

# Trade-Driven Sectoral Upgrading and the Global Imbalances

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## Abstract

This paper analyzes how trade integration may affect international financial flows in a world with heterogeneous financial development. In the presence of financial frictions and sector-specific minimum investment requirements, the static gains from trade triggers the cross-sector investment reallocation on **the extensive margin**. If trade-driven sectoral shifts allow the more financially developed country (North) to offshore low-return production activities and upgrade to high-return activities along the value chains, deepening trade integration may amplify the global imbalances (a phenomenon of large financial flows from developing to developed countries observed in the recent decades). This way, **trade-driven sectoral upgrading** in North becomes as a critical mechanism through which the substantial decline in trade and communication costs and the resulting boom in supply-chain trade may contribute to the global imbalances in the recent decades. This finding complements Antras and Caballero (2009, *Journal of Political Economy*).

**Keywords:** financial frictions, global imbalances, minimum investment requirements, sectoral shifts, supply-chain trade, trade integration

**JEL Classification:** F11, F41

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# 1 Introduction

The recent wave of globalization has two prominent features. First, emerging economies (especially China and other emerging Asian economies) have witnessed large current account surplus, while advanced economies (notably, the United States) have incurred persistent current account deficits in the past two decades. Accordingly, financial flows have been “uphill” from the poor to the rich countries (Kose et al., 2010; Prasad, Rajan, and Subramanian, 2006). Such global imbalances differ significantly from the previous episodes and are in stark contrast to the predictions of neoclassical theories. Second, due to technological progress and the removal of trade barriers, the costs of transportation, communication, and coordination have declined dramatically since 1990s, which accelerates international fragmentation of production (Baldwin, 2013b; Grossman and Rossi-Hansberg, 2006; Timmer et al., 2014). Nowadays, international production, trade and investments are organized within global value chains, which have substantially transformed the landscape of world production networks and influenced trade policies (Amador and Cabral, 2016; Antràs, 2015; Park, Nayyar, and Low, 2013).

Traditionally, trade and capital flows have been analyzed separately in the literature and economists have put little research effort on their interactions. Recent works suggest that such a separation is not always innocuous (Eaton et al., 2016; Ghironi and Melitz, 2005; Jin, 2012; Ju, Shi, and Wei, 2014). In a critical contribution to this literature, Antras and Caballero (2009) show that if the global imbalances are an equilibrium response to heterogeneous degrees of financial development across the world, deepening trade integration raises the return to capital in less financially developed countries (South) and resolves the global imbalances, while trade protectionism may backfire. Their findings have profound implications to the political debates at the peak of global imbalances in 2007-2008. However, the recent global imbalances have emerged and accelerated in parallel to world-wide trade liberalization, which seems at odds with their predictions.

Instead of focusing on the interest rate responses of South to trade flows, I emphasize the impacts of trade integration on the industrial structure in the more financially developed country (North). If trade-driven sectoral shifts allow North to offshore low-return activities and upgrade to high-return activities, trade integration may further raise the interest rate in North, which amplifies the global imbalances. The intuition is as follows.

Consider a two-sector, overlapping-generation model where firms hire physical capital<sup>1</sup> and labor to produce two goods which are combined in the Cobb-Douglas aggregator for consumption and investment. The two sectors are symmetric, except that individual investment is subject to the minimum investment requirements (hereafter, MIR) in a particular sector. For simplicity, the sector with no MIR is called sector 0, while the one with the MIR is called sector 1. In the absence of financial frictions, domestic savings are allocated efficiently across sectors. As a result, the output price is equalized in the two sectors and so is the sectoral rate of return.

In the presence of financial frictions and the heterogeneity in individual wealth, the agents with sufficiently high net wealth can meet the MIR and invest in sector 1, and

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<sup>1</sup>“Physical capital” should be interpreted broadly to include human capital as well as any tangible or intangible capital goods used in production.

they are called entrepreneurs; other agents have to invest in sector 0 or lend to the credit market, and they are called households. If financial frictions are sufficiently severe, the mass of entrepreneurs is inefficiently low and so are the investment and the output in sector 1. Thus, the rate of return and the output price are higher in sector 1 than in sector 0. The lower the level of financial development, the tighter the borrowing constraints, the smaller the mass of entrepreneurs, the larger the distortions on sectoral investment and output, the larger the sectoral rate-of-return and price differentials, the lower the income per capita. Here, the *extensive margin of investment*<sup>2</sup> is a key channel through which financial frictions and the sector-specific MIR distort the aggregate investment efficiency.

Consider a small open economy (North) which is more financially developed than the rest of the world (South). At the autarkic steady state, the rate of return in sector 0 and the income per capital are higher in North than in South. As investing in sector 0 and lending are perfect substitutes, the interest rate is coupled with rate of return in sector 0. Thus, the interest rate is higher in North. If allowed, financial flows are “uphill” from South to North. This way, the global imbalances arise here as an equilibrium response to heterogeneous financial development across the world. Besides, at the autarkic steady state, the output price in sector 1 (0) is lower (higher) in North than in South, implying that financial development is a determinant of comparative advantage for trade.

Starting from autarkic steady state, free trade in the two goods induces North to specialize towards sector 1 and the static gains raise its national income, which affects the sectoral investment in two ways. First, a higher national income allows each agents to invest more so that the investment scale in each sector tends to rise along **the intensive margin**. The decreasing MRK (marginal revenue of capital) effect then **dampens** the rise in the national income in the next period. Second, a higher national income allows more agents to overcome the MIR so that domestic investment is reallocated towards sector 1 along **the extensive margin**. It reinforces North’s comparative advantage and the enhanced specialization **amplifies** the rise in the national income in the next period. As long as the investment reallocation effect dominates the decreasing MRK effect, North’s national income rises over time until the mass of entrepreneurs becomes so large that their total borrowing capacity exceeds the entire household saving. In that case, North offshores sector 0 and specializes fully in sector 1. The investment reallocation effect is then mute and the decreasing MRK effect alone brings North to a new steady state. This way, the *extensive margin of investment* is the key channel through which trade may induce North to offshore the low-return sector, which fundamentally changes the way the interest rate is determined, as explained below.

When North specializes **partially** towards sector 1, trade reverses cross-country differences in the rate of return in sector 0 through the mechanism described in Antras and Caballero (2009). As sector 0 is still active, the interest rate is coupled with its rate of return, implying that the interest rate is lower in North than in South, a result Antras and Caballero (2009) calls “**the interest rate reversal**”. If allowed, financial flows are “downhill” from North to South. In this case, trade resolves the global imbalances.

When North eventually offshores sector 0 and specializes **fully** in sector 1, the interest

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<sup>2</sup>In each sector, the total investment depends on the investment size of individual investors (the intensive margin) as well as the mass of investors (the extensive margin).

rate is **decoupled** from the rate of return in sector 0 and the mechanism of Antras and Caballero (2009) ceases to work. Then, the interest rate in North is coupled with the rate of return in sector 1, which is higher than its autarkic steady-state level, a result I call “**the interest rate re-reversal**”. In this case, trade amplifies the global imbalances.

To sum up, whether trade resolves or amplifies the global imbalances depends on how far it reshapes the sectoral composition in the more financially developed country.

Then, I embed the core mechanism into a multi-sector setting where sectors differ in the MIR. If the borrowing constraints are sufficiently tight, the high-MIR sector is subject to more severe underinvestment problem, implying the high sectoral rate of return. If cross-sector heterogeneity in the MIR and cross-country heterogeneity in financial development are sufficiently large, trade may allow North to **sequentially** offshore the low-MIR, low-return sectors and **gradually** upgrade to the high-MIR, high-return sectors. Every time when North offshores the sector with the lowest return, the interest rate is decoupled from the rate of return in that sector and coupled with the rate of return in the next lowest-return sector. This way, the interest rate in North becomes an inverse sawtooth wave so that *the interest rate reversal and re-reversal may arise recurrently along the convergence path of North towards its steady state under trade integration.*

In the following, I use this mechanism to explain that the recent boom in supply-chain trade may contribute to the global imbalances.

## Supply-Chain Trade and the Global Imbalances

In manufacturing industries, upstream activities (such as R&D, product design, or the manufacturing of key parts and components) and downstream activities (such as marketing, brand building, and customer service) constitute a large share of value-added, while the intermediate production stages (such as component fabrication and final assembly) account for a small value-added share (Kimura, 2003). Stan Shih, the founder of Acer, introduced the smile curve to feature such an “U-shaped” value-added pattern along the production chain (Baldwin, Ito, and Sato, 2014; Ye, Meng, and Wei, 2015). Since the 1970s, fabrication and assembly activities account for an increasingly lower value-added share, in OECD countries, compared with upstream and downstream activities (Baldwin, 2013a; Baldwin and Lopez-Gonzalez, 2015; Gereffi, 1999; Koopman, Wang, and Wei, 2014; OECD, 2013). These facts can be replicated in my model as follows.

Fabrication and assembly are involved intensively with standardized, routine tasks, which require mainly the input of tangible investment; upstream and downstream activities are mostly involved with knowledge-intensive, non-routine tasks, which require heavily the input of intangible investment. Compared with tangible investment, intangible investment has a higher MIR (particularly in terms of human capital) and faces more severe financial frictions. In my model, “sectors” can be interpreted broadly as production stages or tasks, while investment can be tangible or intangible. One can map fabrication and assembly activities into “sector 0” and map upstream and downstream activities into “sector 1”.<sup>3</sup> My model predicts that the output price and the investment

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<sup>3</sup>One can disaggregate upstream and downstream activities into multiple stages, according to their respective MIR. As shown in the multi-sector setting, high-MIR stages have the higher rates of return.

return are higher in upstream and downstream activities than in fabrication and assembly. Thus, one can use the smile curve to characterize the rate-of-return pattern across stages.<sup>4</sup>

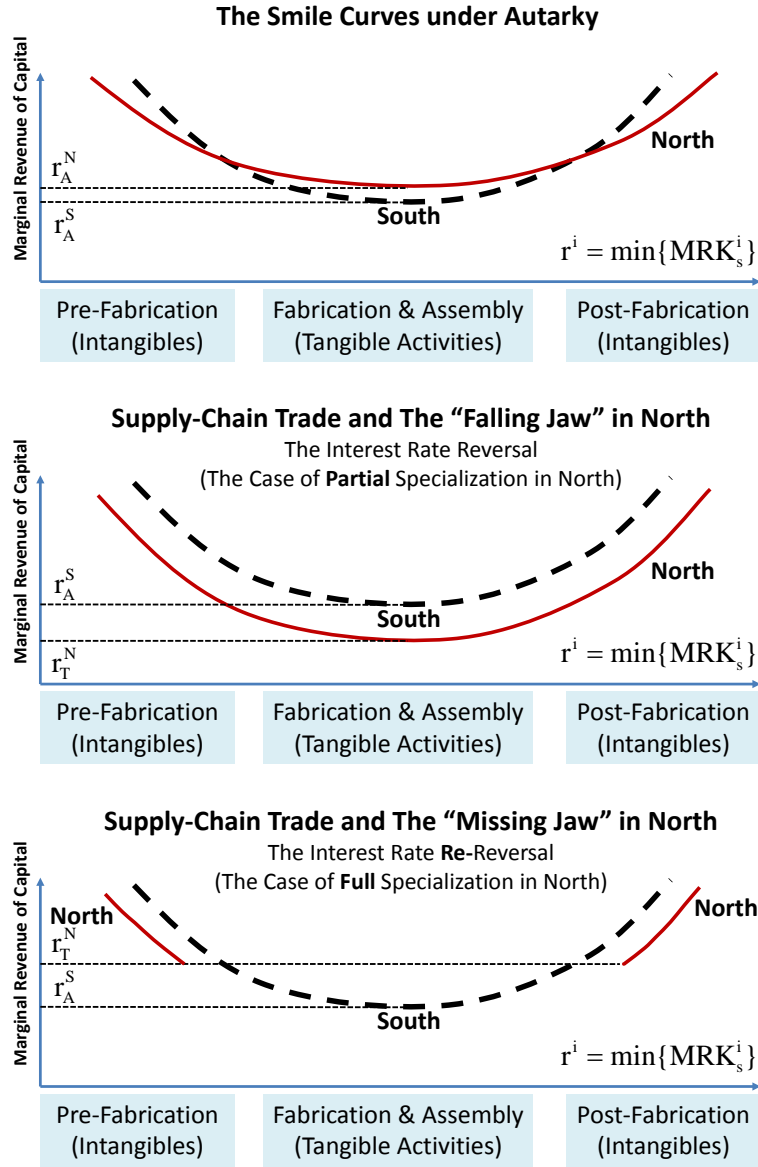


Figure 1: The “Falling vs. Missing Jaw” of the Smile Curve in North

In my model, advanced economies (North) are more developed than emerging economies (South) in financing intangible investment (Corrado et al., 2013). Thus, the sectoral rate-of-return differential is smaller and the smile curve is flatter in North than in South. See the upper panel of figure 1. Besides, the interest rate is coupled with the MRK in the active, lowest-return production stages/tasks,  $r^i = \min\{MRK_s^i | K_s^i > 0\}$ , where superscript  $i \in \{N, S\}$  is the country index and subscript  $s$  is the index of production stage. At the autarkic steady state, the interest rate is higher in North than in South,  $r_A^N > r_A^S$ .

<sup>4</sup>The original smile curve shows the value-added pattern along the production chain. Since the production stages/tasks that account for the high value-added shares usually have the high MRK, I use the smile curve to show the MRK pattern along the value chain.

This way, cross-country heterogeneity in financial development helps explain the “uphill” capital flows and the global imbalances. Meanwhile, North (South) has a comparative advantage in upstream and downstream (fabrication and assembly) activities.

Consider North as a small open economy. In the presence of high trade costs, the incentive of offshoring is small. A marginal decline in trade costs only allows North to offshore *part* of fabrication and assembly, leading to a “**falling jaw**” of its smile curve, as shown in the middle panel of figure 1. This is consistent with the dynamics of value-added shares in OECD countries. Since the interest rate in North is still coupled with the MRK in fabrication and assembly, the *interest rate reversal* happens,  $r_T^N < r_A^S < r_A^N$ .

Since the 1990s, technological progress and world-wide economic liberalization have substantially reduced the costs of transportation, communication, and coordination, leading to the boom of supply-chain trade and the expansion of global production networks. Vertically and horizontally linked production stages/tasks have been increasingly conducted in different countries. In my model, a substantial decline in trade costs allows North to offshore *all* fabrication/assembly and specialize fully in upstream/downstream activities. Then, North witnesses a “**missing jaw**” of its smile curve so that the interest rate is coupled with the MRK in upstream and downstream activities. As shown the lower panel of figure 1, supply-chain trade leads to the *interest rate re-reversal*, which amplifies the initial interest rate differential,  $r_T^N > r_A^N > r_A^S$ .

To sum up, if supply-chain trade allows North to sequentially offshore the low-return activities and upgrade along the value chain, the cross-country interest rate differential can be maintained or even amplified.

This way, **trade-driven sectoral upgrading** can be a critical mechanism through which the recent boom in supply-chain trade may contribute to the global imbalances. It complements the findings of Antras and Caballero (2009). Undoubtedly, various factors, e.g., globalization, technology progress, industrial policies, and etc., may contribute to the sectoral upgrading of advanced economies along the value chain. No matter what the causes are, as long as they offshore the low-return activities and climb up the value chain, the structural shifts in industrial composition may fundamentally change the way the interest rate is determined and the patterns of international financial flows. The core implication of this paper is that one should take into account the underlying shifts of sectoral composition when analyzing international capital flows.

## Related Literature

This paper is closely related to the recent literature on explaining the global imbalances as an equilibrium response to heterogeneous financial development across countries (Caballero, Farhi, and Gourinchas, 2008; Gourinchas and Rey, 2014; Ju and Wei, 2010; Mendoza, Quadrini, and Rios-Rull, 2009; von Hagen and Zhang, 2014; Zhang, 2017). Building upon this literature, Antras and Caballero (2009) show that deepening trade integration leads to the interest rate reversal, which resolves the global imbalances. They focus on the interest rate response to trade flows in South where both constrained and unconstrained sectors are always active. However, they do not explore explicitly whether and under what conditions trade flows may allow North to offshore the low-return sector.

By introducing sector-specific MIR and wealth heterogeneity into the basic model of

Antras and Caballero (2009), I endogenize the extensive margin of investment. It is exactly through this channel that the static gains from trade triggers cross-sector investment reallocation, leading to the possibility of North offshoring the low-return activities and the interest rate re-reversal.

Grossman and Rossi-Hansberg (2006) and Baldwin (2013a,b) show that the recent wave of offshoring and supply-chain trade has changed the composition of world trade and transformed the industrial structures in advanced economies and in emerging economies. My paper emphasizes the critical role of supply-chain trade in affecting the industrial structures in advanced economies and the patterns of international financial flows.

Kletzer and Bardhan (1987) were the first to show that better access to capital becomes a source of comparative advantage. It was then followed by a strand of theoretical literature on financial development and international trade (Antras and Caballero, 2009; Beck, 2002; Chesnokova, 2007; Ju and Wei, 2005).<sup>5</sup> Matsuyama (2005) introduces sector-specific borrowing constraints in a static model and shows that trade allows the rich (poor) country to fully specialize in the sector with tighter (looser) borrowing constraints. Wynne (2005) argues that a country's wealth can be a determinant of comparative advantage when access to credit differs across sectors, i.e., wealthier nations exhibit a comprehensive advantage towards goods produced in sectors facing more severe financial frictions. Ju and Wei (2011) point out that, in the countries with low-quality institutions, the quality of financial system is an independent source of comparative advantage. Building upon this literature, I analyze the joint determination of trade and capital flows.

Jin (2012) integrates factor-proportions-based trade and financial flows in an OLG model and shows that capital tends to flow to countries that become more specialized in capital-intensive industries. Ju, Shi, and Wei (2014) incorporate two tradeable sectors with different factor intensity in a small-open-economy setting and show that the current account adjustment to exogenous shocks depends on factor market flexibility. Instead of introducing sector-specific financial frictions or sector-specific factor intensity, I focus on a real friction, i.e., the sector-specific MIR.

