which takes the same form as that under autarky, i.e., equation (14). The allocations under these two scenarios differ in the determination of $M_t$ and $r_t$. Given $\mu_{t+1} < 1$, combine equations (10) with (56) and (57) to get

$$\mu_{t+1} = (r^*)^{\frac{\eta}{1-\alpha(1-\eta)}},$$

$$w_{t+1} = (\mu_{t+1} \eta r^*)^{\frac{\eta}{\eta r^*}}.$$

Set $\mu_{t+1} = 1$ to get the threshold value $\bar{w}_F \equiv \bar{w}_F \left( \frac{\eta \rho}{w} \left( r^* + \alpha \right) \right)^{\theta}$. For $w_t < \bar{w}_F$, $\mu_{t+1} < 1$ and $q_{0,t+1} > q_{0,t} > r_t$ so that the borrowing constraints are binding; for $w_t \geq \bar{w}_F$, the borrowing constraints are slack and combine $\mu_{t+1} = 1$ with equation (57) to get the law of motion for wage flat at $w_{t+1} = (r^*)^{-\rho}$.

Given $\theta \in (0, 1)$, the law of motion for wage is log-linear with a slope less than unity for $w_t < \bar{w}_F$ and is flat at $w_{t+1} = (r^*)^{-\rho}$ for $w_t \geq \bar{w}_F$. Thus, there exists a unique steady state under financial integration.

**Step 2: Short-Run vs. Long-Run Impacts of Financial Integration on Allocative Efficiency**

In period 0, financial integration lowers the interest rate $r_0 = r^* < r_A$ and, given $w_0 = w_A$, combine equation (10) and (56)-(58) to get (59) and (19) describing the investment size effect and the allocative efficiency effect in period 0 as well as the income dynamics in period 1.

$$\frac{\partial \ln M_0}{\partial \ln r_0} = \frac{1 - \delta_0}{1 - \alpha(1 - \eta)}, \quad \frac{\partial \ln \Gamma_0}{\partial \ln r_0} = \frac{\eta - \delta_0}{1 - \alpha(1 - \eta)}, \quad \frac{\partial \ln \mu_0}{\partial \ln r_0} = \frac{\eta - \delta_0}{1 - \alpha(1 - \eta)}.$$

From period 1 on, the interest rate is constant at $r_1 = r^*$. Combine equation (10), (56), and (58) to get equations (60) and (20) describing the investment size effect and the allocative efficiency effect in period $t \geq 1$ as well as the income dynamics in period $t + 1$.

$$\frac{\partial \ln \delta_t}{\partial \ln w_t} = \frac{\hat{\theta}(1 - \delta_t)}{\rho \eta}, \quad \frac{\partial \ln M_t}{\partial \ln w_t} = \frac{1}{\theta} - \frac{\partial \ln \delta_t}{\partial \ln w_t} = \frac{\hat{\theta}}{\theta} \left( 1 + \frac{\delta_t}{\rho \eta} \right), \quad \frac{\partial \ln \Gamma_t}{\partial \ln \delta_t} = \frac{\eta - \delta_t}{1 - \delta_t}.$$
For \( \theta \in (\hat{\theta}, \alpha) \), the fall in the interest rate raises allocative efficiency in the long run, \( \Gamma_F > \Gamma_A \); for \( \theta \in (\alpha, 1) \), the opposite applies, \( \Gamma_F < \Gamma_A \).

**Step 3: Solve for the Normalized Financial Flow**

For \( w_t < \tilde{w}_F \), the borrowing constraints are binding. According to equation (44),

\[
\mu_{t+1} = \frac{\delta_l M_l}{(1-\delta_l)M_l} = \left( \frac{w_t}{\bar{w}} \right)^{\frac{1-\theta}{\rho}} \frac{1-\eta}{\eta} - \frac{\alpha-\theta}{1-\alpha} \ln r_A + \ln \mu_{t+1} = \left[ 1 - \frac{\eta}{\theta} \right] (\frac{w_t}{\bar{w}})^{\frac{1-\theta}{\rho}} - 1.
\]

For \( w_t > \tilde{w}_F \), the borrowing constraints are slack in sector 1, \( q_{1,t+1} = r^* = q_{0,t+1} \),

\[
(r^*)^{-\rho} = w_{t+1} = \left( \frac{M_l}{\rho \bar{L}} \right)^{\alpha}, \quad \phi_t = \frac{M_l}{w_t L} - 1 = \frac{\rho}{w_t (r^*)^{1-\rho}} - 1. \tag{62}
\]

**Proof of Lemma 2**

*Proof.* The proof consists of three steps.

**Step 1: Solve for the Threshold Value \( \tilde{\lambda}_F \)**

Under financial integration, for \( \lambda = \tilde{\lambda}_F, w_F = \tilde{w}_F \) so that the borrowing constraints are weakly binding at the steady state, \( \mu_F = 1 \) and \( w_F = (r^*)^{-\rho} \). Combine it with \( \tilde{w}_F \equiv \left( \eta \rho w^1 - \theta \right) \eta \mu^1 \) to get

\[
\frac{\alpha-\theta}{1-\alpha} \ln r^* + \theta \ln \eta r^{1-\rho} = 0. \tag{62}
\]

In the autarkic steady state, \( \mu_{t+1} = \mu_t = \mu_A \) and \( M_A = w_A L \). Combine it with equation (17), (18), (10), and (14) to get equation (63) and (64) specifying the autarkic steady state in country N and the rest of the world, respectively.

\[
\theta \ln \eta r^{1-\rho} + \rho \ln \mu_A + \frac{\alpha-\theta}{1-\alpha} \ln r_A = 0, \tag{63}
\]

\[
\theta \ln \eta r^{1-\rho} + \rho \ln \mu_A + \frac{\alpha-\theta}{1-\alpha} \ln r^*_A = \ln \frac{1-\lambda}{1-\lambda^*}. \tag{64}
\]

Combine \( r^* = r^*_A \) with equations (62) and (64) to get \( \tilde{\lambda}_F \) as a function of \( \lambda^* \).

\[
\ln(1 - \tilde{\lambda}_F) = \ln(1 - \lambda^*) + \rho \frac{\theta - \hat{\theta}}{\theta} \ln \mu^*_A. \tag{65}
\]

**Step 2: Compare \( \tilde{\lambda}_F \) and \( \tilde{\lambda}_A \)**

Combine equations (63) and (64) to get

\[
\rho \frac{\theta - \hat{\theta}}{\theta} (\ln \mu_A - \ln \mu^*_A) + \frac{\alpha-\theta}{1-\alpha} (\ln r_A - \ln r^*_A) + \ln \frac{1-\lambda}{1-\lambda^*} = 0. \tag{66}
\]

Under autarky, if \( \lambda = \tilde{\lambda}_A \), the borrowing constraints are weakly binding at the steady state, \( \mu_A = 1 \) and \( r_A = \rho > r^*_A \). Combine them with equations (65)-(66) to get

\[
\ln(1 - \tilde{\lambda}_F) = \ln(1 - \tilde{\lambda}_A) + \frac{\alpha-\theta}{1-\alpha} (\ln \rho - \ln r^*_A). \tag{67}
\]

For \( \theta \in (\hat{\theta}, \alpha) \), given \( \lambda^* \in (0, \tilde{\lambda}_A) \), \( \tilde{\lambda}_F < \tilde{\lambda}_A \), as shown by the flat curve in the middle panel of figure 5. For \( \theta \in (\alpha, 1) \), given \( \lambda^* \in (0, \tilde{\lambda}_A) \), \( \tilde{\lambda}_F > \tilde{\lambda}_A \) and hence the right panel of figure 5 does not show \( \tilde{\lambda}_F \).
Step 3: Financial Integration and Comparative Advantage

Given \( \theta \in (\hat{\theta}, 1) \) and \( \lambda > \lambda^* \), combine equations (47), (61) and (64) to get

\[
\frac{\eta}{1 - \alpha} \frac{\theta - \hat{\theta}}{\hat{\theta}} (\ln \chi_F - \ln \chi_A^*) + \ln \frac{1 - \lambda}{1 - \lambda^*} = 0, \quad \Rightarrow \chi_F > \chi_A^*.
\]

(68)

Under financial integration, country N still has a comparative advantage in sector 1 in the steady state. \( \square \)

Proof of Proposition 3

Proof. The proof consists of three steps.

Step 1: Derive the Law of Motion for Wage under Dual Integration

Consider first the case where country N fully specializes in sector 1 and the borrowing constraints are binding under dual integration, i.e., \( \delta_t = \zeta_{t+1} = 1 \) and \( q_{1,t+1} > r^* > q_{0,t+1} \). Combine \( \chi_{t+1} = \chi^* < 1 \) with equation (45) and (48) to get (22) specifying the law of motion for wage. The normalized financial flow is

\[
\phi_t = \frac{M_t}{M_t} - 1 = \left( \frac{w_t}{\bar{w}} \right)^{\frac{1}{1 - \alpha}} - 1.
\]

Combine equation (22) and (43) to get

\[
\ln q_{1,t+1} - \ln r^* = -(1 - \eta) \ln \chi^* - \frac{1 - \alpha}{\theta} \ln w_t - \ln r^* + (1 - \alpha) \ln \rho \left( \frac{1 - \theta}{\bar{w}} \right)^{\theta - \alpha}.
\]

(69)

\[
\ln q_{0,t+1} - \ln r^* = \ln q_{1,t+1} \mu^* - \ln r^* = \ln q_{1,t+1} - \ln r^* + \ln \mu^*.
\]

(70)

If \( q_{1,t+1} = r^* \), the borrowing constraints are slack; if \( q_{0,t+1} = r^* \), sector 0 is also active. Combine them with equations (69) and (70) to get two threshold values,

\[
\bar{w}_D \equiv \rho^\theta \bar{w}^{1-\theta} \left[ r^*(\chi^*)^{1-\eta} \right]^\frac{\theta}{\eta-\theta}, \quad \text{and} \quad \underline{w}_D \equiv \bar{w}_D (\mu^*)^{\frac{\theta}{\eta-\theta}} < \bar{w}_D.
\]

(71)

Combine equation (45)-(48) to get

\[
w_t = \left( \frac{q_{0,t+1}}{q_{1,t+1}} \right)^{\theta} = \left( \frac{\mu_{t+1}}{q_{0,t+1}} \right)^{\frac{\theta}{1 - \eta}} = \left( \frac{\mu_{t+1}^{\eta - 1}}{\eta} \right)^{\frac{\theta}{1 - \eta}}.
\]

(72)

For \( w_t > \bar{w}_D \), \( q_{1,t+1} = r^* \) and \( \mu_{t+1} = \mu^* \). Combine them with equations (22) and (72). The law of motion for wage is flat at \( w_{t+1} = \left( \frac{r^*(\chi^*)^{1-\eta}}{\alpha} \right) \)

\[
(\chi^*)^{\eta-1} \left[ \frac{(1 + \phi_t) w_t}{\rho} \right]^\alpha = w_{t+1} = \left[ r^*(\chi^*)^{1-\eta} \right]^{-\rho}, \quad \Rightarrow \phi_t = \frac{\rho}{w_t} \left[ \frac{r^*(\chi^*)^{1-\eta}}{1 - \eta} \right]^\frac{1}{\rho} - 1.
\]

For \( w_t < \underline{w}_D \), \( q_{0,t+1} = r^* \) and \( \mu_{t+1} = \mu^* \). Combine them with equations (22), (45)-(48), and (72). The law of motion for wage is flat at \( w_{t+1} = w^* = \left( \frac{\mu^*}{r^*} \right)^{\theta} \) and the normalized financial flow is

\[
\phi_t = \frac{\rho}{w_t} \left[ \frac{r^*(\chi^*)^{1-\eta}}{1 - \eta} \right]^\frac{1}{\rho} - \left( \frac{1}{\mu^*} - 1 \right) \left( \frac{w_t}{\bar{w}} \right)^{\frac{1}{\mu^*} - 1} - 1,
\]

The law of motion for wage is piecewise under dual integration.

- For \( \theta \in (\hat{\theta}, \alpha) \), the law of motion for wage is log-linear with a slope larger than unity in the interval of \( w_t \in (\underline{w}_D, \bar{w}_D) \) and is flat in the intervals of \( w_t < \underline{w}_D \) and \( w_t > \bar{w}_D \). If there is a steady state in the interval of \( w_t \in (\underline{w}_D, \bar{w}_D) \), it is unstable and there are multiple steady states; otherwise, there exists a unique steady state.
• For \( \theta \in (\alpha, 1) \), the law of motion for wage is log-linear with a slope less than unity in the interval of \( w_t \in (w_D, \bar{w}_D) \) and is flat in the intervals of \( w_t < w_D \) and \( w_t > \bar{w}_D \). Thus, there exists a unique steady state.

**Step 2: Trade and the Short-Run Reversal of Financial Flows**

Given \( \lambda \in (\lambda^*, \min\{\hat{\lambda}_A, \hat{\lambda}_F\}) \), country N is initially in the steady state under financial integration where the borrowing constraints are binding. In period 0, \( w_0 = w_F < w_D < w_F \) and \( \mu_1 = \mu^* \leq 1 \) imply that the borrowing constraints are binding. If country N specializes fully in sector 1,

\[
\delta_0 = 1, \quad M_0 = \left( \frac{w_0}{\bar{w}} \right) \frac{1 - \rho}{\sigma}, \quad \phi_0 = M_0 \left( \frac{w_0}{\bar{w}} \right) \frac{1 - \rho}{\sigma} - 1 = \left( \frac{w_0}{\bar{w}} \right) \frac{1 - \rho}{\sigma} - 1
\]

As \( w_0 = w_F < \bar{w}_F, \bar{w}_F < \bar{w} \) is a sufficient condition for the reversal of financial flows in period 0. For \( \lambda = \hat{\lambda}_F \), the following conditions hold in the steady state under financial integration,

\[
w_F = (r^*)^{-\rho}, \quad \delta_F = \eta, \quad M_{F, F, L} = \rho^2, \quad \phi_0 = \left( \frac{w_F}{\bar{w}} \right) \frac{1 - \rho}{\sigma} - 1 = \delta_F M_{F, F, L} - 1 = \frac{\eta \rho}{r^*} - 1.
\]

Combine it with equation (18),

\[
\phi_0 < 0 \iff \eta \rho < r^* = \rho(1 - \eta + \eta \mu^*_A), \quad \Rightarrow \quad \mu^*_A > 2 - \frac{1}{\eta}.
\]

Given \( \mu^*_A \in (0, 1) \), trade unambiguously leads to the reversal of financial flows in period 0 if \( \eta \in (0, 0.5) \). For \( \eta \in (0.5, 1) \), it is the case if \( \lambda^* \) is so high that \( \mu^*_A > 2 - \frac{1}{\eta} \) holds. If \( \eta \in (0.5, 1) \) and \( \mu^*_A < 2 - \frac{1}{\eta} \), country N may still receives financial inflows in period 0.

**Step 3: Solve for the Boundary Conditions of Region IR, GR, and RR**

For (\( \lambda, \lambda^* \)) in region IR of the middle panel of figure 5, dual integration allows country N to converge from the steady state of financial integration immediately to the new steady state in period 1. Technically, it is the case if \( w_F \leq w_D \), as shown in the left panel of figure 6. Combine \( w_F = w_D \) with equations (58), (64), and (71) to get the threshold value \( \hat{\lambda}_{IR} \) as the function of \( \lambda^* \),

\[
\ln(1 - \hat{\lambda}_{IR}) = \ln(1 - \lambda^*) + (\theta - \hat{\theta}) \ln \eta
\]

It specifies the border between region IR and GR in the middle and the right panels of figure 5.

• Given \( \theta \in (\hat{\theta}, \alpha) \), \( \frac{\partial \ln w_{t+1}}{\partial \ln w_t} > 1 \) so that, if there exists a steady state U with \( w_U \in (w_D, \bar{w}_D) \), it must be unstable. As long as \( w_F \geq w_U \), country N eventually converges from the initial to the new steady state where the borrowing constraints are slack. Combine \( w_{t+1} = w_t = w_U = w_F \) with equation (22), (58), (64) to get the threshold value \( \hat{\lambda}_{RR} \) as the function of \( \lambda^* \),

\[
\ln(1 - \hat{\lambda}_{RR}) = \ln(1 - \lambda^*) + \frac{\theta - \hat{\theta}}{1 - \frac{\eta}{\alpha}} \ln \eta
\]

It specifies the border between region GR and RR in the middle panel of figure 5.

• Given \( \theta \in (\alpha, 1) \), \( \frac{\partial \ln w_{t+1}}{\partial \ln w_t} < 1 \) so that, if there exists a steady state FT with \( w_D \in (w_D, \bar{w}_D) \), it is stable and unique. Combine \( w_{t+1} = w_t = w_D \) with equation (22), (58), (64) to get the threshold value \( \hat{\lambda}_{RR} \) as the function of \( \lambda^* \) such that, for \( \lambda = \hat{\lambda}_{RR}, w_D = w \) and \( \phi_D = 0 \),

\[
\ln(1 - \hat{\lambda}_{RR}) = \ln(1 - \lambda^*) + \frac{\theta - \hat{\theta}}{1 - \frac{\eta}{\alpha}} \ln \eta
\]

It specifies the border between region GR and RR in the right panel of figure 5.
Given $\theta \in (\alpha, 1)$, figures 9-10 and 23 show the laws of motion for wage as well as the impulse responses of the wage rate and the normalized financial flow, when $(\lambda^*, \lambda)$ are in region GR, RR, and IR of the right panel of figure 5, respectively.