In the literature, the MIR is used to capture the investment indivisibility at the individual level, which is an important feature of business ideas, physical and human capital (Aghion and Bolton, 1997; Banerjee and Moll, 2010; Banerjee and Newman, 1993; Galor and Zeira, 1993; Piketty, 1997; Zhang, 2017). Recently, Erosa and Hidalgo-Cabrillana (2008), Barseghyan and DiCecio (2011), Buera, Kaboski, and Shin (2011), Manova (2013), and Midrigan and Xu (2014) introduce the fixed cost or the entry cost at the firm level and show that the individual investment is above a minimum scale in equilibrium. In my

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<sup>5</sup>Recently, a booming literature has documented the extensive empirical evidence on the relationship between financial development and trade patterns. Financially developed countries export more in sectors that require more external finance and in sectors with fewer tangible assets (Beck, 2003; Hur, Raj, and Riyanto, 2006; Svaleryd and Vlachos, 2005). Manova (2008) shows that equity market liberalization increases exports disproportionately more in sectors that require more external funds or employ fewer collateralizable assets. Manova (2013) further decomposes the trade effect of weak financial markets and show that financially developed countries serve more destination markets and export more products, in more financially vulnerable sectors. Chor and Manova (2012) analyze the collapse of international trade flows during the global financial crisis and show that credit conditions were an important channel through which the financial crisis affected trade volumes.

model, assuming the MIR rather than the fixed cost allows us to characterize analytically the dynamic properties in the entire parameter space.

I also revisit the factor price equalization (hereafter FPE) theorem in the presence of financial and real frictions. Deardorff (2001) shows that trade leads to the FPE if countries stay in the same cone of diversification. In the dynamic model, the multi-cone equilibrium may arise and the FPE holds within cones, but not between them. In Antras and Caballero (2009), although trade alone does not lead to the FPE, allowing both trade and capital flows can do, which may be understood in Deardorff's framework. In my model, if trade has induced North to offshore the lower-return sector, adding capital mobility in the free-trade equilibrium does not achieve the FPE, which is opposite to the finding of Antras and Caballero (2009). In this sense, *the timing and the sequence of economic integration* are critical for the FPE in my model.

The rest of the paper is structured as follows. Section 2 sets up the model and section 3 analyzes the autarkic equilibrium. Section 4 discusses the impacts of free trade on aggregate income and the interest rate, given the world economy is initially in the autarkic steady state. Section 5 explores the channel for the interest rate re-reversal, discusses the implications of trade costs and the initial conditions in South. Section 5.3 generalizes the two-sector setting into the multi-sector setting. Section 6 makes some final remarks. Technical proofs and other related materials are in the appendix.

## 2 The Model Setting

The world consists of two countries, North and South, which are inherently identical except for the population size and the level of financial development, as specified later. In this section, I first describe the economic setting of North and then use the asterisk superscript to denote the variables and the parameters in South.

In North, a continuum of agents are born every period and live for two periods, young and old. In each generation, the population size is constant at one and agents are indexed by  $j \in [0, 1]$ . Agent  $j$  is endowed with  $l_j = (1 - \theta)\epsilon_j$  units of labor when young, where  $\epsilon_j \in (1, \infty)$  follows the Pareto distribution,  $G(\epsilon_j) = 1 - \epsilon_j^{-\frac{1}{\theta}}$  and  $\theta \in (0, 1)$ .<sup>6</sup> Agents only consume when old and they supply the labor endowment inelastically to the market when young. Every period, the aggregate labor supply is constant at  $L = \int_1^\infty l_j dG(\epsilon_j) = 1$ .

North has two sectors, indexed by  $s \in \{0, 1\}$ . In period  $t$ ,  $K_{s,t}$  units of physical capital and  $L_{s,t}$  units of labor are hired in sector  $s$  to produce  $Y_{s,t}$  units of good  $s$ . Physical capital fully depreciates after use. Sectoral outputs are tradable, while labor and physical capital are not.  $V_{0,t}$  units of good 0 and  $V_{1,t}$  units of good 1 are combined to produce  $Y_t$  units of final goods which are used for consumption and investment.<sup>7</sup> The markets for goods and

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<sup>6</sup>The inverse of  $\theta$  is the tail index of the Pareto distribution. Pareto distribution is widely used in the literature to feature the income and wealth distribution (Atkinson, Piketty, and Saez, 2011; Gabaix, 2009; Jones, 2015). The top tail of income distribution is very well approximated by a Pareto distribution (Kuznets and Jenks, 1953; Piketty, 2014; Piketty and Saez, 2003).

<sup>7</sup>Under autarky, domestic absorption is equal to domestic output at the sectoral level,  $V_{s,t} = Y_{s,t}$ , while it is not the case with trade flows,  $V_{s,t} \neq Y_{s,t}$ .



productive factors are competitive and final goods serve as the numeraire<sup>8</sup>. There is no uncertainty in the model economy. Let  $w_t$  denote the wage rate. Let  $p_{s,t}$  and  $q_{s,t}$  denote respectively the output price and the rental price of capital in sector  $s$ .

$$Y_{s,t} = \left(\frac{K_{s,t}}{\alpha}\right)^\alpha \left(\frac{L_{s,t}}{1-\alpha}\right)^{1-\alpha}, \quad q_{s,t}K_{s,t} = \alpha p_{s,t}Y_{s,t}, \quad w_t L_{s,t} = (1-\alpha)p_{s,t}Y_{s,t}, \quad (1)$$

$$Y_t = \left(\frac{V_{1,t}}{\eta}\right)^\eta \left(\frac{V_{0,t}}{1-\eta}\right)^{1-\eta}, \quad p_{1,t}V_{1,t} = \eta Y_t, \quad p_{0,t}V_{0,t} = (1-\eta)Y_t. \quad (2)$$

where  $\alpha, \eta \in (0, 1)$ . In order to focus on the interest rate response to trade, I exclude international capital flows so that domestic investment is funded by domestic saving and national income is equal to domestic output. Combine (1)-(2) to get  $w_t = (1-\alpha)\frac{Y_t}{L}$ . Thus, the wage rate can serve as a proxy of national income. Later, I use the law of motion for wage to analyze the model dynamics and the steady-state properties.

As labor is perfectly mobile across sectors and the labor market is frictionless, the marginal revenue of labor (hereafter MRL) is equalized across sectors by the wage rate. As described below, physical capital is sector-specific and sectoral investment is subject to frictions. Thus, the marginal revenue of capital (hereafter, MRK) may differ across sectors, leading to the sectoral rental-price-of-capital wedge.

In period  $t$ , young agents invest their entire labor income  $w_t L$  in the two sectors, which yields in period  $t+1$   $K_{s,t+1}$  units of physical capital in sector  $s$ . Let  $\delta_t$  and  $\zeta_{t+1}$  denote respectively the fractions of domestic saving ( $w_t L$ ) and labor endowment ( $L$ ) allocated for the production of good 1 in period  $t+1$ .

$$K_{1,t+1} = \delta_t w_t L, \quad L_{1,t+1} = \zeta_{t+1} L, \quad (3)$$

$$K_{0,t+1} = (1-\delta_t)w_t L, \quad L_{0,t+1} = (1-\zeta_{t+1})L. \quad (4)$$

Let  $\chi_t \equiv \frac{p_{0,t}}{p_{1,t}}$  and  $\mu_t \equiv \frac{q_{0,t}}{q_{1,t}}$  denote respectively the sectoral output-price ratio and the sectoral rental-price-of-capital ratio. Combine equations (1) and (3)-(4) to get

$$p_{s,t} = (q_{s,t})^\alpha w_t^{1-\alpha}, \quad \Rightarrow \quad \chi_t = \mu_t^\alpha, \quad (5)$$

$$\zeta_{t+1} = \frac{\delta_t}{(1-\delta_t)\chi_{t+1}^\frac{1}{\alpha} + \delta_t} = \frac{\delta_t}{(1-\delta_t)\mu_{t+1} + \delta_t}. \quad (6)$$

Given aggregate investment  $w_t L$  and labor input  $L$ , equation (6) implicitly describes the production possibility frontier (PPF, hereafter) for North in period  $t+1$ ,<sup>9</sup> reflecting the sectoral output supply. Equation (2) represents the isoquant, featuring the sectoral output demand. In the next sections, I will combine the isoquant and the PPF to show the equilibrium allocations under autarky and under trade integration, respectively. In the following, I first derive explicitly the PPF, given sectoral investment distortions.

<sup>8</sup>In Antras and Caballero (2009), capital and labor are hired to produce two final goods which are combined in the Cobb-Douglas aggregator for consumption and investment, while one final good is chosen as the numeraire. In Ju and Wei (2011), the two final goods are combined to produce a composite good in a Cobb-Douglas fashion, while the composite good serves as the numeraire and is used for consumption and investment. The results of my model do not depend on the choice of numeraire.

<sup>9</sup>See the proof of Lemma 1 for derivation.

Agent  $j$  born in period  $t$  has three options of saving its labor income: investing in the two sectors and lending to the credit market. By investing  $k_{j,0,t+1} > 0$  units of final goods in period  $t$ , the agent gets in period  $t+1$   $k_{j,0,t+1}$  units of physical capital for sector 0 and the rate of return to its investment is  $q_{0,t+1}$ . It gets the same one-to-one investment outcome in sector 1, if its investment size meets the MIR,  $k_{j,1,t+1} \geq \mathbf{m} > 0$ ; otherwise, its investment in sector 1 yields zero outcome.

Except for the MIR, the two sectors are symmetric in terms of the investment and production technologies. Let  $\mathbb{Y}_{t+1} \equiv \left(\frac{w_t L}{\alpha}\right)^\alpha \left(\frac{L}{1-\alpha}\right)^{1-\alpha}$  denote a sector's maximum possible output if domestic saving and labor are fully allocated in that sector. If credit markets were perfect so that everyone could meet the MIR, investment in the two sectors would be perfect substitutes and the sectoral rates of return would equalize under autarky,  $q_{0,t+1} = q_{1,t+1}$  and  $\mu_{t+1} = 1$ . In this case, for the output of  $Y_{1,t+1}$ , productive factors would be allocated in sector 1 in equal proportions,  $\zeta_{t+1} = \delta_t = \frac{Y_{1,t+1}}{\mathbb{Y}_{t+1}}$ , according to equations (6), (3), and (1). Thus, the PPF is linear and has a constant marginal rate of transformation (MRT, hereafter) at unity,

$$Y_{1,t+1} = \delta_t \mathbb{Y}_{t+1}, \quad Y_{0,t+1} = (1 - \delta_t) \mathbb{Y}_{t+1}, \quad MRT_{0,1} \equiv -\frac{\partial Y_{1,t+1}}{\partial Y_{0,t+1}} = \mu_{t+1}^\alpha = \chi_{t+1} = 1. \quad (7)$$

However, due to limited commitment, agent  $j$  can borrow only up to a fraction  $\lambda$  of its investment in sector 1 and has to use its own funds to cover the gap,

$$b_{j,t} \leq \lambda k_{j,1,t+1}, \quad \text{and} \quad k_{j,1,t+1} - b_{j,t} \leq n_{j,t}, \quad (8)$$

where  $\lambda \in [0, 1)$  measures the level of financial development.<sup>10</sup> If the borrowing constraints are sufficiently tight, the mass of agents who can meet the MIR is inefficiently low and so is the total investment in sector 1. In this case, financial frictions and sector-specific MIR jointly distort sectoral investment, leading to the sectoral return wedge,  $q_{1,t+1} > q_{0,t+1}$ . According to equation (6),  $\mu_{t+1} < 1$  implies that capital and labor are not allocated in sector 1 in equal proportions,  $\zeta_{t+1} > \delta_t$ . I derive the PPF for this case as follows.

Since all agents can freely invest in sector 0 and lend to the credit market, these two options are perfect substitutes. As good 0 is essential for the final good production,  $K_{0,t+1} > 0$  holds under autarky, implying that the interest rate is coupled with the rate of return in sector 0. As shown in section 4, trade may induce North to offshore sector 0 and specialize fully in sector 1. If so, the interest rate is decoupled from (coupled with) the rate of return in sector 0 (1).

$$r_t = \begin{cases} q_{0,t+1}, & \text{if } K_{0,t+1} > 0 \text{ or equivalently } \delta_t < 1; \\ q_{1,t+1}, & \text{if } K_{0,t+1} = 0 \text{ or equivalently } \delta_t = 1. \end{cases} \quad (9)$$

Consider first the case where investment frictions lead to the sectoral return wedge,  $q_{1,t+1} > q_{0,t+1}$  and sector 0 is still active  $K_{0,t+1} > 0$ . In period  $t$ , the positive rate-of-return spread induces agent  $j$  to invest its entire labor income in sector 1 and borrow to the limit,

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<sup>10</sup>Antras and Caballero (2009) use the same form of borrowing constraints and offer detailed micro-foundations. The credit markets would be perfect and the borrowing constraints would be slack if  $\lambda = 1$ .

if it can meet the MIR,  $k_{j,1,t+1} = \frac{n_{j,t}}{1-\lambda} \geq \mathbf{m}$ . In period  $t+1$ , the agent gets the investment return, repays the debt, and consumes the rest. Let  $\Omega_{j,t} \equiv \frac{q_{1,t+1}k_{j,1,t+1} - r_t b_{j,t}}{k_{j,1,t+1} - b_{j,t}}$  denote agent  $j$ 's equity rate, i.e., the rate of return to its own funds. The binding borrowing constraints imply that the equity rate is common for all agents who meet the MIR and the agent's investment size in sector 1 is linear in its net wealth.

$$\Omega_{j,t} = \Omega_t \equiv q_{1,t+1} + (q_{1,t+1} - r_t)\left(\frac{1}{1-\lambda} - 1\right) > q_{1,t+1} > r_t, \quad (10)$$

$$k_{j,1,t+1} = \frac{n_{j,t}}{1-\lambda} = \frac{w_t}{1-\lambda}(1-\theta)\epsilon_j \geq \mathbf{m}, \Rightarrow \epsilon_j \geq \underline{\epsilon}_t \equiv \frac{1-\lambda}{w_t} \frac{\mathbf{m}}{1-\theta}, \quad (11)$$

where  $\underline{\epsilon}_t$  is a cutoff value. Agents with  $\epsilon_j \geq \underline{\epsilon}_t$  can meet the MIR and they are called *entrepreneurs*, with the mass of  $\tau_t = \underline{\epsilon}_t^{-\frac{1}{\theta}}$ . When young, they invest in sector 1 with their entire labor income  $n_{j,t}$  and borrow to the limit  $b_{j,t} = \frac{\lambda n_{j,t}}{1-\lambda}$ ; when old, they consume  $c_{j,t+1}^e$ ,

$$n_{j,t} = w_t l_j \quad \text{and} \quad c_{j,t+1}^e = n_{j,t} \Omega_t. \quad (12)$$

In period  $t$ , the fraction of domestic saving allocated in sector 1 is

$$\delta_t = \frac{\int_{\underline{\epsilon}_t}^{\infty} k_{j,1,t+1} dG(\epsilon_j)}{w_t L} = \frac{\underline{\epsilon}_t^{-\frac{1-\theta}{\theta}}}{1-\lambda} = \frac{\tau_t^{1-\theta}}{1-\lambda} = \left(\frac{w_t}{\bar{w}}\right)^{\frac{1-\theta}{\theta}}, \quad \text{where } \bar{w} \equiv \frac{\mathbf{m}}{1-\theta}(1-\lambda)^{\frac{1}{1-\theta}}. \quad (13)$$

Agents with  $\epsilon_j \in [1, \underline{\epsilon}_t)$  cannot meet the MIR and are called *households*. When young, they invest  $k_{j,0,t+1}$  in sector 0 and lend out  $n_{j,t} - k_{j,0,t+1}$ ; when old, they consume  $c_{j,t+1}^h$ ,

$$k_{j,0,t+1} + (n_{j,t} - k_{j,0,t+1}) = n_{j,t} = w_t l_j \quad \text{and} \quad c_{j,t+1}^h = n_{j,t} r_t. \quad (14)$$

The markets for credit, labor, sector-specific capital, and final goods clear every period,

$$\int_{\underline{\epsilon}_t}^{\infty} (k_{j,1,t+1} - n_{j,t}) dG(\epsilon_j) = \int_1^{\underline{\epsilon}_t} (n_{j,t} - k_{j,0,t+1}) dG(\epsilon_j), \quad L_{0,t} + L_{1,t} = L, \quad (15)$$

$$K_{1,t} = \delta_{t-1} w_{t-1} L, \quad K_{0,t} = (1 - \delta_{t-1}) w_{t-1} L, \quad (16)$$

$$\int_{\underline{\epsilon}_t}^{\infty} c_{j,t}^e dG(\epsilon_j) + \int_1^{\underline{\epsilon}_t} c_{j,t}^h dG(\epsilon_j) + K_{1,t+1} + K_{0,t+1} = Y_t. \quad (17)$$

Given the binding borrowing constraints in period  $t$ ,  $\delta_t$  is predetermined by equation (13) from the perspective of period  $t+1$ . Thus, the PPF in period  $t+1$  can be specified as

$$\left[ \frac{Y_{0,t+1}}{(1-\delta_t)^\alpha} \right]^{\frac{1}{1-\alpha}} + \left( \frac{Y_{1,t+1}}{\delta_t^\alpha} \right)^{\frac{1}{1-\alpha}} = \mathbb{Y}_{t+1}^{\frac{1}{1-\alpha}}, \quad MRT_{0,1} \equiv -\frac{\partial Y_{1,t+1}}{\partial Y_{0,t+1}} = \mu_{t+1}^\alpha = \chi_{t+1} < 1. \quad (18)$$

Consider then the case of  $q_{1,t+1} = q_{0,t+1}$ . Due to the zero spread, the agents who can meet the MIR do not have the incentive to invest their entire labor income in sector 1 or to borrow to the limit. Despite the indeterminacy at the individual level,  $\mu_{t+1} = 1$  implies that the PPF in period  $t+1$  is specified by equation (7).

Let  $\tilde{\delta}_t \equiv \min\left\{\left(\frac{w_t}{\bar{w}}\right)^{\frac{1-\theta}{\theta}}, 1\right\}$  denote the maximum possible share of domestic investment in sector 1 when all entrepreneurs borrow and invest to the limit.

**Lemma 1.** *Let  $\tilde{Y}_{1,t+1} \equiv \tilde{\delta}_t \mathbb{Y}_{t+1}$ . Given  $w_t < \bar{w}$ ,  $\tilde{\delta}_t < 1$  and the PPF in period  $t+1$  is piecewise and consists of two parts:*

- a linear part for  $Y_{1,t+1} \leq \tilde{Y}_{1,t+1}$ , as specified by (7) with  $\delta_t = \frac{Y_{1,t+1}}{\mathbb{Y}_{t+1}}$ , and
- a concave part for  $Y_{1,t+1} > \tilde{Y}_{1,t+1}$ , as specified by (18) with  $\delta_t = \left(\frac{w_t}{\bar{w}}\right)^{\frac{1-\theta}{\theta}} < 1$ .

Given  $w_t \geq \bar{w}$ ,  $\tilde{\delta}_t = 1$  and the PPF in period  $t + 1$  is linear, as specified by equation (7).

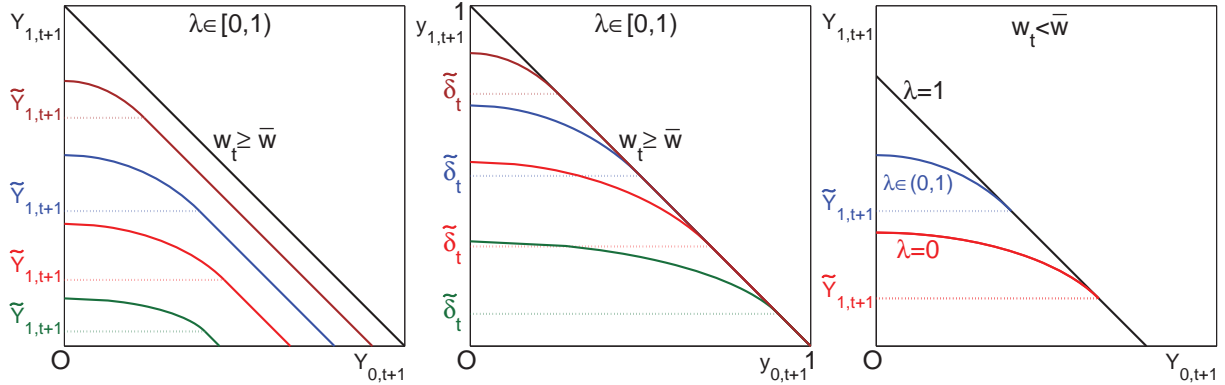


Figure 2: The Production Possibility Frontiers

$\tilde{\delta}_t$  depends positively on the level of financial development ( $\lambda$ ) and the level of national income (with  $w_t$  as a proxy).

- For the output of  $Y_{1,t+1} \leq \tilde{Y}_{1,t+1}$ , equation (7) specifies the efficient fraction of investment in sector 1,  $\hat{\delta}_t = \frac{Y_{1,t+1}}{\mathbb{Y}_{t+1}}$ , which is feasible  $\hat{\delta}_t \leq \tilde{\delta}_t = \frac{\tilde{Y}_{1,t+1}}{\mathbb{Y}_{t+1}}$ . In equilibrium, the sectoral investment is efficient,  $\delta_t = \hat{\delta}_t \leq \tilde{\delta}_t$ , and the sectoral rates of return equalize,  $\mu_{t+1} = 1$ . A marginal rise in  $Y_{1,t+1}$  requires the reallocation of both labor and investment from sector 0 to 1 in equal proportions,  $\zeta_{t+1} = \delta_t$ . Then,  $Y_{0,t+1}$  declines by the same amount, implying a linear PPF, as specified by equation (7).
- For the output of  $Y_{1,t+1} > \tilde{Y}_{1,t+1}$ , the efficient fraction of investment in sector 1 is not feasible,  $\hat{\delta}_t = \frac{Y_{1,t+1}}{\mathbb{Y}_{t+1}} > \tilde{\delta}_t = \frac{\tilde{Y}_{1,t+1}}{\mathbb{Y}_{t+1}}$ . In equilibrium, the sectoral investment in sector 1 is inefficiently low,  $\delta_t = \tilde{\delta}_t < \hat{\delta}_t$ , leading to the sectoral return wedge,  $q_{1,t+1} > q_{0,t+1}$ . Given a predetermined  $\delta_t$ , a marginal rise in  $Y_{1,t+1}$  requires the labor reallocation from sector 0 to 1 more than proportionally. Then,  $Y_{0,t+1}$  falls by a larger amount, implying a concave PPF, as specified by equation (18). If labor is fully allocated in sector 1, the maximum output in sector 1 is  $Y_{1,t+1} = \tilde{\delta}_t^\alpha \mathbb{Y}_{t+1} < \mathbb{Y}_{t+1}$ .

As shown in the left panel of figure 2, given  $w_t < \bar{w}$ , a rise in  $w_t$  affects the PPF in two ways. First, the higher the  $w_t$ , the higher the national income, the higher the domestic saving and investment, the higher the  $\mathbb{Y}_{t+1}$ . I call it the *investment scale effect*, which shifts the PPF parallel away from the origin. Second, the higher the  $w_t$ , the lower the cutoff value  $\underline{\epsilon}_t$ , the larger the mass of entrepreneurs  $\tau_t$ , the higher the  $\tilde{\delta}_t$ . I call it the *investment composition effect*, which extends the linear fraction of the PPF.

By normalizing sectoral outputs in equations (7) and (18) by  $\mathbb{Y}_{t+1}$ , one can explicitly highlight the investment composition effect,

$$y_{0,t+1} + y_{1,t+1} = 1, \quad \text{and} \quad \left[ \frac{y_{0,t+1}}{(1 - \tilde{\delta}_t)^\alpha} \right]^{\frac{1}{1-\alpha}} + \left( \frac{y_{1,t+1}}{\tilde{\delta}_t^\alpha} \right)^{\frac{1}{1-\alpha}} = 1, \quad \text{where} \quad y_{s,t+1} \equiv \frac{Y_{s,t+1}}{\mathbb{Y}_{t+1}}.$$

As shown in the middle panel of figure 2, given  $w_t < \bar{w}$ , a rise in  $w_t$  raises  $\tilde{\delta}_t$  so that the linear part accounts for a larger fraction in the normalized PPF.

For  $w_t \geq \bar{w}$ , the mass of entrepreneurs is so high that domestic saving can be fully invested in sector 1,  $\tilde{\delta}_t = 1$ . For the output of  $Y_{1,t+1} \leq \bar{Y}_{t+1}$ , the efficient fraction of investment in sector 1 is always feasible,  $\hat{\delta}_t = \frac{Y_{1,t+1}}{\bar{Y}_{t+1}} \leq \tilde{\delta}_t = 1$ . Thus, the entire PPF becomes linear, as shown by the diagonal lines in the left and middle panels of figure 2.

The right panel of figure 2 shows that, given  $w_t < \bar{w}$ , the higher the  $\lambda$ , the lower the cutoff value  $\underline{\epsilon}_t$ , the larger the mass of entrepreneurs, the higher the  $\tilde{\delta}_t$  and  $\tilde{Y}_{1,t+1}$ , the larger the linear part of the PPF. For  $\lambda = 1$ ,  $\tilde{\delta}_t = 1$  and the entire PPF becomes linear.

Under autarky, the sectoral output markets clear domestically,

$$V_{s,t} = Y_{s,t}. \quad (19)$$

**Definition 1.** *Under autarky, a market equilibrium is a set of choices of agents  $\{n_{j,t}, c_{j,t}^e, c_{j,t}^h, k_{j,s,t+1}\}$  and aggregate variables  $\{Y_t, Y_{s,t}, K_{s,t}, L_{s,t}, V_{s,t}, \chi_t, \mu_t, w_t, r_t, \Omega_t, \underline{\epsilon}_t\}$ , satisfying equations (1)-(2), (9)-(12), (14)-(16), and (19), where  $s \in \{0, 1\}$ .<sup>11</sup>*

Free trade aligns the sectoral price ratio to the world level and trade is balanced,

$$\chi_t = \chi_t^*, \quad \chi_t^*(V_{0,t} - Y_{0,t}) + (V_{1,t} - Y_{1,t}) = 0. \quad (20)$$

**Definition 2.** *Under free trade, a market equilibrium is a set of choices of agents  $\{n_{j,t}, c_{j,t}^e, c_{j,t}^h, k_{j,s,t+1}\}$  and aggregate variables  $\{Y_t, Y_{s,t}, K_{s,t}, L_{s,t}, V_{s,t}, \mu_t, w_t, r_t, \Omega_t, \underline{\epsilon}_t\}$ , satisfying equations (1)-(2), (9)-(12), and (14)-(16), where  $s \in \{0, 1\}$ , while the sectoral price ratio  $\chi_t$  is determined at the world level by equation (20).*

In period  $t$ , domestic investment is financed by domestic saving,  $\sum_{v=0}^1 K_{v,t+1} = w_t L$ ; in period  $t+1$ , the aggregate investment return is  $\sum_{v=0}^1 q_{v,t+1} K_{v,t+1} = \frac{\alpha}{1-\alpha} w_{t+1} L$ , according to equations (1)-(2). Define the social rate of return as

$$\Upsilon_t \equiv \frac{\sum_{v=0}^1 q_{v,t+1} K_{v,t+1}}{\sum_{v=0}^1 K_{v,t+1}} = (1 - \delta_t) q_{0,t+1} + \delta_t q_{1,t+1} = \rho \frac{w_{t+1}}{w_t}, \quad \text{where } \rho \equiv \frac{\alpha}{1 - \alpha}. \quad (21)$$

### 3 The Autarkic Equilibrium

As the labor market is frictionless and labor is perfectly mobile across sectors, the sectoral allocation of labor is efficient and, under autarky, it is equal to the sectoral share in the production of final goods,  $\zeta_{t+1} = \eta$ . It would also apply to the sectoral allocation of physical capital  $\delta_t = \eta$ , if sectoral investment were efficient, i.e.,  $\mu_{t+1} = 1$ .

Start with the benchmark case where the credit markets are perfect, i.e.,  $\lambda = 1$ . In the absence of financial frictions, sectoral investment is efficient and sectoral prices equalize,  $\chi_{t+1} = \mu_{t+1} = 1$ . The autarkic equilibrium is represented by the tangent point of the isoquant and the PPF. In figure 3, the dashed, diagonal line shows the normalized PPF and point **B** represents the autarkic equilibrium in the benchmark setting. Given the

<sup>11</sup>According to the Walras' law, if the markets for labor, credit, sector-specific physical capital, and sectoral outputs clear, the final good market must clear. In this sense, equation (17) is redundant.

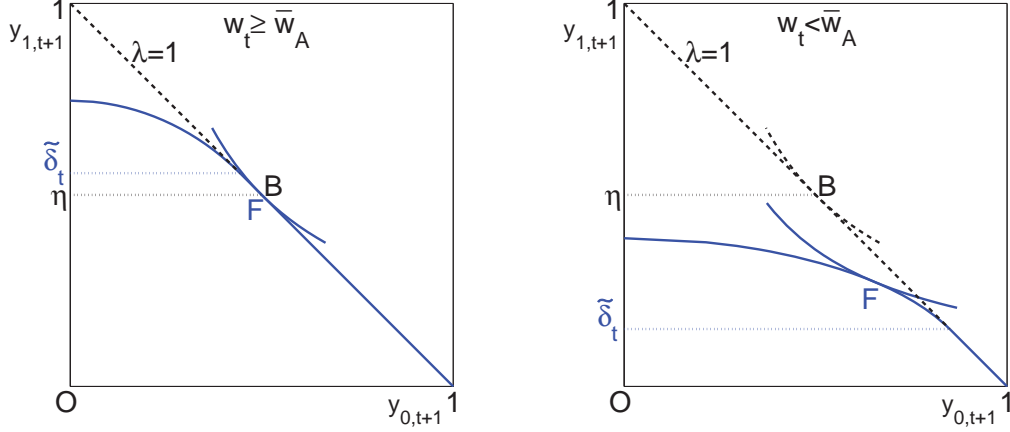


Figure 3: Equilibrium Allocations under Autarky

Cobb-Douglas production functions at the sectoral and at the aggregate levels, capital has the decreasing MRK so that the law of motion for wage is concave.

$$w_{t+1} = \left(\frac{w_t}{\rho}\right)^\alpha, \Rightarrow \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} < 1, \quad \text{where } \rho \equiv \frac{\alpha}{1-\alpha}. \quad (22)$$

As a convergence force, the decreasing MRK drives North to a unique steady state with the wage rate  $w_B = \rho^{-\rho}$ . Subscript **B** refers to the **b**enchmark case.<sup>12</sup> The smaller the  $\alpha$ , the stronger the decreasing MRK effect, the faster the convergence.

Consider now the case with financial frictions,  $\lambda \in [0, 1)$ . Let  $\bar{w}_A \equiv \eta^{\frac{\theta}{1-\theta}} \bar{w} < \bar{w}$ . If  $w_t \geq \bar{w}_A$ , the efficient investment allocation is feasible,  $\eta \leq \tilde{\delta}_t$ . In equilibrium,  $\delta_t = \zeta_{t+1} = \eta$  and  $\chi_{t+1} = \mu_{t+1} = 1$  hold. In the left panel of figure 3, the solid, concave curve shows the normalized PPF, while point **F** shows the autarkic equilibrium with financial frictions, which coincides with point **B**. As the equilibrium allocation is efficient in this case, the income dynamics are also characterized by equation (22).

If  $w_t < \bar{w}_A$ , the efficient investment allocation is infeasible,  $\eta > \tilde{\delta}_t$  and the under-investment in sector 1,  $\delta_t = \tilde{\delta}_t < \zeta_{t+1} = \eta$ , leads to the sectoral return wedge,  $\mu_{t+1} < 1$ . Given  $w_t$ , the income dynamics are featured by  $\{\delta_t, \mu_{t+1}, w_{t+1}\}$  satisfying (13), (23)-(24),<sup>13</sup>

$$w_{t+1} = \left(\frac{w_t \Gamma_t}{\rho}\right)^\alpha, \quad \text{where } \Gamma_t \equiv \frac{\mu_{t+1}^\eta}{1 - \eta(1 - \mu_{t+1})} < 1, \quad \text{and } \frac{\partial \Gamma_t}{\partial \mu_{t+1}} > 0, \quad (23)$$

$$\mu_{t+1} = \frac{\frac{1}{\eta} - 1}{\frac{1}{\delta_t} - 1}, \Rightarrow \frac{\partial \mu_{t+1}}{\partial w_t} = \frac{\partial \mu_{t+1}}{\partial \delta_t} \frac{\partial \delta_t}{\partial w_t} > 0, \quad (24)$$

where  $\Gamma_t$  is the aggregate efficiency indicator. Sectoral investment distortion ( $\mu_{t+1} < 1$ ) creates aggregate inefficiency ( $\Gamma_t < 1$ ).<sup>14</sup> In the right panel of figure 3, the solid, concave curve shows the normalized PPF, while the equilibrium allocation represented by point **F** is on an isoquant lower than in the benchmark case. The distance between the two isoquants reflects the efficiency losses in percentage terms,  $(1 - \Gamma_t^\alpha)$ .

<sup>12</sup>The dashed curves in figure 5 show the law of motion for wage under autarky and point **B** represents the steady state in the benchmark case.

<sup>13</sup>See the proof of proposition 1 in appendix B for derivation.

<sup>14</sup>In the benchmark case,  $\mu_{t+1} = 1$  implies that the aggregate efficiency index is constant at  $\Gamma_t = 1$ .

A rise in national income raises the wage rate, which affects the sectoral investment in two ways. First, it allows agents to invest more so that sectoral investment rises on the *intensive margin*. The decreasing MRK effect *dampens* the initial income change. Second, it allows more agents to meet the MIR and become entrepreneurs, which shifts domestic investment towards sector 1 on the *extensive margin*. This investment composition effect improves aggregate efficiency and *amplifies* the initial income change.

$$\begin{aligned} \frac{\partial \ln w_{t+1}}{\partial \ln w_t} &= 1 - \underbrace{(1 - \alpha)}_{\text{decreasing MRK effect}} + \underbrace{\frac{\partial \ln w_{t+1}}{\partial \ln \Gamma_t} \frac{\partial \ln \Gamma_t}{\partial \ln \mu_{t+1}} \frac{\partial \ln \mu_{t+1}}{\partial \ln w_t}}_{\text{investment composition effect}} \\ &= \alpha + \alpha\eta(1 - \mu_{t+1}) \frac{1 - \theta}{\theta}. \end{aligned} \quad (25)$$

Let  $X_A$  denote the steady-state value of variable  $X_t$  under autarky. If there is a steady state with  $w_A \in (0, \bar{w}_A)$ , the slope of the law of motion for wage at this steady state is  $\frac{\partial w_{t+1}}{\partial w_t}|_{w_A} = \alpha + \alpha\eta(1 - \mu_A) \frac{1 - \theta}{\theta}$ . With  $\mu_A < 1$ , the investment composition effect is positive. The higher the  $\theta$ , the more dispersed the wealth distribution, the less sensitive the mass of entrepreneurs and sectoral investment to income changes, the weaker the investment composition effect; the lower the  $\alpha$ , the stronger the decreasing MRK effect and the weaker the investment composition effect.

**Proposition 1.** Let  $\underline{\theta} \equiv \frac{\alpha}{\alpha + \frac{1-\alpha}{\eta}} < \alpha$ ,  $Z \equiv \frac{\frac{1}{\rho}}{\left(\frac{\eta}{1-\theta}\right)^{\frac{1}{\rho}}}$ , and  $\tilde{\lambda}_A \equiv 1 - Z\rho^{(1-\theta)}$ .

For  $\theta \in [\underline{\theta}, 1)$  and  $\lambda \in [0, \tilde{\lambda}_A)$ , there is a unique, stable steady state under autarky where the borrowing constraints are binding and sectoral investment is inefficient.

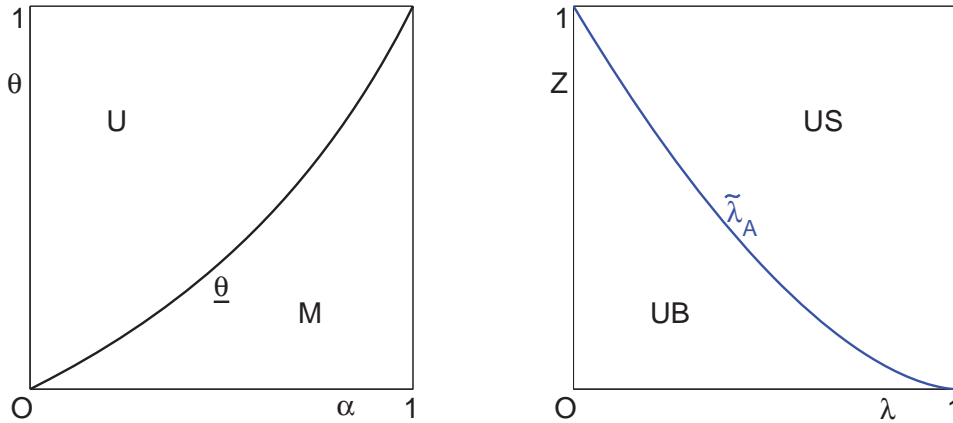


Figure 4: Threshold Values for the Autarkic Equilibrium

Figure 4 shows the two threshold values, i.e.,  $\underline{\theta}$  and  $\tilde{\lambda}_A$ , in the  $\{\alpha, \theta\}$  space and in the  $\{\lambda, Z\}$  space, respectively. For  $\{\alpha, \theta\}$  in region **U** of the left panel and  $\{\lambda, Z\}$  in region **UB** (**US**) of the right panel of figure 4, there exists a **unique** steady state under autarky where  $w_A < \bar{w}_A$  ( $w_A > \bar{w}_A$ ) and the borrowing constraints are **binding** (**slack**).<sup>15</sup>

In figure 5, the solid curves show respectively the laws of motion for wage in these two cases, while the dashed curves show those in the case of  $\lambda = 1$ . For  $w_t \in (0, \bar{w}_A)$ , the solid curve lies below the dashed curve and the gap reflects the efficiency losses,  $(1 - \Gamma_t^\alpha) \left(\frac{w_t}{\rho}\right)^\alpha$ .