For simplicity, we assume that only the agents investing in sector 1 are subject to the borrowing constraints, while those investing in sector 0 are not. Under dual integration, $w_{t+1} = w^*$ for $w_t < w_D$.

As discussed in footnote 30, one can assume alternatively that all agents are subject to the same borrowing constraints, while those investing in sector 0 are not. Under dual integration, $w^*_t < q_{0,t+1} = q_{1,t+1}^* < q_{1,t+1}$. The law of motion for wage is specified by equation (79).

$$\frac{Y_{0,t+1}}{Y_{1,t+1}} = 1 - \frac{\delta_t}{\delta_t} \left( \frac{\Phi_{0,t}}{\Phi_{1,t}} \right)^{\alpha - 1} = 1 - \frac{\delta_t}{\delta_t} (\mu^*)^{1-\alpha}, \quad w_{t+1} = \frac{w^* (\mu^*)^{1-\alpha}}{\delta_t} + 1 = \frac{1}{\delta_t} \left( \frac{w^*}{\rho} \right) + 1 = 1 \left( \frac{w^*}{\rho} \right) + 1 \left( \frac{w^*}{\rho} \right)$$

Proof of Proposition 4

Proof. Given the Cobb-Douglas production function at the aggregate level,

$$Y_{t+1} = \Pi_{s=0}^{2} \left( \frac{V_{s,t+1}}{\eta_s} \right)^{\eta_s}, \quad MRS_{x,s} = \frac{\eta_{x}}{\eta_s} \frac{V_{s,t+1}}{V_{x,t+1}}, \quad \text{where} \ x, s \in \{0, 1, 2\}, \ s \neq x.$$
Thus, the agents with labor input in sector $s$ kinks at $w_{s,t+1} < \eta_s$. Given the Cobb-Douglas production function at the sectoral level, the output markets clear domestically, $V_s = Y_{s,t}$. Combine it with $MRT_{x,s} = MRS_{x,s}$ to get the share of labor input in sector $s$ at $\zeta_{s,t+1} = \eta_s$. Given the Cobb-Douglas production function at the sectoral level, the autarkic steady state is unique. In the following, we focus on the case of $\theta > \theta_3$.

Under autarky, $V_s = Y_{s,t}$. Combine it with $MRT_{x,s} = MRS_{x,s}$ to get the share of labor input in sector $s$ at $\zeta_{s,t+1} = \eta_s$. Given the Cobb-Douglas production function at the sectoral level,

$$
q_{s,t+1} \delta_{s,t} w_{s,t+1} L = p_{s,t+1} Y_{s,t+1} = \frac{w_{t+1} \zeta_{s,t+1} L}{1 - \alpha}, \quad \frac{q_{s,t+1} \delta_{s,t}}{\zeta_{s,t+1}} = \frac{\rho w_{t+1}}{w_t} = \frac{q_{x,t+1} \delta_{x,t}}{\zeta_{x,t+1}}.
$$

(82)

Thus, a sector’s rate of return is inversely related to its capital-labor ratio.

**First, solve for the sectoral investment shares $\delta_{s,t}$ in three cases.**

**Case 1:** if the borrowing constraints are binding in sector 1 and 2, the sectoral investment shares are specified by equations (24)-(26). Given $\zeta_{s,t+1} = \eta_s$, assumption 2 ensures $\frac{\delta_{1,t}}{\eta_1} > \frac{\delta_{2,t}}{\eta_2}$. Combine it with equation (82) to get $q_{1,t+1} < q_{2,t+1}$. Thus, the agents with $\epsilon_j \geq \epsilon_2$ borrow to the limit and invest in sector 2. Besides, $w_t < \bar{w}_{1,A}$ ensures $\frac{\delta_{1,t}}{\eta_1} > \frac{\delta_{2,t}}{\eta_2}$. Combine it with equation (82) to get $q_{1,t+1} < q_{2,t+1}$. Thus, the agents with $\epsilon_j \in [\epsilon_1, \epsilon_2)$ borrow to the limit and invest in sector 1. Thus, assumption 2 and $w_t < \bar{w}_{1,A}$ are the necessary and sufficient condition for this case to arise. **Case 2:** use the same logic to prove that $w_t > \bar{w}_{2,A}$ ensure the efficient sectoral investment shares, i.e., $\delta_{s,t} = \eta_s$.

**Second, derive the conditions for the unique, autarkic steady state.**

Under autarky, $V_s = Y_{s,t}$. Combine it with $\zeta_{s,t+1} = \eta_s$ and equations (24)-(26) and (81) to get,

$$
w_{t+1} = \left(\frac{w_t}{\rho} \Gamma_t \right)^\alpha, \quad \text{where} \quad \Gamma_t = \begin{cases} \Pi_{s=0}^2 \left( \frac{\delta_s}{\theta_s} \right)^{\eta_s} \left( 1 - \frac{\delta_s}{\eta_s + \eta_{s,t}} \right)^{\eta_{s,t}} \left( \frac{\delta_{s,t}}{\eta_{s,t}} \right)^{\eta_{s,t}}, & \text{if } w_t < \bar{w}_1; \\ 1, & \text{if } w_t > \bar{w}_2. \end{cases}
$$

(83)

$$
\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \alpha + \frac{\partial \ln \Gamma_t}{\partial \ln w_t} \quad \text{and} \quad \frac{\partial \ln \Gamma_t}{\partial \ln w_t} = \begin{cases} \frac{1 - \theta}{\theta} \left( 1 - \frac{\eta_{s,t}}{\eta_{s,t}} \right), & \text{if } w_t < \bar{w}_1; \\ \frac{1 - \theta}{\theta} \left( 1 - \frac{\eta_{s,t}}{1 - \delta_{s,t}} \right), & \text{if } w_t \in [\bar{w}_1, \bar{w}_2); \\ 1, & \text{if } w_t > \bar{w}_2. \end{cases}
$$

Given $\theta > \theta_3$, $\frac{\partial \ln w_{t+1}}{\partial \ln w_t} < 1$ holds strictly. As the slope of the law of motion for wage at any steady state is less than unity, the autarkic steady state is unique. In the following, we focus on the case of $\theta > \theta_3$.

**Third, derive the threshold value $\lambda_3$ such that for $\lambda < \lambda_3$, the borrowing constraints are binding in sector 1 and 2 at the autarkic steady state.**

As the law of motion for wage is a piecewise function with two kinks at $w_t = \bar{w}_{1,A}$ and $w_t = \bar{w}_{2,A}$, one can solve $\lambda_3$ by keeping the first kink point as a steady state,
i.e., \( w_A = \bar{w}_{1,A} \) and \( \frac{\delta_{0,A}}{\rho} = \frac{\delta_{1,A}}{\rho} = 1 - \frac{\delta_{2,A}}{\rho} \). Let \( D_1 = (1 - \lambda_3) \frac{1}{1 - \rho} \). Thus, \( \bar{w}_2 = m_2 D_1 \) and \( \bar{w}_1 = \gamma \bar{w}_2 \). Combine them with \( w_A = \bar{w}_{1,A} \), the definition of \( \bar{w}_{1,A} \), and equations (24)-(26) and (83),

\[
\delta_{2,A} = \left( \frac{w_A}{\bar{w}_2} \right)^{\frac{1}{\rho}} = \left( \frac{\bar{w}_{1,A}}{\bar{D}_1} \right)^{\frac{1}{\rho}} m_2^{\frac{1}{\rho}}
\]

\[
\delta_{0,A} + \delta_{1,A} = 1 - \delta_{2,A} = \left( \frac{\bar{w}_{1,A}}{\bar{D}_1} \right)^{\frac{1}{\rho}} m_2^{\frac{1}{\rho}} \left( \frac{\eta_0 + \eta_1}{\eta_0} \right) \left( \frac{1 - \eta_0}{\eta_1} \right)
\]

\[
\rho w_A^{\frac{1}{\rho}} = \sum_{A} \left( \frac{\delta_{0,A} + \delta_{1,A}}{\eta_0 + \eta_1} \right) \left( \frac{\delta_{2,A}}{\eta_2} \right)
\]

\[
\bar{w}_{1,A} \frac{1}{\bar{D}_1} = \frac{\bar{w}_{1,A} \gamma m_2}{\bar{w}_1} = \frac{\gamma m_2}{1 + \frac{m_2}{\eta_1} (1 - \gamma)} \left( 1 - \frac{1}{\eta_2} \right)
\]

\[
\bar{D}_1^{\frac{1}{\rho}} = \left( \frac{\bar{w}_{1,A}}{\bar{D}_1} \right)^{1 - \frac{1}{\rho}} = \frac{1}{\rho} m_2^{\frac{1}{\rho}} \left( \frac{1 - \eta_0}{\eta_1} \right) \left( 1 - \eta_2 \right) \left( 1 - \frac{1}{\eta_2} \right)
\]

\[
(1 - \lambda_3)^{\frac{1}{1 - \rho}} = \frac{Z_3}{F}, \quad \tilde{\lambda}_3 = 1 - \left( \frac{Z_3}{F} \right)^{1 - \rho}
\]

where \( Z_3 \) and \( F < 1 \) are defined in proposition 4.