<sup>15</sup>The proof of proposition 1 analyzes the case of  $\{\alpha, \theta\}$  in region M where multiple steady states arise.

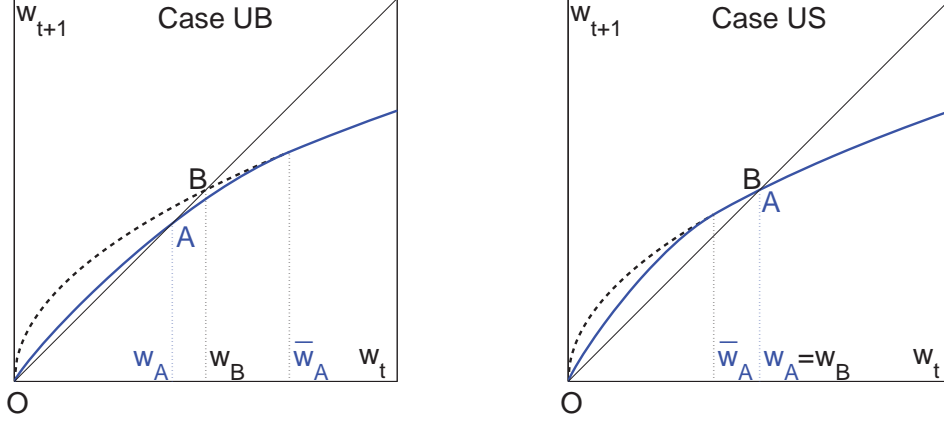


Figure 5: Laws of Motion for Wage under Autarky:  $\theta \geq \underline{\theta}$

### 3.1 The World Economy under Autarkic Equilibrium

North and South are inherently identical, except that North is more financially developed and its population share in the world economy is negligible.

**Assumption 1.**  $0 < \lambda^* < \lambda < \tilde{\lambda}_A$  and  $\frac{L}{L+L^*} \rightarrow 0$ , given  $\theta > \underline{\theta}$ .<sup>16</sup>

Given assumption 1, the parameter configurations of both countries are in region U of the left panel and in region UB of the right panel of figure 4. In each country, there has a unique, autarkic steady state where the borrowing constraints are binding.

Under autarky, the interest rate is coupled with the rate of return in sector 0,

$$r_t = q_{0,t+1} = \Upsilon_t[1 - \eta(1 - \mu_{t+1})] < \Upsilon_t. \quad (26)$$

**Lemma 2.**  $w_A^* < w_A < w_B$ ,  $\chi_A^* < \chi_A < 1$ ,  $\mu_A^* < \mu_A < 1$ , and  $r_A^* < r_A < \rho$ .

A higher level of financial development not only allows each entrepreneur to borrow and invest more, but also allows more agents to meet the MIR and invest in sector 1. The improvement of investment composition  $\frac{\partial \mu_{t+1}}{\partial \lambda} > 0$  enhances the aggregate efficiency,  $\frac{\partial \Upsilon_t}{\partial \lambda} > 0$ . In period  $t + 1$ , higher aggregate output implies higher aggregate saving, which further improves investment composition and production efficiency. In the long run, national income is higher  $\frac{\partial w_A}{\partial \lambda} > 0$  and so is the sectoral price ratio,  $\frac{\partial \chi_A}{\partial \lambda} > 0$ .

The dashed curve in figure 6 shows the steady-state sectoral outputs as the functions of  $\lambda \in [0, \tilde{\lambda}_A)$ . At the autarkic steady state, the (absolute) gradient of the PPF measures the sectoral output-price ratio, which rises in  $\lambda$ . This way, **the level of financial development is a determinant of comparative advantage** in this model, i.e., good 1 (0) is cheaper in North (South) at the autarkic steady state.

Financial development affects the interest rate in three ways. First, it allows each entrepreneur to borrow more as well as allows more agents to become entrepreneurs. The higher (lower) aggregate credit demand (supply) on the *intensive* and *extensive margins* leads to a higher interest rate. I call it the **IEM** effect. Second, by reallocating domestic investment towards the sector with the higher return, financial development improves

<sup>16</sup>By definition, the composite parameter  $Z$  is independent of the level of financial development and the population size. Thus,  $Z$  takes the same value for both countries.



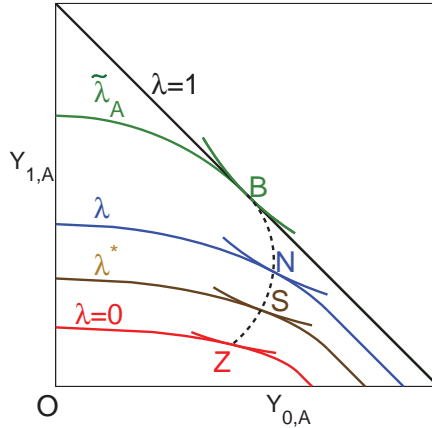


Figure 6: The Steady-State Pattern of Sectoral Outputs under Autarky:  $\lambda \in [0, 1]$

aggregate efficiency and raises the social rate of return, leading to a higher interest rate in period  $t$ . Third, higher aggregate output in period  $t + 1$  implies higher domestic saving and investment. Due to the decreasing MRK effect, the social rate of return declines in the long run and so does the interest rate.

At the autarkic steady state, the decreasing MRK effect and the aggregate efficiency effect cancel out.<sup>17</sup> Due to the positive **IEM** effect, the interest rate is higher in North than in South. Starting from the autarkic steady state, if agents are allowed to borrow and lend abroad<sup>18</sup>, financial flows are “uphill” from South to North, which widens the cross-country income gap. Thus, the “global imbalances” arise in this model as an equilibrium response to heterogeneous financial development across countries.

## 4 Can Free Trade Resolve the Global Imbalances?

The world economy is initially at the autarkic steady state. In period 0, the two countries announce that goods 0 and 1 will be freely traded from period 1 on.<sup>19</sup> Due to its negligible world population share, North is a small open economy. Thus, free trade does not affect the equilibrium allocation in South. In the absence of trade costs, the sectoral price ratio in North is aligned to the world level from period  $t = 1$  on,  $\chi_t = \chi^* = \chi_A^*$  and so is the sectoral return ratio,  $\mu_t = \chi_t^{\frac{1}{\alpha}} = (\chi^*)^{\frac{1}{\alpha}} = \mu^*$ .<sup>20</sup> Without international factor mobility, trade is balanced and the social rate of return is still characterized by equation (21).

<sup>17</sup>Due to inelastic aggregate saving  $w_t L$  and the Cobb-Douglas production functions, the social rate of return at the autarkic steady state is constant at  $\rho$ , independent of  $\lambda$ . See equation (21).

In von Hagen and Zhang (2014), aggregate saving is elastic and the extensive margin is mute. At the autarkic steady state, the decreasing MRK effect dominates the aggregate efficiency effect so that the social rate of return declines in the level of financial development. Nevertheless, the intensive-margin effect dominates so that the interest rate is still higher in the more financially developed country.

<sup>18</sup>In this case, the final good is tradeable and serves as the vehicle for international borrowing/lending.

<sup>19</sup>If free trade is announced and implemented in period 0,  $\chi_0$  is aligned immediately with  $\chi_A^*$ , which unexpectedly affects the investment return of those born in period  $t = -1$ . In the two-period OLG setting, announcing the policy one period in advance avoids such an uncertainty.

<sup>20</sup>Subsection 5.2 analyzes explicitly the role of trade costs in determining the degree of specialization.

## 4.1 Specialization as An Amplification Mechanism

**Lemma 3.** *If  $w_t < \bar{w}$  ( $w_t \geq \bar{w}$ ), North specializes partially (fully) in sector 1 under trade.*

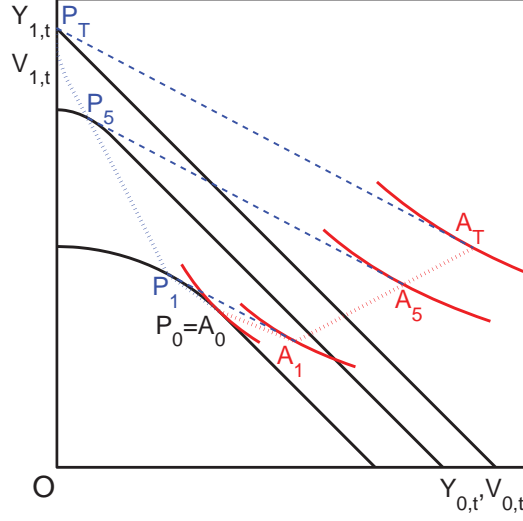


Figure 7: The Equilibrium Dynamics in North under Free Trade

Figure 7 shows the sectoral dynamics in North, where  $P_t \equiv (Y_{0,t}, Y_{1,t})$  denotes domestic production and  $A_t \equiv (V_{0,t}, V_{1,t})$  denotes domestic absorption of the two sectors. In period 0, North is at the autarkic steady state with  $A_0 = P_0$  and  $MRS_{0,1} = MRT_{0,1} = \chi_A$ . The announcement of free trade does not affect the sectoral investment in period 0,  $\delta_0 = \left(\frac{w_0}{\bar{w}}\right)^{\frac{1-\theta}{\theta}} = \delta_A$ , so that the PPF in period 1 is the same as before. By aligning the sectoral price ratio to the world level  $\chi_1 = \chi^* < \chi_A$ , free trade induces North to import (export) good 0 (1) in period 1, while the decline (rise) in the MRL in sector 0 (1) reallocates labor from sector 0 to 1,  $\zeta_1 = \frac{1}{1+(\frac{1}{\delta_A}-1)(\chi^*)^{\frac{1}{\alpha}}} > \zeta_A$ . Thus, domestic production moves from point  $P_0$  to  $P_1$ . Trade decouples domestic absorption from domestic production and the national income line (the dashed line linking  $P_1$  and  $A_1$ ) is flatter than in period 0,  $\chi_1 = \chi^* < \chi_0 = \chi_A$ . The static gains from trade raise national income, as shown by the fact that domestic absorption  $A_1$  is on the isoquant higher than  $A_0$ . It allows more agents to overcome the MIR and invest in sector 1,  $\delta_1 = \left(\frac{w_1}{\bar{w}}\right)^{\frac{1-\theta}{\theta}} > \delta_0$ , which enhances North's comparative advantage and induces it to specialize further towards sector 1 along the labor margin in period 2,  $\zeta_2 = \frac{1}{1+(\frac{1}{\delta_1}-1)(\chi_A^*)^{\frac{1}{\alpha}}} > \zeta_1$ .

This way, the static gains from trade in period 1 trigger a dynamic, virtuous cycle, i.e., the rise in national income improves North's comparative advantage along the extensive margin of sectoral investment and the enhanced specialization in the next period raises national income further. I call it the *trade-driven investment reallocation effect*. As long as it dominates the decreasing MRK effect, such an amplification process propagates over time until the mass of entrepreneurs becomes so large that their borrowing capacity exceeds the entire household labor income and sector 0 vanishes in North, i.e.,  $w_t \geq \bar{w}$  and  $\delta_t = 1$ . From then on, the decreasing MRK effect brings North to a new steady state.

As explained in section 2, higher national income affects the PPF in the next period through the investment scale effect and the investment composition effect. The former

moves the PPF parallel away from the origin, while the latter extends the linear part of the PPF by raising  $\tilde{\delta}_t$ , given  $w_t \leq \bar{w}$ . Let  $X_T$  denote the steady-state value of variable  $X_t$  under trade integration. In figure 7, the dotted line linking  $P_1$  and  $P_T$ , i.e., the path of domestic sectoral production, eventually aligns with the vertical axis, implying that North offshores sector 0 at some point in time. The dotted line linking  $A_1$  and  $A_T$  features the path of domestic sectoral absorption, showing the rise in national income over time.

The dynamic impacts of trade on national income can be illustrated by the law of motion for wage, which is piecewise and characterized by equation (27),

$$w_{t+1} = \left( \frac{1}{\rho} w_t \Gamma_t \right)^\alpha, \text{ where } \Gamma_t = (\mu^*)^\eta [1 + (\frac{1}{\mu^*} - 1)\delta_t], \delta_t = \begin{cases} (\frac{w_t}{\bar{w}})^{\frac{1-\theta}{\theta}}, & \text{if } w_t < \bar{w}; \\ 1 & \text{if } w_t \geq \bar{w}. \end{cases} \quad (27)$$

$$\frac{\partial \ln w_1}{\partial \ln \chi^*} = \frac{\partial \ln \Gamma_0}{\partial \ln \mu^*} = \frac{\mu^* - \mu_A}{\frac{\mu^*}{\eta} + \frac{\mu_A}{1-\eta}}, \quad (28)$$

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} + \underbrace{\alpha \frac{\partial \ln \Gamma_t}{\partial \ln \delta_t} \frac{\partial \ln \delta_t}{\partial \ln w_t}}_{\text{investment reallocation effect}} = \alpha + \alpha \frac{\frac{\partial \ln \delta_t}{\partial \ln w_t}}{1 + \frac{1}{(\frac{1}{\mu^*} - 1)\delta_t}}, \quad (29)$$

$$\text{where } \frac{\partial \ln \delta_t}{\partial \ln w_t} = \begin{cases} \frac{1-\theta}{\theta}, & \text{if } w_t < \bar{w}; \\ 0 & \text{if } w_t \geq \bar{w}. \end{cases} \quad (30)$$

Given  $\mu^* = \mu_A^* < \mu_A$ ,  $\frac{\partial \ln w_1}{\partial \ln \chi^*} < 0$ . Thus, the static gains from trade raise national income in period 1  $w_1 > w_0$  by a *downward alignment* of  $\mu_1 = \mu^* < \mu_A$ . From period 1 on, national income is driven by the competition of the investment reallocation effect and the decreasing MRK if  $w_t < \bar{w}$ , while it is driven by the decreasing MRK effect if  $w_t \geq \bar{w}$ .

**Proposition 2.** *Free trade may induce North to converge from the autarkic steady state to a unique steady state where it offshores sector 0 and fully specializes in sector 1.*

Case **UF1** denotes the one described in proposition 2. It arises if the static gains from trade in period 1 and the subsequent investment reallocation effect are sufficiently large.

- The larger the cross-country differences in financial development  $\lambda - \lambda^*$ , the larger the cross-country differences in the sectoral price ratio  $\chi_A - \chi_A^*$ , the larger the trade flows and the static gains in period 1. See equation (28).
- The lower the  $\lambda^*$ , the lower the  $\chi^*$  and the  $\mu^*$ , the larger the sectoral price differential and the sectoral rate-of-return differential on the world market, the larger the gains for North by specializing towards sector 1, the stronger the investment reallocation effect. See equation (29).
- Given  $\theta \geq \underline{\theta}$ , the lower the  $\theta$ , the less dispersed the wealth distribution, the more sensitive the mass of entrepreneurs and sectoral investment to the static gains and income changes, the stronger the investment reallocation effect. See equation (30).

Case UF1 arises for  $\{\theta, Z, \lambda^*, \lambda\}$  in region UF1 of figure 8, i.e.,  $\theta \in [\underline{\theta}, \tilde{\theta})$ ,  $\lambda^* < \tilde{\lambda}_T^*$ , and  $\lambda > \bar{\lambda}_T$  where the threshold value  $\bar{\lambda}_T > \lambda^*$  increases in  $\lambda^*$ . In the following, I focus on case UF1 and analyze the dynamic responses of national income and the interest rate with respect to free trade.<sup>21</sup>

<sup>21</sup>Appendix A gives a complete analysis of the dynamic and steady-state properties under free trade.

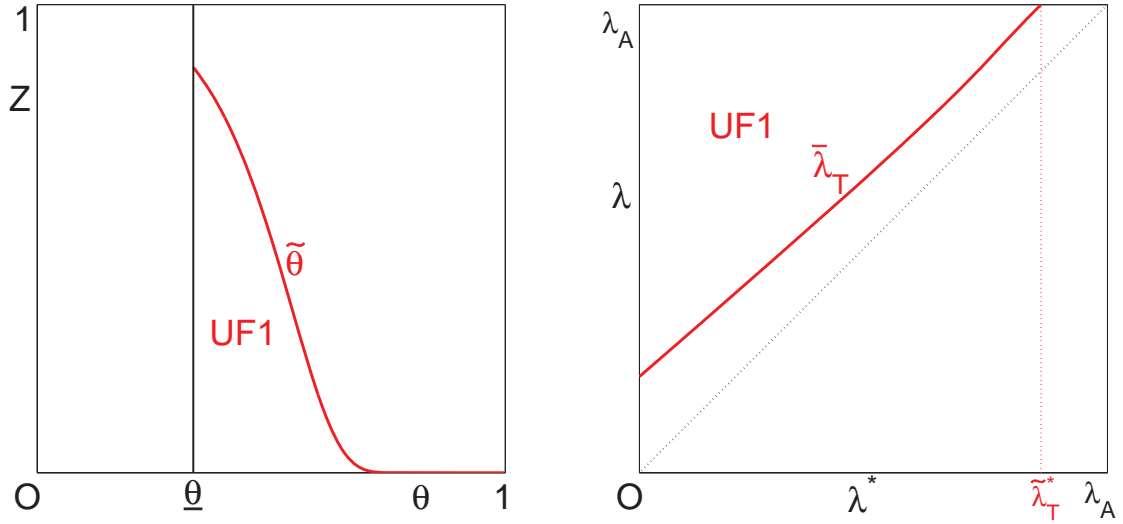


Figure 8: Threshold Values for the Free-Trade Equilibrium

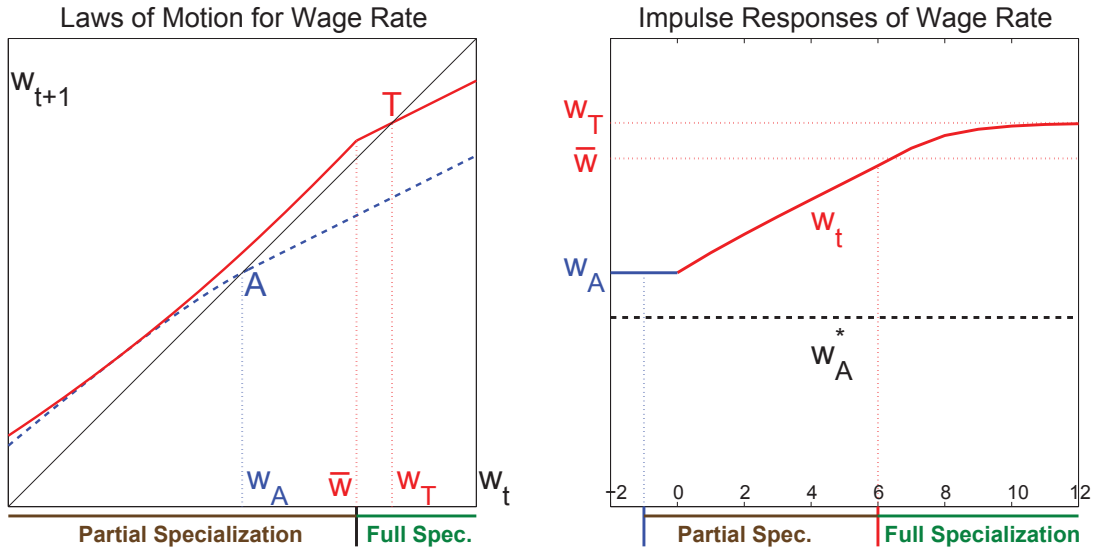


Figure 9: Dynamics of National Income: From Autarkic Steady State to Free Trade

In figure 9, the solid (dashed) curve in the left panel shows the law of motion for wage under trade (autarky), while the solid curve in the right panel shows the impulse responses of wage under trade.<sup>22</sup> For period  $t \leq 6$ ,  $w_t < \bar{w}$  and hence,  $\delta_t < 1$ . As the investment reallocation effect dominates the decreasing MRK effect, national income rises over time. For period  $t \geq 7$ ,  $w_t > \bar{w}$  and North offshores sector 0,  $\delta_t = 1$ . Then, the decreasing MRK effect brings North to steady state T where the wage rate is  $w_T$ .

## 4.2 Interest Rate Reversal and Re-reversal under Free Trade

**Lemma 4.** *Under free trade, the interest rate in North is a piecewise function of national income, depending on whether North offshores the low-return sector.*

- 1.) For  $w_t \geq \bar{w}$ , the mass of entrepreneurs is so large that their total debt capacity

<sup>22</sup>For illustration clarity, the axes in the left panel are scaled in logarithm and so is the vertical axis in the right panel. This scaling approach also applies to figures 10, 14, and 15.

exceeds the entire household labor income and the aggregate credit demand pushes the interest rate equal to the rate of return in sector 1,

$$r_t = q_{1,t+1} = \Upsilon_t = \frac{\rho w_{t+1}}{w_t}, \quad \frac{\partial \ln r_t}{\partial \ln w_t} = - \underbrace{(1 - \alpha)}_{\text{decreasing MRK effect}} < 0. \quad (31)$$

The higher the national income, the higher the domestic saving and investment, the lower the MRK in sector 1, the lower the rate of return in sector 1 and the interest rate.

2.) For  $w_t \in (0, \bar{w})$ , the mass of entrepreneurs is so small that their total debt capacity is below the entire household labor income. Besides lending to entrepreneurs, households invest the rest of their labor income in sector 0. According to the no-arbitrage condition,

$$r_t = q_{0,t+1} = \frac{\Upsilon_t}{1 + \frac{1-\mu^*}{\mu^*} \delta_t}, \quad \frac{\partial \ln r_t}{\partial \ln w_t} = - \underbrace{(1 - \alpha)}_{\text{decreasing MRK effect}} - \underbrace{\frac{(1 - \alpha) \left(\frac{1}{\theta} - 1\right)}{1 + \frac{1}{(\frac{1}{\mu^*} - 1)\delta_t}}}_{\text{reallocation effect}} < 0. \quad (32)$$

In period 1, trade only triggers the labor reallocation, i.e.,  $\zeta_1 > \zeta_A$  and  $\delta_0 = \delta_A$ , leading to a higher capital-labor ratio in sector 0,

$$\frac{K_{0,1}}{L_{0,1}} = \frac{1 - \delta_0}{1 - \zeta_1} \frac{w_0 L}{L} = \left[ 1 + \delta_A \left( \frac{1}{\mu^*} - 1 \right) \right] w_0 > \frac{K_{0,0}}{L_{0,0}} = \left[ 1 + \delta_A \left( \frac{1}{\mu_A} - 1 \right) \right] w_A.$$

Meanwhile, the price of good 0 falls in period 1,  $p_{0,1} = \chi_1^\eta = (\chi^*)^\eta < p_{0,0} = \chi_A^\eta$ . Both effects lead to a fall in the sector 0's rental price of capital,

$$q_{0,1} = p_{0,1} \frac{\alpha Y_{0,1}}{K_{0,1}} = p_{0,1} \left( \frac{K_{0,1}}{\rho L_{0,1}} \right)^{\alpha-1} < q_{0,0} = p_{0,0} \left( \frac{K_{0,0}}{\rho L_{0,0}} \right)^{\alpha-1}.$$

The interest rate falls in period 0, too. I call it the reallocation effect. It dominates the **IEM** effect so that the interest rate in North is even lower than the world level,  $r_0 < r_A^* < r_A$ , a result Antras and Caballero (2009) calls **the interest rate reversal**.

From period  $t = 1$  on, trade triggers sectoral reallocation of labor and investment. Labor reallocation is frictionless, while it is not the case for investment. Besides the scale effect, the rise in national income also raises the capital-labor share in sector 0 through the disproportional sectoral reallocation of labor and investment. Use equation (6) to get

$$\frac{K_{0,t+1}}{L_{0,t+1}} = \frac{1 - \delta_t}{1 - \zeta_{t+1}} \frac{w_t L}{L} = \underbrace{\left[ 1 + \delta_t \left( \frac{1}{\mu^*} - 1 \right) \right]}_{\text{reallocation effect}} \underbrace{w_t}_{\text{scale effect}}.$$

Thus, the reallocation effect and the decreasing MRK effect jointly reduce the MRK in sector 0 over time and the interest rate falls further.

The left panel of figure 10 shows the interest rate as a piecewise function of national income, while the right panel shows the interest rate responses to free trade. In period 0, the announcement of free trade leads to the interest rate reversal,  $r_0 < r_A^* < r_A$ . From period 1 on, the factor reallocation effect and the decreasing MRK effect reduce the interest rate further. This process continues as long as sector 0 is active. In period 7,

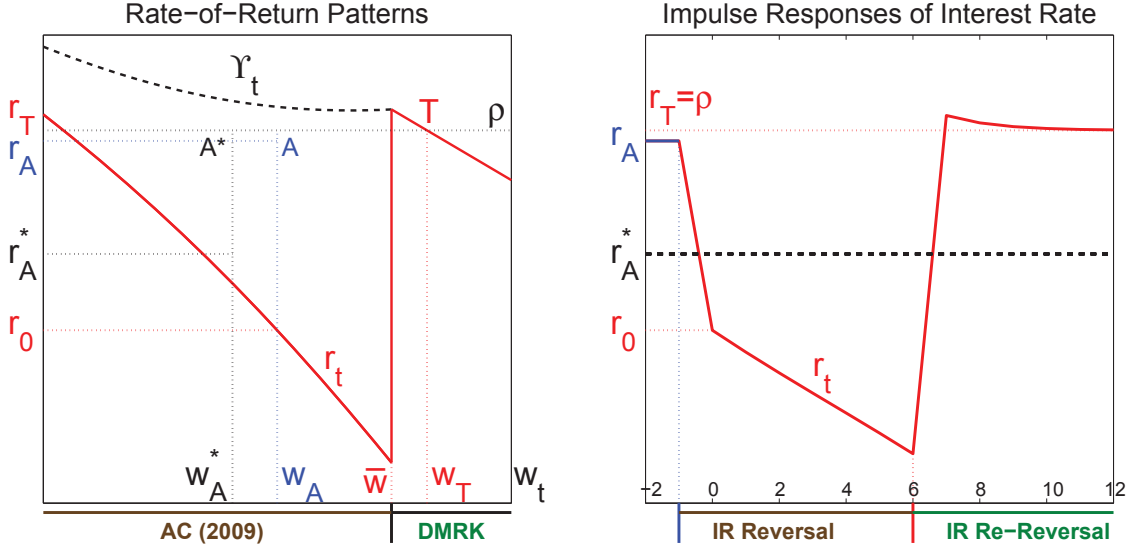


Figure 10: The Interest Rate Dynamics: From Autarkic Steady State to Free Trade

North offshores sector 0 and the interest rate is decoupled from (coupled with) the rate of return to sector 0 (1), featured by an interest rate jump in the right panel. I call it **the interest rate re-reversal**. Then, the decreasing MRK effect reduces the interest rate over time until North reaches the new steady state with  $r_T = \Upsilon_T = \rho > r_A > r_A^*$ .<sup>23</sup>

**Proposition 3.** *In case UF1, North may witness a non-monotonic interest rate pattern during its transition towards the new steady state under free trade. The interest rate in the new steady state is higher than in the autarkic one,  $r_T > r_A > r_A^*$ .*

For the parameter configurations outside region UF1 of figure 8, the static gains and the investment reallocation effect are too weak to ensure that North offshores sector 0, when starting from the autarkic steady state. Then, trade leads to the interest rate reversal, as predicted by Antras and Caballero (2009). To sum up, whether trade lead to the interest rate re-reversal depends on how far it reshapes North's sectoral composition.

### 4.3 FPE and The Timing of Economic Integration

In Antras and Caballero (2009), financial frictions distort domestic allocation along two dimensions. Along the intratemporal dimension, the deviation of the sectoral price ratio from its efficient level ( $1 - \chi_t$ ) reflects the distortions on cross-sector investment allocation; along the intertemporal dimension, the deviation of the interest rate from the social rate of return ( $1 - \frac{r_t}{\Upsilon_t}$ ) reflects the distortion on aggregate credit demand. Trade alone only equalizes across countries the intertemporal distortion, while capital mobility alone only equalizes across countries the intratemporal distortion. In either case, factor prices (i.e., the wage rate, the interest rate, etc) do not equalize across countries. If trade and capital

<sup>23</sup>When North offshores sector 0 and specializes fully in sector 1, the borrowing constraints are slack in North. Section 5.3 generalizes this two-sector setting into a multi-sector setting. When trade induces North to offshore the lowest-return sector, the interest rate is coupled with the rate of return in the second lowest-return sector, while the borrowing constraints are still binding in all other sectors.

flows are allowed simultaneously, the distortions are equalized along both dimensions, which equalizes across countries the factor prices as well as income per capita.

Their results can be shown in my model. According to the right panels of figure 9 and 10, trade induces North to specialize *partially* towards sector 1 until period 6, with the interest rate reversal. If agents can borrow/lend abroad before or in period 6, financial flows are “downhill” from North to South and the interest rate in North is aligned to the world level,  $r_t = r_t^*$ . Due to the interest rate coupling, the rate of return in sector 0 is also equalized,  $q_{0,t+1} = r_t = r_t^* = q_t^{0,*}$ . Meanwhile, trade equalizes the sectoral price ratio  $\chi_t = \chi_t^*$ , which implicitly equalizes the sectoral rental-price-of-capital ratio,  $\mu_t = \chi_t^{\frac{1}{\alpha}} = (\chi_t^*)^{\frac{1}{\alpha}} = \mu_t^*$ . This way, trade and capital mobility jointly equalize the rate of return in sector 1,  $q_{1,t} = \frac{q_{0,t}}{\mu_t} = \frac{q_{0,t}^*}{\mu_t^*} = q_{1,t}^*$ . Despite international labor immobility, the wage rate is equalized across countries and so is income per capita,<sup>24</sup>

$$\frac{Y_t}{2L} = \frac{w_t}{2(1-\alpha)} = \frac{(q_{0,t}^{1-\eta} q_{1,t}^\eta)^{-\rho}}{2(1-\alpha)} = \frac{[(q_{0,t}^*)^{1-\eta} (q_{1,t}^*)^\eta]^{-\rho}}{2(1-\alpha)} = \frac{w_t^*}{2(1-\alpha)} = \frac{Y_t^*}{2L^*}.$$

Here, the factor price equalization (FPE, hereafter) and cross-country income convergence depend critically on the fact that trade only leads to partial specialization and hence, the interest rate is coupled with the sector 0’s rate of return before and in period 6.

Suppose that agents are allowed to borrow/lend abroad only from period 7 on. Since North has offshored sector 0, the interest rate is decoupled from the rate of return there. Thus, the interest rate equalization does not imply the cross-country equalization of sector 0’s rate of return, which invalidates the aforementioned mechanism. In fact, the interest rate is re-reversed from period  $t \geq 7$  on. If allowed, financial flows are “uphill” from South to North, which further widens the cross-country income gap. This is in stark contrast to the findings of Antras and Caballero (2009).

To sum up, whether trade and capital mobility jointly lead to the FPE depends critically on the **timing of integration**, i.e., whether the more financially developed country has offshored the low-return sector upon financial integration.

## 5 Discussions and Extensions

The possibility that trade induces North to offshore sector 0 and leads to the interest rate re-reversal depends critically on the endogenous responses of  $\delta_t$  to the static gains from trade which then depend on the international sectoral price differentials. In this section, I first compare two channels through which the static gains from trade affect  $\delta_t$  and then discuss the roles of trade costs and the initial conditions of South in determining the international sectoral price differentials. In the end, I show in a three-sector model that the interest rate reversal and re-reversal may arise recurrently if trade induces North to sequentially offshore the low-return sectors and upgrade to the high-return sectors.

<sup>24</sup>In period  $t$ , each country is populated with the young and the old of the same mass. Thus, the size of total population is  $2L$  in North and  $2L^*$  in South, respectively.

## 5.1 Endogenous Responses of $\delta_t$ through Two Channels

Let  $W_t^e$  and  $W_t$  denote respectively entrepreneurial wealth and national wealth. In the case of the binding borrowing constraints, the fraction of domestic investment in sector 1,  $\delta_t = \frac{W_t^e}{W_t} = \frac{1}{1-\lambda} \frac{W_t^e}{W_t}$ , is proportional to the leverage multiplier  $\frac{1}{1-\lambda}$  and the entrepreneurial wealth share  $\frac{W_t^e}{W_t}$ . The former is constant by assumption, while the latter is higher if the mass of entrepreneurs is higher (the **extensive** margin) and/or if the average wealth of entrepreneurs rises relative to the national average (the **intensive** margin). As shown below,  $\delta_t$  may respond to the static gains from trade through these two margins.

In my model, agents live for **two** periods; each agent invests its entire labor income when young and consumes when old. Due to the absence of wealth accumulation at the individual level, agent  $j$ 's wealth relevant for investment is just its labor income,  $n_{j,t} = w_t(1-\theta)\epsilon_j$ . One can decompose along two margins the impacts of the static gains from trade on the entrepreneurial wealth share in North,

$$\begin{aligned} \frac{W_1^e}{W_1} - \frac{W_A^e}{W_A} &= \underbrace{\frac{\int_{\underline{\epsilon}_A}^{\infty} n_{j,1} dG(\epsilon_j)}{w_1 L} - \frac{\int_{\underline{\epsilon}_A}^{\infty} n_{j,A} dG(\epsilon_j)}{w_A L}}_{\text{the intensive margin effect}} + \underbrace{\frac{\int_{\underline{\epsilon}_1}^{\underline{\epsilon}_A} n_{j,1} dG(\epsilon_j)}{w_1 L}}_{\text{the extensive margin effect}} \quad (33) \\ &= \underbrace{\frac{w_1 L \tau_A^{1-\theta}}{w_1 L} - \frac{w_A L \tau_A^{1-\theta}}{w_A L}}_{\text{the intensive margin effect (0)}} + \underbrace{\frac{w_1 L (\tau_1^{1-\theta} - \tau_A^{1-\theta})}{w_1 L}}_{\text{the extensive margin effect (+)}}. \quad (34) \end{aligned}$$

Consider the agents who are born in period 1 and would meet the MIR at the autarkic steady state, i.e.,  $\epsilon_j \geq \underline{\epsilon}_A$ . The static gains affect in equal proportions the wealth of these “existing” entrepreneurs,<sup>25</sup>  $n_{j,t} = w_t(1-\theta)\epsilon_j$ , and the national wealth,  $w_t L$ , through the wage rate. Since the static gains from trade do not affect the national wealth share of “existing” entrepreneurs, the intensive margin is mute. Due to financial frictions and sector-specific MIR, the cutoff value  $\underline{\epsilon}_t$  and the mass of entrepreneurs  $\tau_t = \underline{\epsilon}_t^{-\frac{1}{\theta}}$  are endogenous. Thus, the static gains raise  $\frac{W_t^e}{W_t}$  purely through the extensive margin, which then raises  $\delta_t = \frac{1}{1-\lambda} \frac{W_t^e}{W_t}$  and creates the possibility of North offshoring sector 0.

In the static model of Antras and Caballero (2009), the extensive margin of entrepreneurial wealth is mute, due to two assumptions: **(1)** there is no MIR; **(2)** only a fixed mass  $\tau$  of agents can invest in the constrained sector and are called entrepreneurs. In the case of the homogeneous labor endowment,  $l_j = 1$ , one can embed this static model into the two-period OLG setting and the wealth share of entrepreneurs is constant at  $\frac{W_t^e}{W_t} = \frac{\tau w_t L}{w_t L} = \tau$ . Under assumption 1 of Antras and Caballero (2009), the borrowing constraints are so tight that the investment share of the constrained sector is inefficiently low,  $\delta = \frac{\tau}{1-\lambda} < \eta$ . As  $\delta$  is constant and does not respond to the static gains from trade, the unconstrained sector is always active,  $\frac{K_{0,t+1}}{w_t L} = 1 - \delta > 1 - \eta > 0$  and the interest rate reversal never happens. Thus, **the endogenous extensive margin of entrepreneurial wealth** in this two-period OLG model is key to the interest rate re-reversal.