Figure 25: Threshold Values for the Autarkic Steady State in the 3-Sector Setting

In figure 25, the left panel shows \( \hat{\theta}_3 \) in the \( \{\alpha, \theta\} \) space, while the right panel shows \( \bar{\lambda}_{A,3} \) in the \( \{\lambda, Z_3\} \) space. For \( \{\alpha, \theta\} \) in region U of the left panel and \( \{\lambda, Z_3\} \) in region UB12 of the right panel, there is a unique, autarkic steady state with the binding borrowing constraints in sector 1 and 2.

Under autarky, for \( w_t < \bar{w}_{1,A} \), the borrowing constraints are binding in sector 1 and 2. According to equations (24)-(25) and (82), the normalized rate of return in the sector with the binding borrowing constraints is constant, independent of the income level and the degree of financial development.

\[
\mu_{1,t+1} = \frac{q_{1,t+1}}{q_{2,t+1}} = \frac{\delta_{2,t}}{\delta_{1,t}} \zeta_{1,t} \zeta_{2,t} = \mu_1 \equiv \frac{1}{\gamma \eta_1 - 1 \eta_2}
\]

The normalized rate of return in the sector with the slack borrowing constraints increases in the income level and the degree of financial development.

\[
\mu_{0,t+1} = \frac{q_{0,t+1}}{q_{2,t+1}} = \frac{\delta_{2,t}}{\delta_{0,t}} \zeta_{0,t} \zeta_{2,t} = \frac{\gamma \eta_0}{\eta_2} - \frac{\eta_0}{\eta_2} < \mu_1, \text{ given } w_t < \bar{w}_{1,A}
\]
Proof of Lemma 3

Proof. According to proposition 4, there is a unique steady state where \( w_A < \bar{w}_{1,A} \) and the borrowing constraints are binding in sector 1 and 2. Combine \( r_t = q_{0,t+1} \) with equations (82) and (86) to get the steady-state interest rate,

\[
r_A = \frac{\rho}{\delta_{0, A}} = \rho \eta_0 \left( 1 + \frac{1}{H} \frac{1}{\gamma} \right), \quad \frac{\partial r_A}{\partial \lambda} < 0, \quad \Rightarrow \quad \frac{\partial r_A}{\partial \lambda} = \frac{\partial r_A}{\partial H} \frac{\partial H}{\partial \lambda} > 0, \quad \Rightarrow \quad r^*_A < r_A.
\]

Let \( H = \frac{\delta_{0, A}}{\delta_{2, A}} = \theta^{-1} \) \(-\gamma^{-1} \frac{\rho}{\sigma} \). At the autarkic steady state, combine \( w_{t+1} = w_t = \bar{w}_{A} \) with equations (24)-(26) and (83),

\[
\rho \frac{1}{2} \bar{w}^A_2 = \Gamma_A, \quad \rho \theta^{-1} \frac{1}{2} \bar{w}^A_2 = \delta_{2, A} \left( \frac{H}{\eta_0} \right)^{\eta_0} \left( \left( \gamma^{-1} \frac{\rho}{\sigma} - 1 \right) \right)^{\eta_1} \left( \eta_2 \right)^{\eta_2},
\]

\[
\bar{w}^A_2 \left[ H + \gamma^{-1} \frac{\rho}{\sigma} \right] \rho \theta^{-1} \left( \eta_0 \left( \eta_1 \right)^{\eta_1} \left( \eta_2 \right)^{\eta_2} \right),
\]

\[
\frac{1}{\rho} = \left\{ \eta_0 + \left[ \frac{1}{\rho} \left( \frac{1}{\theta} - 1 \right) \right] \delta_{0, A} > \eta_0 \left( 1 - \delta_{0, A} \right), \quad \Rightarrow \quad \frac{\partial \ln H}{\partial \ln \bar{w}} > 0, \quad \Rightarrow \quad \frac{\partial \ln H}{\partial \ln \lambda} \frac{\partial \ln \bar{w}}{\partial \ln \lambda} < 0. \quad (86)
\]

At the autarkic steady state, combine \( \zeta_{s,t+1} = \eta_s \) with equations (24)-(26) and (82),

\[
\mu_{0,A} = \frac{\delta_{0, A}}{\delta_{2, A}} = \frac{\delta_{0, A}}{\delta_{2, A}} \frac{\eta_0}{\eta_2}, \quad \frac{\partial \ln \mu_{0,A}}{\partial \ln \lambda} = \frac{\partial \ln \mu_{0,A}}{\partial \ln \lambda} > 0,
\]

\[
\mu_{1,A} = \frac{\delta_{1, A}}{\delta_{2, A}} = \frac{\delta_{1, A}}{\delta_{2, A}} \frac{\eta_1}{\eta_2}, \quad \frac{\partial \ln \mu_{1,A}}{\partial \ln \lambda} = 0.
\]

Thus, given \( \lambda^* < \lambda < \bar{\lambda}_3, \mu_{0,A} < \mu_{0,A} \) and \( \mu_{1,A} = \mu_{1, A} \) hold. According to equation (27), the ascending sectoral rate of return in each country implies,

\[
\mu_{0,A} < \mu_{0,A} < \mu_{1,A} = \mu_{1,A} < \mu_{2,A} = \mu_{2, A} = 1. \quad (87)
\]

For \( s \in \{0, 1, 2\} \), use equation (82) to get

\[
q_{s,t+1} = \alpha p_{s,t+1} \left( \frac{\delta_{s,t}}{\zeta_{s,t+1}} \right)^{\alpha-1} \chi_{s,t+1}, \quad \mu_{s,t+1} = \frac{q_{s,t+1}}{q_{2,t+1}} = \frac{\delta_{s,t}}{\zeta_{s,t+1}}, \quad \chi_{s,t+1} = \mu_{s,t+1} \chi_{s,t+1}, \quad \Rightarrow \quad \chi_{s,t+1} = \mu_{s,t+1} \chi_{s,t+1}, \quad (88)
\]

\[
\mu_{s,t+1} = \chi_{s,t+1} \left( \frac{\delta_{s,t}}{\zeta_{s,t+1}} \right)^{\alpha-1} = \chi_{s,t+1} \mu_{s,t+1}, \quad \Rightarrow \quad \chi_{s,t+1} = \mu_{s,t+1}^{\alpha-1}, \quad \mu_{s,t+1} = \mu_{s,t+1}^{\alpha-1}, \quad (89)
\]

which holds under autarky as well as under free trade. Combine it with inequality (87) to get the cross-country patterns of the relative sectoral prices,

\[
\chi_{0,A} < \chi_{0,A} < \chi_{1,A} < \chi_{2,A} = \chi_{2, A} = 1.
\]

\[
\square
\]
Proof of Proposition 5

Proof. The rest of the world is in the autarkic steady state with \( \zeta_s^* = \eta_s \) and \( \mu^*_0 < \mu^*_1 = \mu^*_1 \). For simplicity, we drop subscript A for the variables in the rest of the world. The Cobb-Douglas production functions at the sectoral and at the aggregate level imply

\[
\chi_{s,t} = \mu^*_s, t, \quad \Pi_s^2 = p_s^\gamma, t = 1, \quad p^\gamma, t = \chi_{s,0}^\gamma, t_1, t = \mu^*_{0, t} \mu^*_{1, t} \quad (90)
\]

\[
w_{t+1} = (\Pi_s = 0 q_{s, t+1}) - \rho = \left( \frac{u_{s, t+1}}{q_s, t+1, t+1} \right)^{\rho}. \quad (91)
\]

The proof consists of three steps.

Step 1: Derive the Law of Motion for Wage under Dual Integration

Dual integration aligns the sectoral prices and the interest rate to the world levels. The law of motion for wage is flat at \( w_{t+1} = w^* = \left( \frac{u_0}{r - \Pi_0 (\mu^*_0)^\gamma} \right)^{\rho} \). The borrowing constraints are binding in sector 1 and 2, \( q_{0, t+1} = r^* < q_{1, t+1} < q_{2, t+1} \).