Antras and Caballero (2009) embed their static model into a dynamic, continuous-time setting with two key assumptions: **(1)** agents are born at a constant rate per unit of

<sup>25</sup>Referring these agents as “existing” entrepreneurs is purely for ease of exposition. It may be misleading, because they are born in period 1 and did not “exist” in period 0.



time and die at the same rate, **(2)** agents save all their (labor and investment) income and consume only when they die. Due to the law of large numbers, the mass of entrepreneurs is constant and hence, the extensive margin of entrepreneurial wealth is mute. Different from the two-period OLG setting, agents actually accumulate wealth over their lifetime. Due to the privilege of investing in the constrained sector at a higher rate of return, entrepreneurs accumulate wealth at a faster speed than others under autarky and hence, their wealth share  $\frac{W_t^e}{W_t}$  becomes endogenous on the intensive margin. By aligning the sectoral price ratio at the world level, trade raises (reduces) the sectoral rate-of-return differential in North (South), which induces entrepreneurs to accumulate wealth at a faster (slower) speed than in the autarkic steady state. In this case, trade affects the entrepreneurial wealth share along **the intensive margin** and  $\delta_t$  responds accordingly.

Antras and Caballero (2009) focus on the interest rate response to trade flows for the **less** financially developed country. As both sectors are always active there, the interest rate reversal always holds. They do not analyze explicitly whether and under what conditions trade can induce the **more** financially developed country to offshore the low-return sector.<sup>26</sup> In contrast, I highlight the **extensive margin** as a key channel through which the entrepreneurial wealth share responds to the static gains and show that trade may lead to the interest rate re-reversal if it substantially changes the sectoral composition in the **more** financially developed country.<sup>27</sup> It complements their findings.

## 5.2 The Implications of Trade Costs and Initial Conditions

So far, I have made two simplifying assumptions: **(1)** there is no trade cost, and **(2)** the world economy is initially at the autarkic steady state. Given  $\{\theta, Z\}$  in region UF1 of the left panel of figure 8, the left panel of figure 11 shows that case UF1 arises if  $\chi^* < \tilde{\chi}_T^*$  and  $\chi_A > \bar{\chi}_T$ . Let  $\underline{\chi}_A$  denote the sectoral price ratio at the autarkic steady state in the case of  $\lambda = 0$ .<sup>28</sup> It holds that  $\chi_A, \chi_A^* \geq \underline{\chi}_A$ .

Under the two assumptions, the world sectoral price ratio that North faces is  $\chi^* = \chi_A^*$ . Then, one can convert the threshold values ( $\tilde{\chi}_T^*$  and  $\bar{\chi}_T$ ) into the  $\{\lambda^*, \lambda\}$  space, as shown by the right panel of figure 8. In the following, I discuss how trade costs and South's initial condition affect the likelihood of case UF1 for North.

Let us first consider the trade costs. Suppose that the world economy is initially at the autarkic steady state as before. Let  $\sigma \in (0, 1)$  denote the proportional trade costs. Although trade costs do not affect the autarkic relative price in each country, they raise the world sectoral price ratio that North faces upon trade liberalization  $\chi^* = \chi_A^* \frac{1+\sigma}{1-\sigma}$ , which narrows the international sectoral price differential,  $\chi_A - \chi^* = \chi_A - \chi_A^* \frac{1+\sigma}{1-\sigma} < \chi_A - \chi_A^*$ , and reduces the trade flows in period 1. This way, trade costs weaken the static gains from trade and the subsequent investment reallocation effect. For the existence of case UF1,  $\chi_A^*$  needs to be lower than in the absence of trade costs ( $\sigma = 0$ ) and so does  $\lambda^*$ . In the right panel of figure 11, trade costs reduce the threshold value  $\tilde{\lambda}_T^*$  and raise the

<sup>26</sup>It is beyond the scope of my paper to explore this possibility in their dynamic model.

<sup>27</sup>Allowing wealth accumulation at the individual level in my model would strengthen my results through the additional, positive intensive margin effect.

<sup>28</sup>It is the (absolute) gradient of the PPF at point Z in figure 6.

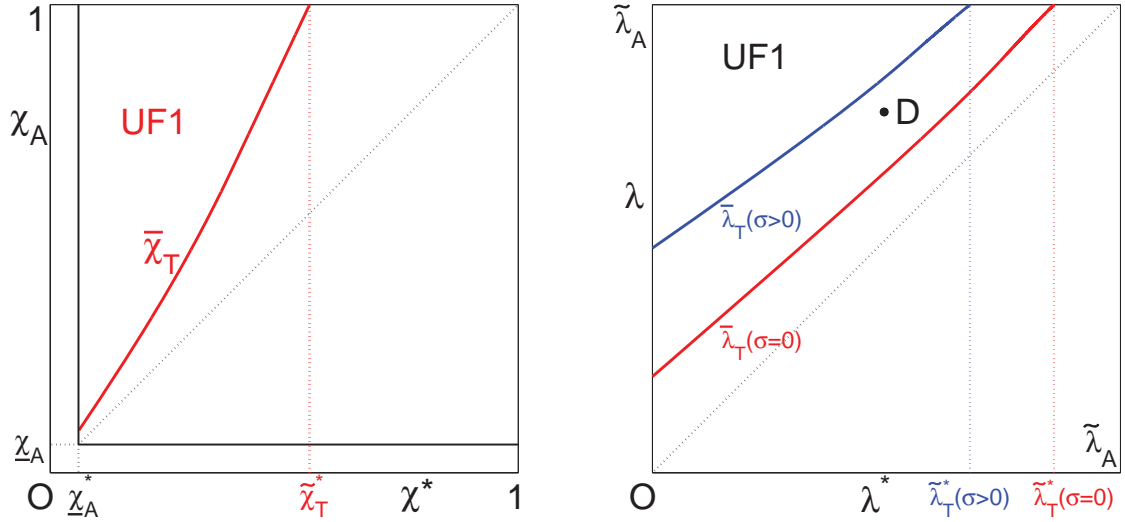


Figure 11: Threshold Values for the Trade Equilibrium with Trade Costs  $\sigma \in (0, \bar{\sigma})$

threshold value  $\bar{\lambda}_T$ . The larger the trade costs, the smaller the region UF1 in the right panel of figure 11, the less likely case UF1 can arise.

**Lemma 5.** *Let  $\bar{\sigma} \equiv \frac{\eta \frac{(1-\alpha)\theta}{(1-\eta)(1-\theta)} - \chi_A}{\eta \frac{(1-\alpha)\theta}{(1-\eta)(1-\theta)} + \chi_A}$ . For  $\sigma \in [\bar{\sigma}, 1]$ , case UF1 does not arise. For  $\sigma \in (0, \bar{\sigma})$ , the interest rate in North may have a non-monotonic pattern with regard to trade costs.*

Take as an example the parameter configuration represented by point D in the right panel of figure 11. For the sufficiently large trade costs, point D is outside of region UF1 and trade does not induce North to offshore sector 0. By reducing  $\chi^* = \chi_A^* \frac{1+\sigma}{1-\sigma}$ , a marginal decline in trade costs widens the international sectoral price differential,  $\chi_A - \chi^* = \chi_A - \chi_A^* \frac{1+\sigma}{1-\sigma}$ , which raises the trade flows and allows North to specialize further towards sector 1. Meanwhile, the decline in trade costs reduces the threshold value  $\bar{\lambda}_T$  and shifts rightwards the border of region UF1, making case UF1 more likely. If the border is far above point D, a marginal decline in trade costs reduces  $\chi^*$  and the interest rate in North falls, as mentioned in subsection 4.2. However, if the decline in trade costs is so large that the border of region UF1 is below point D, North eventually offshores sector 0, leading to the interest rate re-reversal. This way, deepening trade integration may have the non-monotonic effect on the interest rate in North.

Next, let us consider the role of South's initial conditions. Suppose that there is no trade cost ( $\sigma = 0$ ) and North is initially at the autarkic steady state ( $w_0 = w_A$ ) as before, while South is below its autarkic steady state ( $w_0^* < w_A^*$ ). It is a relevant case for many emerging economies, e.g., China and Emerging Asia. Take as an example the parameter configuration represented by point T in the left panel of figure 12. Since point T is outside of region UF1, trade integration with South does not induce North to offshore sector 0, if South is initially at the autarkic steady state. However, if South is initially below the autarkic steady state,  $w_0^* < w_A^*$  implies  $\mu_1^* < \mu_A^*$  and  $\chi_1^* < \chi_A^*$ , according to equations (13) and (24). Then, case UF1 may arise for North in the short run.

The right panel of figure 12 shows the threshold values in the  $\{\chi_t^*, \chi_A\}$  space, while point A and T denote the sectoral price ratio in South in period 1 and in the long run,

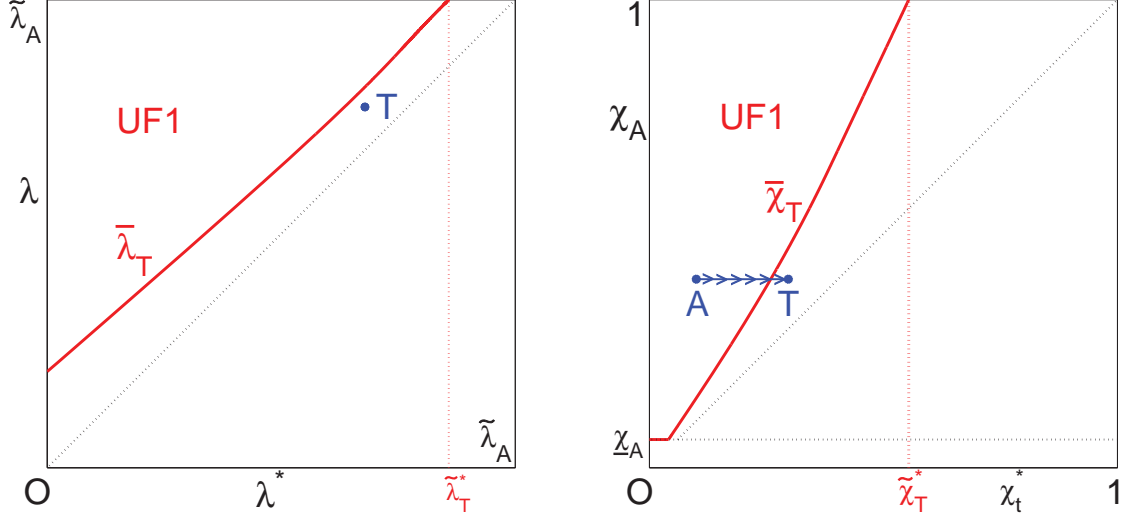


Figure 12: Threshold Values for the Trade Equilibrium with  $w_0^* < w_A^*$

respectively. Starting with point A, trade integration with South may induce North to gradually offshore sector 0 in the medium run if South converges slowly towards its steady state<sup>29</sup> and  $\chi_t^*$  stays in region UF1 for a sufficiently long time. However, when South is sufficiently close to its steady state,  $\chi_t^*$  is so high that trade flows become small. As a result, sector 0 becomes active again in North. In this sense, the economic development in South may affect the sectoral composition in North through trade linkages, which may have significant implications to the global imbalances.

### 5.3 Recurrent Interest Rate Reversal and Re-Reversal

So far, I have shown that the interest rate re-reversal may occur when trade allows North to fully offshore the low-return sector and specialize in the high-return sector. The two-sector setting helps explore the core mechanism. In this section, I embed it into a three-sector setting and show that the interest rate reversal and re-reversal may arise *recurrently* when trade allows North to *sequentially* offshore the low-return sectors.

Each country has three sectors, indexed by  $s \in \{0, 1, 2\}$  and ranked according to the MIR, i.e.,  $\mathbf{m}_2 > \mathbf{m}_1 > \mathbf{m}_0 = 0$ . Let  $\delta_{s,t}$  and  $\zeta_{s,t+1}$  denote the respective shares of domestic saving  $w_t L$  and labor input  $L$  allocated for the production of good  $s$  in period  $t + 1$ ,  $Y_{s,t+1} = \left(\frac{\delta_{s,t} w_t L}{\alpha}\right)^\alpha \left(\frac{\zeta_{s,t+1} L}{1-\alpha}\right)^{1-\alpha}$ . Sectoral outputs are combined to produce final goods,  $Y_{t+1} = \Pi_{s=0}^2 \left(\frac{V_{s,t+1}}{\eta_s}\right)^{\eta_s}$ , where  $\eta_s$  denotes the sectoral share in the final good production and  $\sum_{s=0}^2 \eta_s = 1$ . Let  $\mu_{s,t} \equiv \frac{q_{s,t}}{q_{2,t}}$  and  $\chi_{s,t} \equiv \frac{p_{s,t}}{p_{2,t}}$  denote respectively the rental price of capital and the output price of sector  $s$  relative to those of sector 2.

Let  $\underline{\epsilon}_{s,t} \equiv \frac{1-\lambda}{w_t} \frac{\mathbf{m}_s}{1-\theta}$  and  $\bar{w}_s \equiv \frac{\mathbf{m}_s}{1-\theta} (1-\lambda)^{\frac{1}{1-\theta}}$ . According to equation (11), the agents with  $\epsilon_j \geq \underline{\epsilon}_{s,t}$  can meet the MIR and invest in sector  $v \geq s$ . According to equation (13), for  $w_t \geq \bar{w}_s$ , if the agents with  $\epsilon_j \geq \underline{\epsilon}_{s,t}$  borrow and invest to the limit, the entire domestic saving is invested in sector  $v \geq s$ , while sector  $v < s$  is inactive.

<sup>29</sup>Under assumption 1,  $L^* \gg L$  so that trade integration with North does not affect the income dynamics of South. In the long run, South still converges to its autarkic steady state.

### 5.3.1 Autarkic Equilibrium

As labor is perfectly mobile across sectors, the sectoral labor input is efficient at  $\zeta_{s,t+1} = \eta_s$  under autarky. In the absence of the borrowing constraints, the same would also apply to the sectoral investment input,  $\delta_{s,t} = \zeta_{s,t+1} = \eta_s$  and the cross-sector equalization of the capital-labor ratio  $\frac{\delta_{s,t} w_t L}{\zeta_{s,t+1} L} = w_t$  would lead to the sectoral rate-of-return equalization,  $q_{s,t+1} = r_t$ . As shown below, the borrowing constraints may depress the mass of agents investing in the MIR sectors and the sectoral investment distortions then lead to the sectoral rate-of-return differentials, similar as in the two-sector setting.

**Assumption 2.**  $\gamma \equiv \frac{m_1}{m_2} < \left( \frac{\eta_2}{\eta_1 + \eta_2} \right)^{\frac{\theta}{1-\theta}}$ .

Let  $\bar{w}_{1,A} \equiv \frac{\bar{w}_1}{\left(1 + \frac{\eta_0}{\eta_1} \left[1 - \gamma^{\frac{1-\theta}{\theta}}\right]\right)^{\frac{\theta}{1-\theta}}} < \bar{w}_1$  and  $\bar{w}_{2,A} \equiv \bar{w}_2 \eta_2^{\frac{\theta}{1-\theta}} < \bar{w}_2$ .

For a sufficiently low level of national income  $w_t < \bar{w}_{1,A}$ , the mass of agents investing in sector 1 and 2 is inefficiently low and so is the aggregate credit demand. Thus, the rate-of-return spread,  $r_t < q_{s,t+1}$ , arises in sector  $s \in \{1, 2\}$  and the borrowing constraints are binding there. In this case, the sectoral investment shares are as follows,

$$\delta_{2,t} = \frac{\int_{\epsilon_{2,t}}^{\infty} \frac{(1-\theta)\epsilon_j w_t}{1-\lambda} dG(\epsilon_j)}{w_t L} = \left( \frac{w_t}{\bar{w}_2} \right)^{\frac{1-\theta}{\theta}} < \eta_2, \quad (35)$$

$$\delta_{1,t} = \frac{\int_{\epsilon_{1,t}}^{\epsilon_{2,t}} \frac{(1-\theta)\epsilon_j w_t}{1-\lambda} dG(\epsilon_j)}{w_t L} = \delta_{2,t} \left( \gamma^{-\frac{1-\theta}{\theta}} - 1 \right), \quad (36)$$

$$\delta_{0,t} = 1 - (\delta_{1,t} + \delta_{2,t}) = 1 - \delta_{2,t} \gamma^{-\frac{1-\theta}{\theta}}. \quad (37)$$

Assumption 2 and  $w_t < \bar{w}_{1,A}$  jointly lead to the descending sectoral capital-labor ratio, which implies the ascending pattern of the sectoral rate of return.

$$\frac{\delta_{0,t}}{\eta_0} > \frac{\delta_{1,t}}{\eta_1} > \frac{\delta_{2,t}}{\eta_2}, \quad \Rightarrow \quad r_t = q_{0,t+1} < q_{1,t+1} < q_{2,t+1}. \quad (38)$$

As all sectoral outputs are essential for the production of final goods, all sectors are active under autarky and the interest rate is determined by the lowest-return sector (sector 0). A rise in national income raises the investment shares of sector 1 and 2 in equal proportions through the extensive margin, while that of sector 0 falls accordingly.<sup>30</sup>

$$\text{Let } \underline{\theta}_3 \equiv \frac{\alpha}{\alpha + \frac{1-\alpha}{\eta_1 + \eta_2}}, \quad Z_3 \equiv \frac{\frac{1}{\rho}}{\left( \frac{m_2}{1-\theta} \eta_2^{\frac{\theta}{1-\theta}} \right)^{\frac{1}{\rho}}}, \quad \mathbb{K} \equiv \left( \frac{\gamma^{\frac{1-\theta}{\theta}}}{\eta_2 \left[ 1 + \frac{\eta_0}{\eta_1} \left( 1 - \gamma^{\frac{1-\theta}{\theta}} \right) \right]} \right)^{1 - \frac{\theta}{\rho(1-\theta)}} \left( \frac{\eta_2 (\gamma^{-\frac{1-\theta}{\theta}} - 1)}{\eta_1} \right)^{1-\eta_2}.$$

<sup>30</sup>For a moderate level of national income  $w_t \in (\bar{w}_{1,A}, \bar{w}_{2,A})$ , the mass of agents investing in sector 2 is still so low that the rate-of-return spread,  $r_t < q_{2,t+1}$ , leads to the binding borrowing constraints and the investment share of sector 2 is still specified by equation (35). In contrast, the mass of agents investing in sector 1 is large enough to ensure the zero rate-of-return spread  $r_t = q_{1,t+1}$  and the slack borrowing constraints. The rate of return equalizes in sector 0 and 1,  $q_{1,t+1} = r_t = q_{0,t+1}$ , and so does the capital-labor ratio,  $\frac{\delta_{1,t}}{\eta_1} = \frac{\delta_{0,t}}{\eta_0} = \frac{1-\delta_{2,t}}{\eta_0 + \eta_1}$ . A rise in national income raises the investment share of sector 2 through the extensive margin, while that of sector 0 and 1 falls in equal proportions.

For a high level of national income  $w_t > \bar{w}_{2,A}$ , the mass of agents investing in sector 1 and 2 are large enough to ensure the zero rate-of-return spread, i.e.,  $q_{s,t+1} = r_t$  for  $s \in \{1, 2\}$ . Accordingly, the sectoral investment shares are efficient at  $\delta_{s,t} = \eta_s$ , independent of the change in national income.

**Proposition 4.** For  $\theta > \underline{\theta}_3$ ,  $Z_3 < \frac{1}{\mathbb{K}}$ , and  $\lambda < \tilde{\lambda}_{1,A} \equiv 1 - (\mathbb{K}Z_3)^{\rho(1-\theta)}$ , there is a unique steady state under autarky where the borrowing constraints are binding in sector 1 and 2.

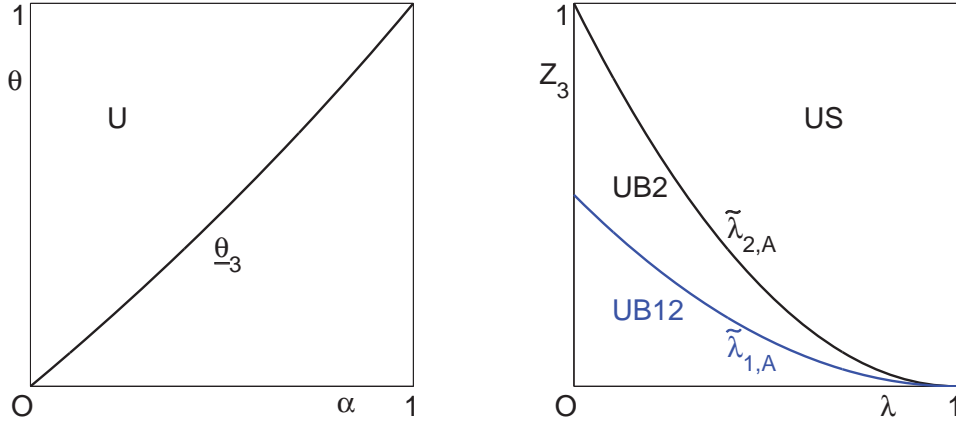


Figure 13: Threshold Values for the Autarkic Steady State: the Three-Sector Setting

Figure 13 shows graphically proposition 4. Given  $\{\alpha, \theta\}$  in region U of the left panel, for  $\{\lambda, Z_3\}$  in region UB12 of the right panel, the level of financial development is so low that the borrowing constraints are **binding** in sector **1** and **2** at the autarkic steady state.

Let  $X_{s,A}$  denote the autarkic steady-state value of variable  $X_{s,t}$ .

**Lemma 6.** Given  $\theta > \underline{\theta}_3$  and  $Z_3 < \frac{1}{\mathbb{K}}$ , for  $0 < \lambda^* < \lambda < \tilde{\lambda}_{1,A}$ ,  $\chi_{0,A}^* < \chi_{0,A} < \chi_{1,A}^* = \chi_{1,A} < 1$ ,  $\mu_{0,A}^* < \mu_{0,A} < \mu_{1,A}^* = \mu_{1,A} < 1$ , and  $r_A^* < r_A < \rho$ .

At the autarkic steady state, the borrowing constraints are binding in sector  $s \in \{1, 2\}$ . According to equations (35)-(36), the relative capital-labor ratio of sector 1 is independent of the level of financial development,  $\frac{\delta_{1,A}}{\delta_{2,A}} = \left( \gamma^{-\frac{1-\theta}{\theta}} - 1 \right) \frac{\eta_2}{\eta_1} > 1$ . Thus,  $\mu_{1,A} = \mu_{1,A}^* < 1$  and  $\chi_{1,A} = \chi_{1,A}^* < 1$ . Besides, the higher the level of financial development, the less severe the overinvestment in sector 0, the higher the relative output price and the relative rate of return in sector 0, i.e.,  $\chi_{0,A}^* < \chi_{0,A}$  and  $\mu_{0,A}^* < \mu_{0,A}$ . Thus, South has a comparative advantage in sector 0, while North has a comparative advantage in sector  $s \in \{1, 2\}$ .

### 5.3.2 Free-Trade Equilibrium

The world economy is at the autarkic steady state when the free trade policy is announced in period 0. According to lemma 6,  $\mu_{0,A}^* < \mu_{1,A}^* < \mu_{2,A}^* = 1$ . From period  $t = 1$  on, the relative sectoral prices in North are aligned to the world levels,  $\chi_{s,t} = \chi_{s,A}^*$ , and so are its relative sectoral rates of return,  $\mu_{s,t} = \mu_{s,A}^*$ , implying that the sectoral rate of return is strictly ascending in North,  $q_{0,t+1} = \mu_{0,A}^* q_{2,t+1} < q_{1,t+1} = \mu_{1,A}^* q_{2,t+1} < q_{2,t+1}$ . Thus, agents in North always invest in the sector with the highest MIR that they can afford.

The static gains from trade raises the national income in North, which then triggers the sectoral investment reallocation along the extensive margin.

For  $w_t < \bar{w}_1$ , the mass of agents investing in sector 1 and 2 is so low that they cannot borrow and invest the entire domestic saving there. Thus, sector 0 is active and the interest rate is coupled with the rate of return there,  $r_t = q_{0,t+1}$ . Given the ascending

sectoral rates of return, the borrowing constraints are binding in sector 1 and 2. Thus, the sectoral investment shares are still featured by equations (35)-(37).

For  $w_t \in (\bar{w}_1, \bar{w}_2)$ , the mass of agents investing in sector 1 and 2 is so high that the entire domestic saving is invested in sector 1 and 2, while sector 0 is inactive,  $\delta_{0,t} = 0$ . The interest rate is then coupled with the rate of return in sector 1,  $r_t = q_{1,t+1}$ , and the borrowing constraints are slack there. Combine it with the ascending sectoral rate of return  $q_{1,t+1} < q_{2,t+1}$  to get the binding borrowing constraints in sector 2. Thus,  $\delta_{2,t}$  is still featured by equation (35), while  $\delta_{1,t} = 1 - \delta_{2,t}$ .

For  $w_t > \bar{w}_2$ , the mass of agents investing in sector 2 is so large that the entire domestic saving is invested there  $\delta_{2,t} = 1$ , while sector 1 is inactive as sector 0,  $\delta_{0,t} = \delta_{1,t} = 0$ .

As shown in subsection 4.2, whether trade leads to the interest rate re-reversal depends critically on the magnitude of the static and the dynamic gains from trade and, ultimately, on the cross-country heterogeneity in financial development.

**Proposition 5.** *For a sufficiently large heterogeneity in financial development, trade integration induces North to offshore sequentially the low-MIR, low-return sectors and upgrade gradually to the high-MIR, high-return sectors.*

In the case of a moderate heterogeneity in financial development, figure 14 shows the dynamics of national income and the interest rate in North, where the notations follow those of figures 9 and 10. As described before, the interest rate has an inverse sawtooth wave pattern with respect to national income. Given  $w_0 = w_A < \bar{w}_1$ , North specializes partially towards sector 1 and 2 in period  $t = 1$ , which leads to the interest rate reversal in period 0,  $r_0 < r_A^* < r_A$ , by the same logic as in subsection 4.2. Then, the rise in national income reduces the interest rate in North until period  $t = 20$ . In period  $t = 21$ , North fully offshores sector 0, leading to the interest rate re-reversal,  $r_{21} > r_A^*$ . Eventually, North converges to the new steady state with  $w_T \in (\bar{w}_1, \bar{w}_2)$  and  $r_T > r_A^*$ . In this case, trade integration does not solve the global imbalances in the long run.

In the case of a large heterogeneity in financial development, figure 15 shows the dynamics of national income and the interest rate in North. In comparison with the previous case, the international relative price differentials are larger and so are trade flows and the gains from trade. Thus, North converges eventually to a new steady state with a higher national income,  $w_T > \bar{w}_2$ . During this process, the interest rate re-reversal happens twice, when North offshores sector 0 and 1 in period 6 and 26, respectively.

To sum up, the interest rate reversal and re-reversal can be an recurrent phenomenon if trade integration allows the more financially develop country to sequentially offshore the low-return sectors and upgrade to the high-return sectors over time.<sup>31</sup>

As discussed in the Introduction, “sectors” in my model can be interpreted as production stages or tasks. The empirical literature has documented extensively firms’ heterogeneity in financial dependence along the dimensions of size, industries, trade exposures, and etc. (Beck et al., 2008; Manova, 2013; Rajan and Zingales, 1998). My paper calls for the empirical analysis of heterogeneous financial dependence across production stages.

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<sup>31</sup>This mechanism can be easily extended into a multi-sector setting. The technical analysis of a generalized, multi-sector model is available upon request.

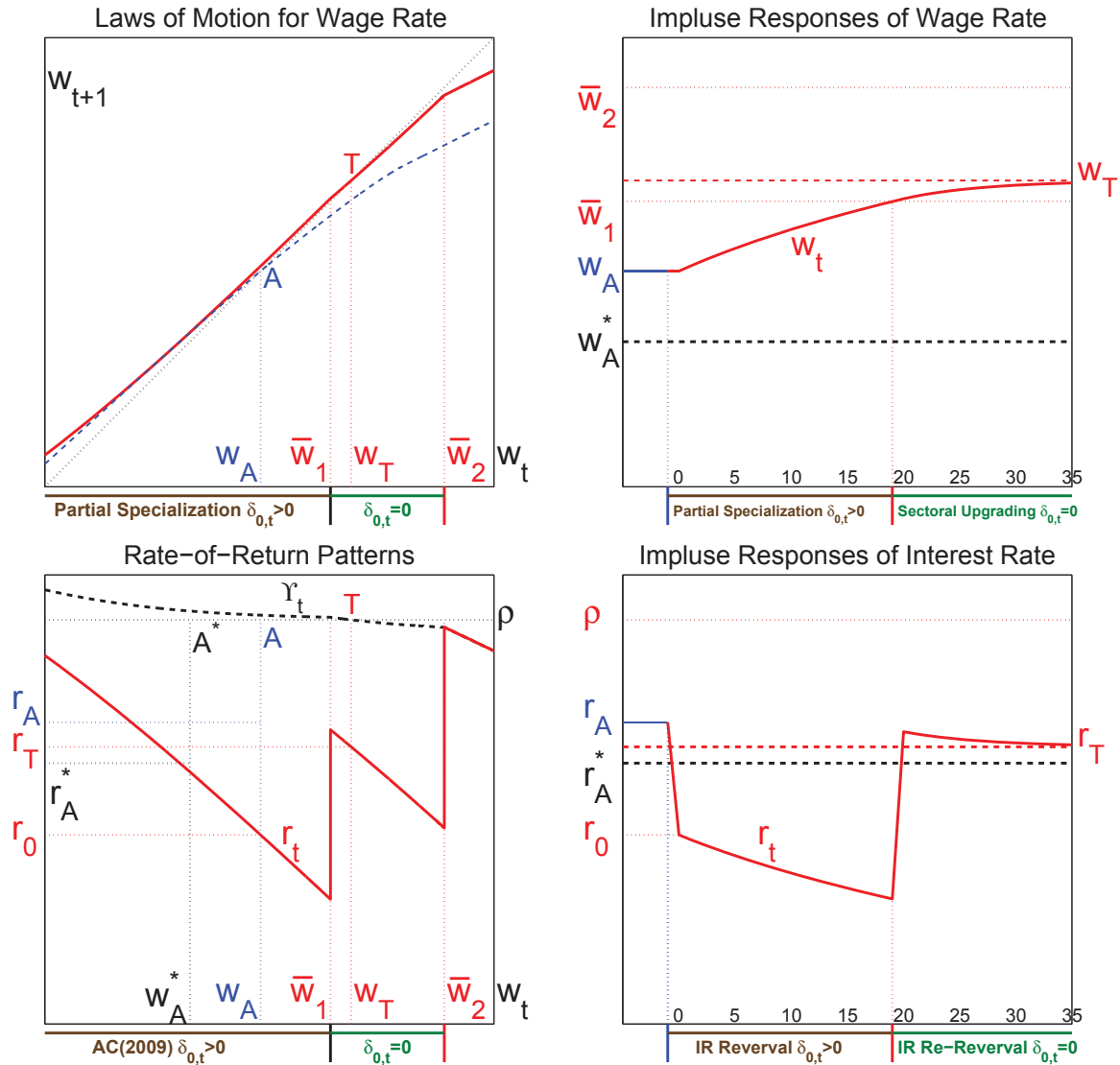


Figure 14: The Case of Moderate Heterogeneity in Financial Development

Given cross-stage heterogeneity in financial dependence, cross-country heterogeneity in financial development can be the driving force behind global production network and supply-chain trade. As shown in figure 1, if supply-chain trade allows North to move partially away from the low-return production stages, the smile curve in North witnesses a falling jaw, which helps resolve the global imbalances. However, if supply-chain trade allows North to fully offshore the low-return activities, the smile curve in North witnesses a missing jaw, which may amplify the global imbalances. In the multi-sector setting, the sequential offshoring of low-return activities in North may have the non-monotone implications to international interest rate differentials. Thus, the global imbalances can be a recurrent phenomenon in the new era of globalization.

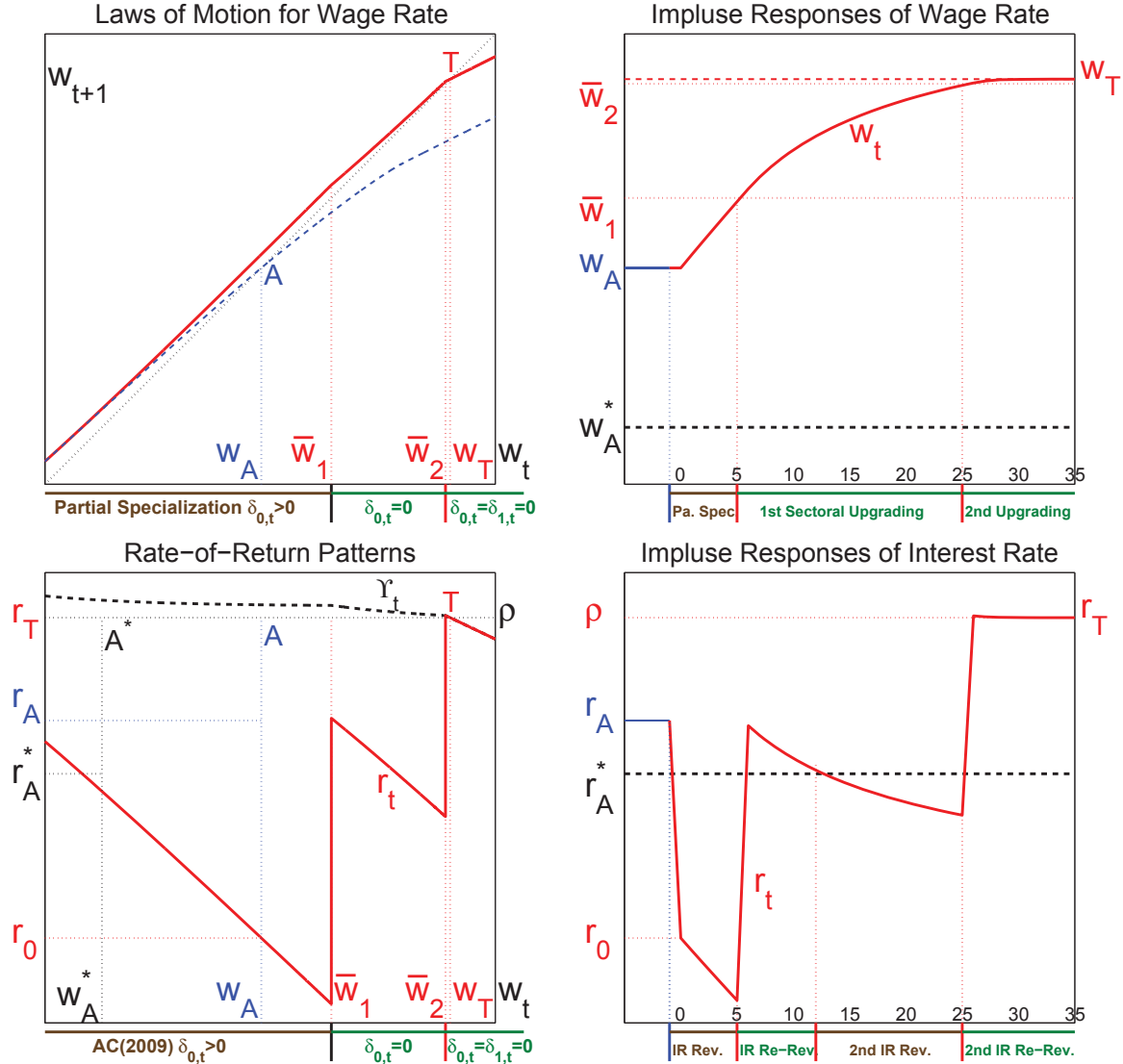


Figure 15: The Case of Large Heterogeneity in Financial Development

## 6 Final Remarks

This paper highlights the extensive margin of sectoral investment as a key channel through which trade integration affects domestic sectoral composition and the incentives for international financial flows in a world with heterogeneous financial development. Whether trade flows amplify or dampen the global imbalances depends on how far trade-driven sectoral shifts reshape the industrial structure in the more financially developed country, which fundamentally complements the findings of Antras and Caballero (2009). The logic of my model results can be applied to the case of supply-chain trade. In particular, supply-chain trade may amplify the global imbalances, if it allows the developed countries to offshore the low-return production activities and upgrade to the high-return activities.

For simplicity, I take the level of financial development as exogenously given and analyze the impacts of economic integration. In fact, trade and capital account liberalization may affect the incentives for financial intermediaries (Alessandria and Qian, 2005; Svaleryd and Vlachos, 2002; Tressel and Verdier, 2011) and the structures of financial



markets from the political economy perspective (Braun and Raddatz, 2008; Rajan and Zingales, 2003, 2004). Besides, market-reform policies that improve domestic financial sectors and institutional structures are more fundamental to the CEECs' income convergence than simply reducing the barriers to trade or financial flows.<sup>32</sup> Taking into account these factors, developing countries may use trade and capital account liberalization as a triggering device and combine them with other market-reform policies to promote financial development and productivity growth. These topics are kept for future research.

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<sup>32</sup>Since the 2004 EU enlargement, the adoption of EU laws and directives has improved financial sector quality in the CEECs by upgrading their legal, regulatory, and supervisory framework to the same standard as in the Western Europe, while the dominance of foreign banks in the CEECs' financial markets has also improved the quality of domestic banking sectors (Herrmann and Winkler, 2009a,b). As a result, the CEECs have witnessed rapid and massive capital inflows (von Hagen and Zhang, 2014).