For \( w_t < w_{D1} \), all sectors are active and the FPE holds, i.e., \( q_{0, t+1} = q_0^* = r^* \) and \( q_{s, t+1} = q_s^* \). The law of motion for wage is flat at \( w_{t+1} = w^* = \left( \frac{u_0}{r - \Pi_0 (\mu^*_0)^\gamma} \right)^{\rho} \). The borrowing constraints are binding in sector 1 and 2, \( q_{0, t+1} = r^* < q_{1, t+1} < q_{2, t+1} \).

For \( w_t \in (w_{D1}, w_{D2}) \), country N specializes fully in sector 1 and 2, while the borrowing constraints are binding in both sectors, \( q_{0, t+1} < r^* < q_{1, t+1} < q_{2, t+1} \). According to equations (24)-(25) and (82),

\[
\frac{\delta_2, t M_t}{L} = w_t^\alpha \frac{1}{\theta} - \rho, \quad \frac{\delta_1, t M_t}{L} = \frac{\delta_2, t M_t}{L} \left( (1 - \frac{1}{\alpha}) \right) - 1, \quad \mu^*_1 \frac{\delta_t}{\delta_2} = \mu^*_1 (1 - \frac{1}{\gamma}) - 1 = \frac{\eta_2}{\eta_2}, \quad (92)
\]

\[
\zeta_{1, t+1} = \frac{\mu^*_1 \zeta_2, t+1, t}{\delta_2, t}, \quad \zeta_{2, t+1} = \frac{\zeta_{1, t+1} + \zeta_2, t+1, t}{\mu^*_1 \delta_2, t} + 1 = \frac{1}{\eta_2} = \frac{\eta_1 + \eta_2}{\eta_1 + \eta_2}. \quad (93)
\]

Solve for the law of motion for wage in this case,

\[
p_2^* = (\mu^*_0)^{-\gamma}, (\mu^*_1)^{-\gamma}, \quad Y_{1, t+1} = \left( \frac{\delta_1, t + 1}{\delta_2, t} \right)^\alpha \left( \frac{\zeta_{1, t+1} + 1}{\zeta_{2, t+1} + 1} \right)^{\alpha} = (1 - \frac{\eta_2}{\eta_2} + 1) \left( \frac{\eta_1 + \eta_2}{\eta_1 + \eta_2} \right) \quad (94)
\]

\[
Y_{t+1} = \frac{w_t + L}{1 - \alpha} = p_2, t+1 Y_{t+1} \left[ \chi_{1, t+1}^\alpha Y_{2, t+1} + 1 \right] = p_2^2 Y_{t+1} \left[ \frac{\eta_1 + \eta_2}{\eta_1 + \eta_2} \right] \quad (94)
\]

\[
w_{t+1} = p_2^* \left( \frac{\delta_2, t M_t}{\zeta_{2, t+1} + 1} \right)^\alpha \left( \frac{\mu^*_0}{\mu^*_1} \right)^{-\gamma} \left( \frac{\eta_2}{\eta_1 + \eta_2} \right)^{\frac{\gamma}{\eta_2}} \right)^{\frac{\gamma}{\eta_2}}, \quad \partial \ln w_{t+1} = \frac{\alpha}{\theta} \quad (94)
\]

\[
\ln q_{1, t+1} - \ln r^* = -\alpha \eta_1 \ln \mu^*_1 - (\alpha \eta_1 - 1) \ln \mu^*_1 - \frac{1 - \alpha}{\theta} \ln w_t - \ln r^* + (1 - \alpha) \ln \left( \frac{\eta_2}{\eta_2} \right)^{\frac{1}{\alpha}} \quad (94)
\]

\[
\ln q_{0, t+1} - \ln r^* = -\alpha \eta_0 \ln \mu^*_0 - (\alpha \eta_1 - 1) \ln \mu^*_1 - \frac{1 - \alpha}{\theta} \ln w_t - \ln r^* + (1 - \alpha) \ln \left( \frac{\eta_2}{\eta_2} \right)^{\frac{1}{\alpha}} \quad (94)
\]

By setting \( q_{0, t+1} - \ln r^* = 0 \) and \( q_{1, t+1} - \ln r^* = 0 \), we get two threshold values

\[
w_{D1} = \left[ \frac{\mu^*_0}{\mu^*_1} \left( \frac{\eta_2}{\eta_1 + \eta_2} \right)^{\frac{1 - \alpha}{\gamma}} \right]^{\frac{1 - \alpha}{\gamma}}, \quad w_{D2} = w_{D1} \left( \frac{\mu^*_1}{\mu^*_0} \right)^{\frac{1 - \alpha}{\gamma}} \quad (95)
\]

50
• For \( w_t \in (\bar{w}_{D1}, \bar{w}_{D2}) \), country N specializes fully in sector 1 and 2, while the borrowing constraints are slack (binding) in sector 1 (2), \( q_{1,t+1} = r^* < q_{2,t+1} \). According to equation (91), the law of motion for wage is flat at \( w_{t+1} = w^* (\frac{\mu_0}{\mu_1})^{(1-\alpha)} > w^* \).

• For \( w_t \in (\bar{w}_{D2}, \bar{w}_{D2}) \), country N specializes fully in sector 2 with the binding borrowing constraints, \( q_{1,t+1} < r^* < q_{2,t+1} \). The law of motion for wage is

\[
w_{t+1} = p_{2,t+1} + \left( \frac{\delta_{2,t} M_t}{\rho L} \right)^{\alpha} w_t^\theta \left[ (\mu_{0,A}^{(\mu_{0,A}^{\eta}))} \right]^{(1-\theta)} \eta \ln \left( \frac{\mu_{0,A}^{\eta} \mu_{1,A}^{1-\eta}}{\rho \bar{w}_{2}^{\eta}} \right)^{\frac{\theta}{1-\theta}} \]

(96)

\[
\ln q_{2,t+1} - \ln r^* = -\alpha \eta_0 \ln \mu_{0,A}^* - \alpha \eta_1 \ln \mu_{1,A}^* - \frac{1-\alpha}{\theta} \ln w_t - \ln r^* + (1-\alpha) \ln \rho \bar{w}_{2}^{\eta}.
\]

\[
\ln q_{1,t+1} - \ln r^* = -\alpha \eta_0 \ln \mu_{0,A}^* - (\alpha \eta_1 - 1) \ln \mu_{1,A}^* - \frac{1-\alpha}{\theta} \ln w_t - \ln r^* + (1-\alpha) \ln \rho \bar{w}_{2}^{\eta}.
\]

By setting \( q_{1,t+1} - r^* = 0 \) and \( q_{2,t+1} - r^* = 0 \), we get the other two threshold values

\[
\bar{w}_{D2} = \bar{w}_{D1} \left( \frac{\eta_1 + \eta_2}{\eta_2} \right)^{1-\theta} > \bar{w}_{D1}, \quad \bar{w}_{D2} = \bar{w}_{D2} \left( \frac{\mu_1^*}{\mu_0^*} \right)^{\frac{\theta}{1-\theta}} > \bar{w}_{D2}.
\]

(97)

• For \( w_t > \bar{w}_{D2} \), country N specializes fully in sector 2 where the borrowing constraints are slack, \( q_{2,t+1} = r^* \). According to equation (91), the law of motion for wage is flat at \( w_{t+1} = w^* (\frac{\mu_0}{\mu_1})^{(1-\alpha)} \).

To sum up, under dual integration, if the borrowing constraints are slack in one active sector, the law of motion for wage is flat; if the borrowing constraints are binding in all active sectors, the law of motion for wage (in logarithm) has a slope of \( \frac{\partial \ln w_t}{\partial \ln w_t} = \frac{\alpha}{\theta} \). For \( \theta \in (\alpha, 1) \), there exists a unique steady state; for \( \theta \in (\hat{\theta}, \alpha) \), multiple steady states may arise.

Given \( \theta \in (\hat{\theta}, \alpha) \), if dual integration induces country N to fully offshore sector 0 and aggregate income rises in period 1, the reversal and the re-reversal of financial flows occur. As shown in figure 12, if the kink point at \( w_t = \bar{w}_{D2} \) is below the 45° line, country N converges to the steady state M where sector 0 vanishes and the borrowing constraints are slack (binding) in sector 1 (2). In this case, the re-reversal of financial flows occurs once. As shown in figure 13, if the kink point at \( w_t = \bar{w}_{D2} \) is above the 45° line, country N sequentially offshores sector 0 and 1 before converging to the steady state H with full specialization in sector 2 and the slack borrowing constraints. The border between region RR1 and RR2 in figure 12 specifies the combinations of \( (\lambda^*, \lambda) \) that ensure

\[
\bar{w}_{D2} = w^* (\frac{\mu_1^*}{\mu_0^*})^{\frac{\theta}{1-\theta}}
\]

(98)

Proof of Lemma 4

**Proof.** For \( w_t < \bar{w} \), the maximum possible share of investment in sector 1 is less than unity. In this case, sector 0 is active \( K_{0,t+1} > 0 \) and the interest rate coupling gives \( r_t = q_{0,t+1} \). Thus, the borrowing constraints are binding in country N, \( \mu_{t+1} = \frac{q_{0,t+1}}{q_{1,t+1}} = \frac{r_t}{q_{1,t+1}} = \mu_{1,A}^* < 1 \) and \( \delta_t = \tilde{\delta}_t < 1 \) and \( \zeta_{t+1} = \frac{1}{(\delta_t - 1)^{\mu_{1,A}^*}} \in (\delta_t, 1) \).

For \( w_t \geq \bar{w} \), country N offshores sector 0, \( K_{0,t+1} = 0 \), implying the interest rate decoupling from the rate of return in sector 0. The credit market competition leads to the interest rate coupling with the rate of return in sector 1. Combine them with equations (1) and (3)-(4) to get (28).

Proof of Proposition 6
Case M: under free trade, multiple steady states arise if four conditions are satisfied simultaneously, i.e., 

\[ \{ \theta, Z, \lambda^*, \lambda \} \] space that characterize the four cases. Then, we show the law of motion for wage in each case. 

**Proof.** For \( \theta \in (\hat{\theta}, 1) \), there exists a unique, autarkic steady state. Under trade integration, the static gains raise aggregate income in country N so that the law of motion for wage lies strictly above the one under autarky. For \( w_t > \bar{w} \), the law of motion for wage in logarithm has a slope of \( \alpha < 1 \). Thus, there exists at least one stable steady state. In the following, we first summarize four possible cases under free trade before deriving the relevant threshold conditions.

**Step 1: the summary of four possible cases under trade integration.**

Given \( \theta \in (\hat{\theta}, 1) \) and \( \lambda^*, \lambda \in (0, \bar{\lambda}_A) \), there are four cases under free trade, \( \chi^* = \chi^*_A \).

- **Case M**: free trade leads to multiple steady states in country N, i.e., two stable steady states denoted by H and L, and one unstable steady state denoted by M. The steady states are ranked in terms of income level, \( w_H > \bar{w} > w_M > w_L > w_A \).

- **Case UF1**: country N converges to a unique steady state where it fully specializes in sector 1;

- **Case UP1**: country N converges to a unique steady state where it partially specializes in sector 1;

- **Case UP0**: country N converges to a unique steady state where it partially specializes in sector 0;

Define some threshold values:

\[
\hat{Z} \equiv (1 - g)^{\frac{\theta}{1 - \rho(1 - \theta)}} \quad \text{and} \quad \hat{\lambda}^*_T \equiv 1 - \left( \frac{Z}{Z} \right)^{1 - \theta}, \text{ where } g \equiv \frac{\theta}{\rho(1 - \theta)} \quad (99) 
\]

\[
\tilde{Z} \equiv \eta^{\theta(1 - \theta)/(1 - \sigma)} \left( 1 - \eta + \frac{\theta - 1}{\eta - 1} \right)^{-\theta} \quad \text{and} \quad \tilde{\lambda}^*_T \equiv 1 - \left( \frac{Z}{Z} \right)^{1 - \theta} \quad (100) 
\]

In the following, we first show the threshold conditions in the \( \{ \theta, Z, \lambda^*, \lambda \} \) space that characterize the four cases. Then, we show the law of motion for wage in each case.

**Case M**: under free trade, multiple steady states arise if four conditions are satisfied simultaneously,

- \( \theta \in (\hat{\theta}, 1) \), i.e., \( \{ \alpha, \theta \} \) in region M of the upper-left panel of figure 26, and

- \( Z < \hat{Z} \), i.e., \( \{ \theta, Z \} \) in region M of the upper-right panel of figure 26, and

- \( \lambda^* < \hat{\lambda}_T \) and \( \lambda \in (\hat{\lambda}_T, \hat{\lambda}_T) \), i.e., \( \{ \lambda^*, \lambda \} \) in region M of the lower-left panel of figure 26.

**Case UF1**: section A focuses on case UF1 which arises if four conditions are satisfied jointly, i.e.,

- given \( \theta \in (\hat{\theta}, 1) \) and \( Z < \hat{Z} \), \( \{ \theta, Z \} \) are in region M of the upper-right panel of figure 26 and \( \{ \lambda^*, \lambda \} \) are in region UF1 of the lower-left panel of figure 26.

- given \( \theta \in (\hat{\theta}, 1) \) and \( Z \in (\hat{Z}, \bar{Z}) \), \( \{ \theta, Z \} \) are in region UF of the upper-right panel of figure 26 and \( \{ \lambda^*, \lambda \} \) are in region UF1 of the lower-middle panel of figure 26.

**Case UP1 and Case UP0**: given \( \theta \in (\hat{\theta}, 1) \), free trade induces country N to move from the autarkic steady state to a unique steady state where it specializes partially in sector 1 (0) if parameters are in region region UP1 (UP0) of the lower panels of figure 26 which correspond to \( \{ \theta, Z \} \) in the three regions of the upper-right panel of figure 26.

In figure 27, the solid (dashed) curve in each panel shows the law of motion for wage under free trade (autarky) in the each case, respectively. Next, let us derive the relevant threshold conditions.

**Step 2: derive the condition under which trade leads to multiple steady states in country N.**

---

\(^{52}\)The proof of proposition 6 provides the technical derivation for these threshold values.
Figure 26: Threshold Values under Free Trade

If there exists an unstable steady state \( M \), \( w_M \in (0, \bar{w}) \) should hold and

\[
\frac{\partial w_{t+1}}{\partial w_t} \bigg|_{w_M} = \alpha + \alpha \frac{1 - \theta}{\mu^* \delta_M + 1} > 1 \Rightarrow \frac{\alpha}{1 - \alpha} - \frac{\theta}{1 - \theta} > \frac{\theta}{1 - \theta (1 - \mu^*) \delta_M}. \tag{101}
\]

For \( \theta \geq \alpha \), condition (101) does not hold and the law of motion for wage in logarithm has the slope less than unity. Thus, there exists a unique, stable steady state. Next, we focus on the case of \( \theta \in (\hat{\theta}, \alpha) \).

Combine condition (101) with \( \delta_M = \left( \frac{w_M}{\bar{w}} \right)^{\frac{\theta}{1 - \theta}} < 1 \) to get an upper bound for \( \mu^* \).

\[
\mu^* \frac{\frac{\theta}{1 - \mu^*}}{1 - \frac{\theta}{1 - \mu^*}} < \delta_M < 1, \quad \Rightarrow \quad \mu^* \leq \hat{\mu}^* \equiv 1 - g, \quad \text{where} \quad g \equiv \frac{1}{\theta} - 1 \in (\eta, 1). \tag{102}
\]

According to equations (54), \( \mu_A \) is an increasing function of \( \lambda \). As the rest of the world has the same economic structure as country \( N \), equations (54) also specify \( \mu^*_A \) as an increasing function of \( \lambda^* \in [0, \hat{\lambda}_A] \).

Combine (102) and (54) to get the upper bound for \( \lambda^* \).

\[
\lambda^* \leq \hat{\lambda}^*_T \equiv 1 - \left( \frac{Z}{\hat{Z}} \right)^{1 - \theta}, \quad \text{where} \quad \hat{Z} \equiv (1 - g) \frac{\theta}{1 - \theta} \rho \eta^{1 - \theta} < 1. \tag{103}
\]

\( Z \leq \hat{Z} \) ensures \( \hat{\lambda}^*_T \geq 0 \) and the existence of multiple steady states. \( \hat{Z} \) is a function of \( \theta \), as shown by the curve between region \( M \) and UF in the upper-right panel of figure 26.

Given \( \{ \theta, Z \} \) in region \( M \) of the upper-right panel of figure 26, derive the condition in the \( \{ \lambda^*, \lambda \} \) space under which the law of motion for wage is tangent with the 45° line at a steady state \( M \),

\[
\frac{\partial w_{t+1}}{\partial w_t} \bigg|_{w_M} = \alpha + \alpha \frac{1 - \theta}{\mu^* \delta_M + 1} = 1, \quad \Rightarrow \quad \delta_M = \frac{g}{1 - g (1 - \mu^*)}. \tag{104}
\]

\[\]
Combine equations (28) with (104) and get a threshold value of $\hat{\lambda}_T$ as a function of $\mu^*$,

$$w_M = \left[\frac{(\mu^*)^\eta}{\rho} \left(1 + \frac{1 - \mu^*}{\mu^*} \delta_M \right)\right]^\rho = \bar{w}_M = \left(1 - \hat{\lambda}_T\right)^{1-\theta} \frac{m}{1 - \theta} \left[\frac{\eta g}{1 - \eta (1 - \mu^*)}\right]^{\theta/(1-\sigma)}$$

$$\hat{\lambda}_T = 1 - \left\{\frac{Z}{\left(1 - \mu^*\right)\eta} \left(\mu^*\right)^{\sigma/(1-\sigma)} - \rho \eta (1 - g)\rho^{-\sigma}\right\}^{1-\theta}. \quad (105)$$

For $\lambda^* \in [0, \hat{\lambda}_A^*]$, use (54) to solve for $\mu^*_A$. Then, combine $\mu^* = \mu^*_A$ with (105) to solve for $\hat{\lambda}_T$. The curve between region M and UF1 in the lower-left panel of figure 26 shows $\hat{\lambda}_T$ as a function of $\lambda^* \in [0, \hat{\lambda}_T^*]$.

**Step 3: derive the condition under which country N fully offshores sector 0 before reaching the stable steady state T.**

Given $\mu^*$, use equation (28) to get a lower bound for $\lambda$,

$$w_T = \left[\frac{(\mu^*)^{\eta-1}}{\rho}\right]^\rho \geq \bar{w} = \frac{m(1 - \lambda)^{1-\sigma}}{1 - \theta}, \Rightarrow \lambda \geq \tilde{\lambda}_T = 1 - \left[\frac{Z\eta^{\sigma/(1-\sigma)}}{(\mu^*)^{\rho(1-\eta)}}\right]^{1-\theta}. \quad (106)$$

Given $\lambda \in [0, \tilde{\lambda}_A]$, combine equation (106) with $\tilde{\lambda}_A \equiv 1 - Z^{1-\theta}$ to get an upper bound for $\mu^*$,

$$\tilde{\lambda}_T \leq \tilde{\lambda}_A \Rightarrow \mu^* < \tilde{\mu}_T^* = \eta^{\rho(1-\eta)/(1-\sigma)} \cdot (107)$$

By the same logic as mentioned above, combine (107) and (54) to get an upper bound for $\lambda^*$,

$$\lambda^* < \tilde{\lambda}_T^* \equiv 1 - \left(\frac{Z}{Z}\right)^{1-\theta}, \text{ where } \tilde{Z} \equiv \eta^{\frac{(\sigma-\rho)}{\rho(1-\eta)}} (1 - \eta + \frac{\mu^*}{\eta^{\rho(1-\eta)}})^{\rho(1-\eta)}.$$
\( Z \leq \bar{Z} \) ensures \( \bar{\lambda}_T \geq 0 \). \( \bar{Z} \) is a function of \( \theta \) and shown by the curve between region UF and UP in the upper-right panel of figure 26.

Given \( \{\theta, Z\} \) in region UF and M in the upper-right panel of figure 26, for each \( \lambda^* \in [0, \bar{\lambda}_T] \), use equation (54) to solve for \( \mu_A^t \) and then combine \( \mu^* = \mu_A^* \) with (106) to solve for \( \bar{\lambda}_T \).

For \( \{\theta, Z\} \) in region M of the upper-right panel of figure 26, the curve denoted by \( \bar{\lambda}_T \) in the lower-left panel of figure 26 shows \( \lambda^* \) as a function of \( \lambda^* \in [0, \bar{\lambda}_T] \). For \( \{\theta, Z\} \) in region UF of the upper-right panel of figure 26, the curve between region UF1 and UP1 in the lower-middle panel of figure 26 shows \( \lambda^* \) as a function of \( \lambda^* \in [0, \bar{\lambda}_T] \).

**Proof of Lemma 5**

**Proof.** Following the approach shown in the proof of lemma 1 and taking into account equation (28), one can derive the interest rate as the piecewise function of aggregate income characterized by (32)-(33). □

**Proof of Lemma 6**

**Proof.** Under autarky, sectoral output is equal to sectoral absorption, \( V_{s,t+1} = Y_{s,t+1} \), while the sectoral fraction of labor input is efficient and equal to the sector share in the production of final goods, \( \zeta_{s,t} = \eta_s \), as shown in the proof of proposition 1. Combine these two conditions with equations (35) to get

\[
\frac{q_{s,t+1} \delta_{s,t}}{\zeta_{s,t+1}} = \frac{q_{v,t+1} \delta_{v,t}}{\zeta_{v,t+1}}, \quad \text{for } s \neq v \quad \text{and} \quad s, v \in \{0, 1, ..., S - 1\}. \tag{108}
\]

**Step 1: derive the condition for the ascending sectoral rate of return.**

Suppose that the borrowing constraints are binding in sector \( s \in \{z + 1, ..., S - 1\} \) and the sectoral rate of return is ascending, \( q_{s-1,t+1} < q_{s,t+1} \). Thus, agents always invest in the sector with the highest MIR they can afford and borrow to the limit. Let \( \zeta_{s,t} = \frac{1 - \lambda}{w_t} m_s \). The investment share of sector \( s \) is

\[
\delta_{s,t} = \delta_{s,t} = \frac{f_{s+1,t} \eta_s}{w_t L} = \frac{-\eta_s}{w_t} \frac{\mu_{s+1,t}}{1 - \lambda} = \frac{1 - \eta_s}{w_t} \kappa_s, \quad \text{where } \kappa_s = \frac{w_s - 1 - \eta_s}{w_s - 1 - \eta_s}. \tag{109}
\]

Combine \( \zeta_{s,t} = \eta_s \) with equation (108) to get the condition for the ascending sectoral rate of return,

\[
q_{s,t+1} < q_{s+1,t+1}, \quad \Leftrightarrow \quad \frac{\delta_{s+1,t+1}}{\delta_{s,t}} < \frac{\zeta_{s+1,t+1}}{\zeta_{s,t}}, \quad \Leftrightarrow \quad \frac{m_{s+1} - 1 - \eta_s}{w_t} m_{s+1} < \frac{\eta_s}{w_t}, \tag{110}
\]

which is guaranteed by assumption 4.53

**Step 2: derive the threshold values of \( \bar{w}_z \) and \( \bar{w}_{z_a} \).**

Suppose that the borrowing constraints are slack in sector \( s \in \{0, ..., z\} \). The sectoral rate of return equals, \( q_{s,t+1} = r_t = q_{z,t+1} \) and so does the sectoral capital-labor ratio, \( \delta_{s,t} = \delta_{s,t+1} = \sum_{v=0}^{S} \delta_{v,t} \), according to equation (108). Technically, one can pool these sectors into a new one and call it sector \( \tau \) (unconstrained). Let \( \kappa_{s,t} = \frac{w_t - 1 - \eta_s}{w_t - 1 - \eta_s} \). The investment and the labor shares of sector \( \tau \) are

\[
\delta_{s,t} = \sum_{v=0}^{z} \delta_{v,t} = 1 - \sum_{v=z+1}^{S-1} \delta_{v,t} = 1 - \left( \frac{w_t}{w_t + \eta_s} \right) \frac{1 - \eta_s}{w_t} = w_t \frac{1 - \eta_s}{w_t} \kappa_{s,t}, \quad \text{and} \quad \tag{111}
\]

\[
\zeta_{s,t+1} = \sum_{v=0}^{z} \zeta_{v,t+1} = \eta_s = \sum_{v=0}^{z} \eta_v. \tag{112}
\]

53If assumption 4 does not hold for two neighboring sectors, the rate of return equalizes between them and so does the capital-labor ratio, according to equation (108). In that case, one can pool the two sectors together and redefine a new sector so as to maintain the ascending rate-of-return pattern. In other words, the sectors in this setting are specified in terms of their rate-of-return pattern rather than their physical characteristics.
In the boundary case where the borrowing constraints are slack in sector $s \in \{0, ..., z-1\}$ and weakly binding in sector $z$, the rate of return equalizes across these sectors

$$q_{s,t+1} = q_{u,t+1} = q_{z,t+1} = r_t \iff \frac{\delta_{u,t}}{\delta_{z,t}} = \frac{\zeta_{u,t+1}}{\zeta_{z,t+1}}.$$  \hfill (113)

Combine $\zeta_{z,t} = \eta_z$ with (109) and (111)-(113) to get the threshold values $\bar{w}_z$ and $\bar{w}_{z,A}$ as specified in (36).

**Step 3: derive the law of motion for wage under autarky.**

Combine equations (34)-(35) with $\zeta_{s,t+1} = \eta_s$ to get

$$w_{t+1} = (1 - \alpha) \frac{Y_{t+1}}{L} = (1 - \alpha) \frac{\sigma_{t+1}}{\eta_v} \Pi_{v=0}^{S-1} \left( \frac{\delta_{v,t}}{\eta_v} \right)^{\eta_v}.$$  \hfill (114)

For sector $s \in \{0, ..., z\}$, the sectoral capital-labor ratio equalizes and hence,

$$\Pi_{v=0}^{z} \left( \frac{\delta_{v,t}}{\eta_v} \right)^{\eta_v} = \Pi_{v=0}^{z} \left( \frac{\delta_{u,t}}{\eta_u} \right)^{\eta_v}.$$  \hfill (115)

Combine (109), (111), (114)-(115) to get the law of motion for wage as show by equation (37).

**Proof of Proposition 8**

**Proof.** **Step 1: derive the sufficient condition for the unique steady state under autarky.**

For $w_t \in (\bar{w}_{z,A}, \bar{w}_{z+1,A})$, the borrowing constraints are *binding* in sector $b \in \{z+1, ..., S-1\}$ and *slack* in sector $u$.\footnote{See the proof of lemma 6 for the definition of sector $u$.}

\[
\frac{\partial \ln \delta_{b,t}}{\partial \ln w_t} = \frac{1 - \theta}{\theta} > 0, \quad \text{and} \quad \frac{\partial \ln \delta_{u,t}}{\partial \ln w_t} = \frac{1 - \theta}{\theta} - \frac{1 - \theta}{\theta} \frac{1}{\delta_{u,t}} < 0 \hfill (116)
\]

\[
\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \alpha + \alpha \left[ \eta_u \frac{\partial \ln \delta_{u,t}}{\partial \ln w_t} + \sum_{v=z+1}^{S-1} \left( \sigma \frac{\partial \ln \delta_{v,t}}{\partial \ln w_t} \right) \right] = \alpha + \alpha \frac{1 - \theta}{\theta} \left( 1 - \frac{\eta_u}{\delta_{u,t}} \right). \hfill (117)
\]

There exists a unique steady state if $\frac{\partial \ln w_{t+1}}{\partial \ln w_t} < 1$ or equivalently, $1 - \frac{\eta_u}{\delta_{u,t}} < \frac{1}{\theta - 1}$. As $1 - \frac{\eta_u}{\delta_{u,t}} < 1$, the sufficient condition for the unique steady state is $1 - \eta_0 < \frac{1}{\theta - 1}$.

**Step 2: derive the threshold values $\bar{\lambda}_z$.**

Let $\mathbb{D} \equiv \frac{(1-\lambda)\sigma}{1-\theta}$ and then $\bar{\omega}_s = \mathbb{D} m_s$. Let us derive the condition under which the steady-state wage rate is equal to a threshold value under autarky, $w_A = \bar{w}_{z,A}$. In that case, the borrowing constraints are slack in sector $s \in \{0, ..., z-1\}$, weakly binding in sector $s = z$, and strictly binding in sector.
Given across sectors, the total investment share of sector \( v \) prefer to invest in the sector with the highest MIR that they can afford. As the leverage ratio is identical \( \mu \), \( \tilde{\lambda} \) splitting the interval of \([0, 1]\) into \( S \) sub-intervals. Let \( \lambda_0 = 0 \) and \( \lambda_S = 1 \). For \( \lambda \in [\tilde{\lambda}_z, \tilde{\lambda}_{z+1}] \), the borrowing constraints are slack in sector \( s \in \{0, \ldots, z\} \) and binding in sector \( s \in \{z + 1, \ldots, S - 1\} \) in the autarkic steady state where \( w_A \in [\bar{w}_{z,A}, \bar{w}_{z+1,A}] \).

### Proof of Corollary 1

**Proof.** Step 1: derive the sectoral investment share under free trade

For \( z \in \{z^* + 1, \ldots, S - 1\} \), the relative rate of return in sector \( s \in \{z, \ldots, S - 1\} \) is ascending in the rest of the world \( \mu_{s,A} < \mu_{s+1,A} \) under autarky and so is that in country \( N \) under free trade. Agents in country \( N \) prefer to invest in the sector with the highest MIR that they can afford. As the leverage ratio is identical across sectors, the total investment share of sector \( v \in \{z + 1, \ldots, S - 1\} \) is the sum of the MIRs of the sectors. Given \( w_t \in (\bar{w}_z, \bar{w}_{z+1}) \times \sum_{v=z+1}^{S-1} \delta_{v,t} < 1 < \sum_{v=z+1}^{S-1} \delta_{v,t} \) implies that

- sector \( v \in \{z + 1, \ldots, S - 1\} \) is active, the borrowing constraints are binding, \( q_{v,t+1} > r_t \), and the sectoral investment share is specified by equation (42);
- sector \( z \) is active, the borrowing constraints are slack, \( q_{z,t+1} = r_t \), and the sectoral investment share is specified by equation (41);
- sector \( s \in \{0, \ldots, z - 1\} \) is inactive, \( \delta_{s,t} = 0 \).

Combine it with \( \mathbb{D} \equiv \frac{(1 - \lambda) \tau}{1 - \theta} \) to get the threshold value

\[
\lambda_z \equiv 1 - \frac{(1 - \theta)^{1 - \theta} \left( \frac{\Omega_z}{\rho} \right)^{\rho(1 - \theta)}}{\left[ m_z^{1 - \theta} + \frac{\eta_h}{\eta_z} \left( m_{z+1}^{1 - \theta} - m_z^{1 - \theta} \right) \right]^{\rho(1 - \theta) - \theta}}. \tag{118}
\]

For \( z \in \{1, \ldots, S - 1\} \), there are \( S - 1 \) threshold values \( \tilde{\lambda}_z \) splitting the interval of \([0, 1]\) into \( S \) sub-intervals.
Step 2: derive the sectoral capital-labor ratio and output price under free trade

For \( s \in \{z, \ldots, S - 1\} \), use the definition of the relative sectoral rate of return to get

\[
\mu_{s,t+1} = \frac{q_{s,t+1}}{q_{S-1,t+1}} = \frac{\delta_{s-1,t}}{\delta_{s,t}} \mu_{s,A}, \quad \Rightarrow \quad \zeta_{s,t+1} = \frac{\mu_{s,A} \delta_{s,t}}{\delta_{s-1,t}} = \sum_{v=z}^{S-1} \zeta_{v,t+1} = 1,
\]

\[
\frac{\delta_{s-1,t}}{\zeta_{s,t+1}} = \sum_{v=z}^{S-1} \mu_{v,A} \delta_{v,t}, \quad \zeta_{s,t+1} = \frac{\delta_{s,t}}{\delta_{s-1,t}} \mu_{s,A}, \quad \Rightarrow \quad \delta_{s,t} = \frac{\sum_{v=z}^{S-1} \mu_{v,A} \delta_{v,t}}{\mu_{s,A}}.
\]

Use the production function of final goods to calculate the sectoral output price,

\[
Y_{t+1} = \Pi_{v=0}^{S-1} \left( \frac{V_{v,t+1}}{\eta_v} \right)^{\eta_v}, \quad \frac{V_{v,t+1}}{\eta_v} = \frac{Y_{t+1}}{p_{v,t+1}}; \quad \Pi_{v=0}^{S-1} p_{v,t+1} = p_{S-1,A} \Pi_{v=0}^{S-1} (\chi_{v,A})^{\eta_v} = 1
\]

\[
\Rightarrow p_{S-1,A} = \frac{1}{\Pi_{v=0}^{S-1} (\chi_{v,A})^{\eta_v}}; \quad \Rightarrow \quad p_{S-1,A} = \chi_{S-1,A} p_{S-1,A} = \frac{\chi_{s,A}}{\Pi_{v=0}^{S-1} (\chi_{v,A})^{\eta_v}} = \left[ \frac{\mu_{s,A}}{\Pi_{v=0}^{S-1} (\mu_{v,A})^{\eta_v}} \right]^\alpha.
\]

Step 3: derive the law of motion for wage under free trade

Under free trade, for \( w_t \in (\bar{w}_z, \bar{w}_{z+1}) \), sector \( s \in \{z, \ldots, S - 1\} \) is active in period \( t + 1 \) and the law of motion for wage is

\[
p_{s,t+1} Y_{s,t+1} = \left[ \frac{\mu_{s,A}}{\Pi_{v=0}^{S-1} (\mu_{v,A})^{\eta_v}} \right]^\alpha \left( \frac{\zeta_{s,t+1}}{\zeta_{s,t+1}} \right)^\alpha Y_{t+1} = \zeta_{s,t+1} \Gamma_t (w_{t+1}),
\]

\[
w_{t+1} = (1 - \alpha) \frac{\sum_{v=z}^{S-1} \mu_{v,A} \delta_{v,t} + 1}{\sum_{v=z}^{S-1} \mu_{v,A} \delta_{v,t}} = \left( \frac{w_t}{\rho} \Gamma_t \right) \Gamma_t, \quad \text{where} \quad \Gamma_t = \frac{\sum_{v=z}^{S-1} \mu_{v,A} \delta_{v,t}}{\Pi_{v=0}^{S-1} (\mu_{v,A})^{\eta_v}}
\]

\(\square\)