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# Appendix

## A Equilibrium Allocation under Free Trade

For  $\theta \in (\underline{\theta}, 1)$  and  $\lambda, \lambda^* \in [0, \tilde{\lambda}_A)$ , four cases may arise under free trade, given  $\chi^* = \chi_A^*$ :

- **Case M:** multiple steady states arise, i.e., there are two stable steady states denoted by H and L, and one unstable steady state denoted by M. The steady states are ranked in terms of income level,  $w_H > \bar{w} > w_M > w_L > w_A$ .
- **Case UF1:** North converges to a **unique** steady state where it **fully** specializes in sector 1;
- **Case UP1:** North converges to a **unique** steady state where it **partially** specializes in sector 1;
- **Case UP0:** North converges to a **unique** steady state where it **partially** specializes in sector 0;

Define some threshold values as follows,

$$\hat{Z} \equiv (1 - g)^{g-\eta}(1 - \eta g)^{1-g} \quad \text{and} \quad \hat{\lambda}_T^* \equiv 1 - \left(\frac{Z}{\hat{Z}}\right)^{\rho(1-\theta)}, \quad \text{where} \quad g \equiv \frac{\theta}{\rho(1-\theta)} \quad (39)$$

$$\tilde{Z} \equiv \eta^{\frac{g(g-\eta)}{1-\eta}}(1 - \eta + \eta^{\frac{g}{1-\eta}+1})^{1-g} \quad \text{and} \quad \tilde{\lambda}_T^* \equiv 1 - \left(\frac{Z}{\tilde{Z}}\right)^{\rho(1-\theta)} \quad (40)$$

In the following, I show graphically the threshold conditions in the  $\{\theta, Z, \lambda^*, \lambda\}$  space that define the four cases<sup>33</sup> as well as show the law of motion for wage in each case.

Under free trade, multiple steady states arise if four conditions are satisfied simultaneously,

- $\theta \in (\underline{\theta}, \alpha)$ , i.e.,  $\{\alpha, \theta\}$  in region M of the left panel of figure 16, and
- $Z < \hat{Z}$ , i.e.,  $\{\theta, Z\}$  in region M of the right panel of figure 16, and
- $\lambda^* < \hat{\lambda}_T^*$  and  $\lambda \in (\tilde{\lambda}_T, \hat{\lambda}_T)$ , i.e.,  $\{\lambda^*, \lambda\}$  in region M of the left panel of figure 17.

In the upper-left panel of figure 18, the solid (dashed) curve shows the law of motion for wage under free trade (autarky) for case M.

In section 4, I focus on case UF1 which arises if four conditions are satisfied simultaneously, i.e.,  $\theta \in (\underline{\theta}, 1)$  and  $Z < \tilde{Z}$ , i.e.,  $\{\theta, Z\}$  in regions M and UF of the right panel of figure 16,

- given  $\{\theta, Z\}$  in region UF of the right panel of figure 16, it is required that  $\{\lambda^*, \lambda\}$  in region UF1 of the middle panel of figure 17, i.e.,  $\lambda^* < \tilde{\lambda}_T^*$  and  $\lambda \in (\tilde{\lambda}_T, \tilde{\lambda}_A)$ ;
- given  $\{\theta, Z\}$  in region M of the right panel of figure 16, it is required that  $\{\lambda^*, \lambda\}$  in region UF1 of the middle panel of figure 17, i.e.,  $\lambda^* \in (0, \hat{\lambda}_T^*)$  and  $\lambda \in (\hat{\lambda}_T, \tilde{\lambda}_A)$ , or alternatively,  $\lambda^* \in (\hat{\lambda}_T^*, \tilde{\lambda}_T^*)$  and  $\lambda \in (\tilde{\lambda}_T, \tilde{\lambda}_A)$ .

In the upper-right panel of figure 18, the solid (dashed) curve shows the law of motion for wage under free trade (autarky) for case UF1.

<sup>33</sup>The proof of proposition 2 provides the technical derivation for these threshold values.

Given  $\theta \in (\underline{\theta}, 1)$ , free trade induces North to move from the autarkic steady state to a **unique** steady state where it specializes **partially** in sector **1 (0)** if parameters are in region **UP1 (UP0)** of figure 17. The three panels in figure 17 correspond to  $\{\theta, Z\}$  in the three regions of the right panel of figure 16.

In the two bottom panels of figure 18, the solid (dashed) curve shows the law of motion for wage under free trade (autarky) in case UP1 and UP0, respectively.

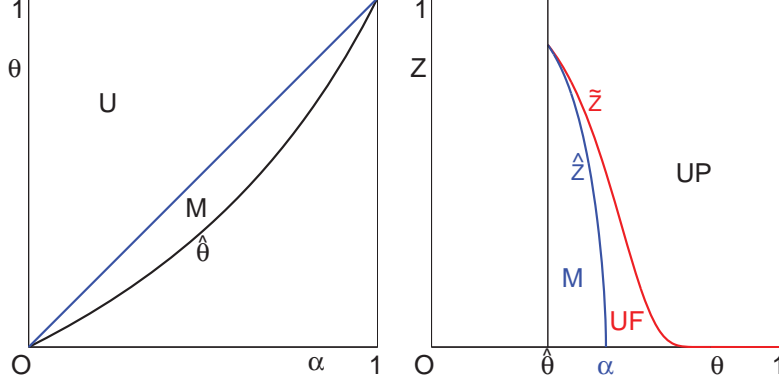


Figure 16: Threshold Values under Free Trade

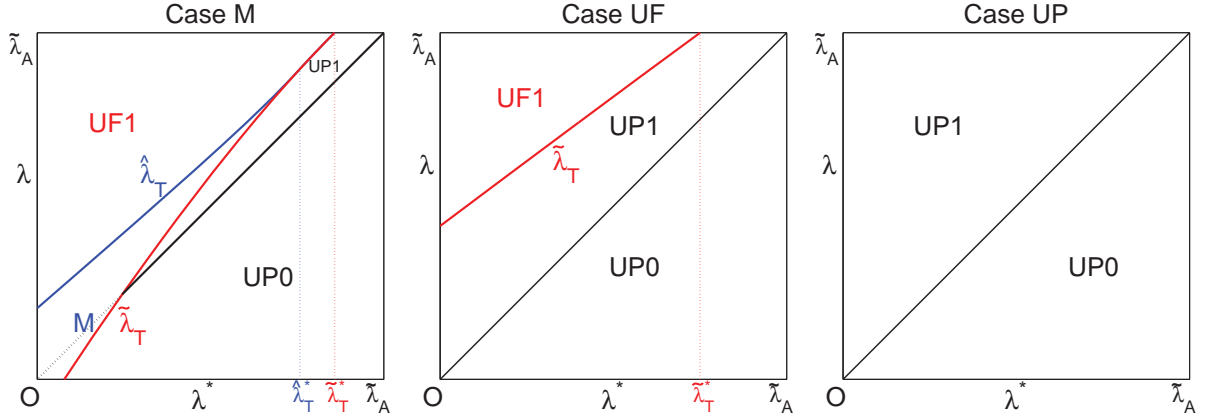


Figure 17: Threshold Values under Free Trade

## B Proofs

### Proof of Lemma 1

*Proof.* First, combine equations (1) and (3)-(4) to get equations (5)-(6),

$$\frac{q_{1,t+1}\delta_t w_t L}{\alpha} = p_{1,t+1} Y_{1,t+1} = \frac{w_{t+1}\zeta_{t+1}L}{1-\alpha}, \quad \frac{q_{0,t+1}(1-\delta_t)w_t L}{\alpha} = p_{0,t+1} Y_{0,t+1} = \frac{w_{t+1}(1-\zeta_{t+1})L}{1-\alpha} \quad (41)$$

$$\frac{w_{t+1}}{q_{1,t+1}} = \frac{\delta_t w_t}{\rho \zeta_{t+1}}, \quad \frac{w_{t+1}}{q_{0,t+1}} = \frac{(1-\delta_t)w_t}{\rho(1-\zeta_{t+1})}, \quad \mu_{t+1} \equiv \frac{q_{0,t+1}}{q_{1,t+1}} = \frac{\frac{\delta_t}{1-\delta_t}}{\frac{\zeta_{t+1}}{1-\zeta_{t+1}}}, \quad \zeta_{t+1} = \frac{\delta_t}{(1-\delta_t)\mu_{t+1} + \delta_t} \quad (42)$$

$$Y_{1,t+1} = \left(\frac{\delta_t w_t L}{\alpha}\right)^\alpha \left(\frac{\zeta_{t+1} L}{1-\alpha}\right)^{1-\alpha} = \left(\frac{p_{1,t+1} Y_{1,t+1}}{q_{1,t+1}}\right)^\alpha \left(\frac{p_{1,t+1} Y_{1,t+1}}{w_{t+1}}\right)^{1-\alpha}, \quad (43)$$

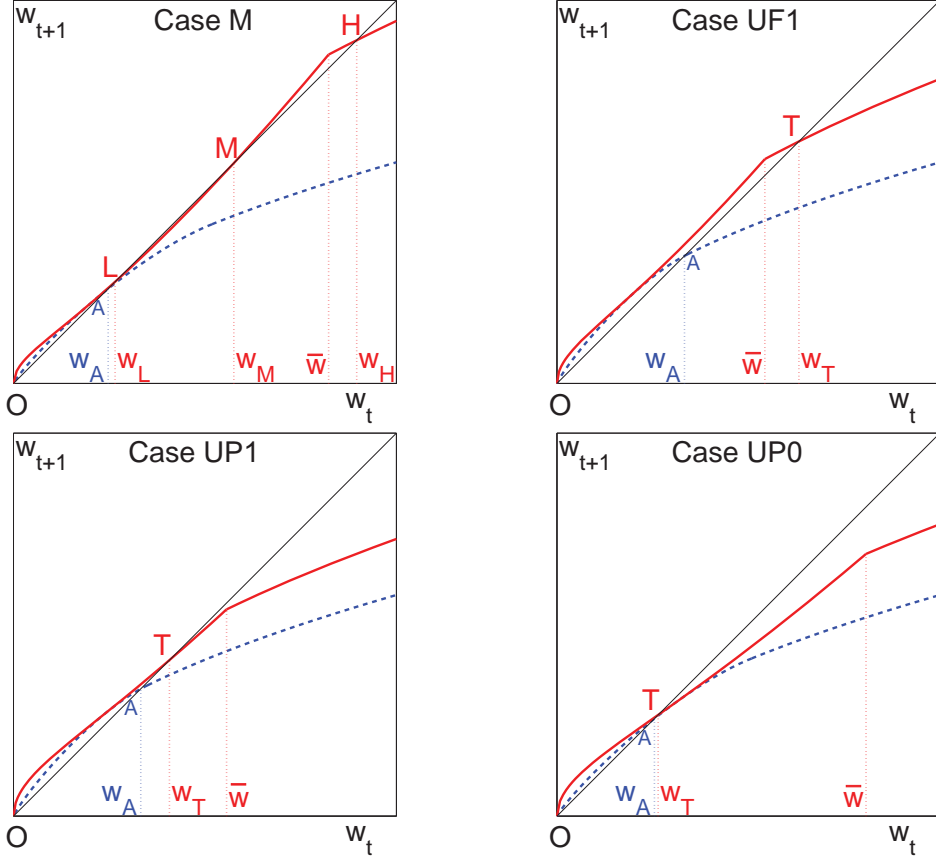


Figure 18: Laws of Motion for Wage under Free Trade:  $\theta \in (\underline{\theta}, \alpha)$

$$Y_{0,t+1} = \left[ \frac{(1 - \delta_t)w_t L}{\alpha} \right]^\alpha \left[ \frac{(1 - \zeta_{t+1})L}{1 - \alpha} \right]^{1-\alpha} = \left( \frac{p_{0,t+1} Y_{0,t+1}}{q_{0,t+1}} \right)^\alpha \left( \frac{p_{0,t+1} Y_{0,t+1}}{w_{t+1}} \right)^{1-\alpha}, \quad (44)$$

$$p_{1,t+1} = (q_{1,t+1})^\alpha w_{t+1}^{1-\alpha}, \quad p_{0,t+1} = (q_{0,t+1})^\alpha w_{t+1}^{1-\alpha}, \Rightarrow \frac{p_{0,t+1}}{p_{1,t+1}} = \left( \frac{q_{0,t+1}}{q_{1,t+1}} \right)^\alpha, \quad \chi_{t+1} = \mu_{t+1}^\alpha. \quad (45)$$

**Second, derive the PPF in two cases.**

- In the case of efficient sectoral investment,  $\mu_{t+1} = 1$  and, according to equation (42), the sectoral shares of labor and capital equalize,  $\zeta_{t+1} = \delta_t$ . Use equations (43)-(44) to get

$$Y_{1,t+1} = \zeta_{t+1} \mathbb{Y}_{t+1}, \quad Y_{0,t+1} = (1 - \zeta_{t+1}) \mathbb{Y}_{t+1}, \quad \text{where } \mathbb{Y}_{t+1} \equiv \left[ \frac{w_t L}{\alpha} \right]^\alpha \left[ \frac{L}{1 - \alpha} \right]^{1-\alpha}. \quad (46)$$

In this case, the PPF is specified by equation (7).

- In the case of inefficient sectoral investment,  $\mu_{t+1} < 1$  and the borrowing constraints are binding. According to equation (13), sector 1's share of physical capital in period  $t + 1$ , i.e.,  $\delta_t$ , is predetermined and depends on  $Y_t$ . Rewrite equations (43)-(44) as

$$\left( \frac{Y_{1,t+1}}{\delta_t^\alpha} \right)^{\frac{1}{1-\alpha}} = \zeta_{t+1} \mathbb{Y}_{t+1}^{\frac{1}{1-\alpha}}, \quad \left[ \frac{Y_{0,t+1}}{(1 - \delta_t)^\alpha} \right]^{\frac{1}{1-\alpha}} = (1 - \zeta_{t+1}) \mathbb{Y}_{t+1}^{\frac{1}{1-\alpha}}. \quad (47)$$

In this case, the PPF is specified by equation (18).

**Third, derive the boundary condition for the two cases.** The level of national income in period  $t$  determines sector 1's maximum share of physical capital in period  $t + 1$ , according to equation (13). Maintaining the efficient sectoral investment  $\mu_{t+1} = 1$ , sector 1's maximum possible share of domestic labor is  $\zeta_{t+1} = \tilde{\delta}_t$ , corresponding to the threshold output  $\tilde{Y}_{1,t+1}$ .

**Fourth, derive the MRT from the gradient of the PPF.** Combine (1) and (3)-(4),

$$\begin{aligned} \ln Y_{1,t+1} &= \alpha \ln \delta_t + (1 - \alpha) \ln \zeta_{t+1} + \ln \mathbb{Y}_{t+1}, \quad \ln Y_{0,t+1} = \alpha \ln(1 - \delta_t) + (1 - \alpha) \ln(1 - \zeta_{t+1}) + \ln \mathbb{Y}_{t+1} \\ \frac{\partial \ln Y_{1,t+1}}{\partial \ln \zeta_{t+1}} &= \alpha \frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1 - \alpha), \quad \frac{\partial \ln Y_{0,t+1}}{\partial \ln \zeta_{t+1}} = \alpha \frac{-\delta_t}{1 - \delta_t} \frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1 - \alpha) \frac{-\zeta_{t+1}}{1 - \zeta_{t+1}} \\ MRT_{0,1} &\equiv -\frac{\partial Y_{1,t+1}}{\partial Y_{0,t+1}} = -\frac{\partial \ln Y_{1,t+1}}{\partial \ln Y_{0,t+1}} \frac{Y_{1,t+1}}{Y_{0,t+1}} = \frac{\alpha \frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1 - \alpha)}{\alpha \frac{-\delta_t}{1 - \delta_t} \frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1 - \alpha) \frac{-\zeta_{t+1}}{1 - \zeta_{t+1}}} \frac{Y_{1,t+1}}{Y_{0,t+1}}. \end{aligned}$$

In the case of efficient sectoral investment,  $\mu_{t+1} = 1$  and combine it with equation (42) to get  $\zeta_{t+1} = \delta_t$  and  $\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} = 1$ . In the case of inefficient sectoral investment,  $\mu_{t+1} < 1$  and, given the predetermined  $\delta_t$ ,  $\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} = 0$ . In either case, the (absolute) gradient of the PPF is

$$MRT_{0,1} = \frac{1 - \zeta_{t+1}}{\zeta_{t+1}} \frac{Y_{1,t+1}}{Y_{0,t+1}}. \quad (48)$$

□

### Proof of Proposition 1

*Proof.* **First, derive the autarkic equilibrium.** Use equation (2) to get the gradient of isoquant,

$$\ln Y_{t+1} = \eta \ln V_{1,t+1} + (1 - \eta) \ln V_{0,t+1} - \ln \eta^\eta (1 - \eta)^{1-\eta}, \quad MRS_{0,1} \equiv -\frac{\partial V_{1,t+1}}{\partial V_{0,t+1}} = \frac{1 - \eta}{\eta} \frac{V_{1,t+1}}{V_{0,t+1}}. \quad (49)$$

In the autarkic equilibrium, the PPF and the isoquant are tangent and the markets for sectoral outputs clear domestically. Combine equations (19), (48), and (49) to get

$$MRS_{0,1} = MRT_{0,1}, \quad \Rightarrow \quad \frac{1 - \eta}{\eta} = \frac{1 - \zeta_{t+1}}{\zeta_{t+1}}, \quad \Rightarrow \quad \zeta_{t+1} = \eta. \quad (50)$$

Thus, the share of domestic labor allocated in a sector under autarky is equal to the sector's share in the production of final goods. Whether it also applies to the investment share depends on how far financial frictions distort the maximum share of domestic investment in sector 1.

$$\delta_t = \begin{cases} \tilde{\delta}_t < \zeta_{t+1} = \eta, & \text{if } \tilde{\delta}_t < \eta; \\ \zeta_{t+1} = \eta, & \text{if } \tilde{\delta}_t \geq \eta. \end{cases} \quad (51)$$

**Second, derive the law of motion for wage** by combining equations (50)-(51) with (1)-(4) and (6) to get (23). In the case of efficient sectoral investment,  $\mu_{t+1} = 1$  and the law of motion for wage degenerates into (22). In the case of inefficient sectoral investment, equation (13) specifies the fraction of domestic investment in sector 1,  $\delta_t$ , as a function of  $w_t$ . Combine equation (6) and (50) to get (24) specifying the sectoral return wedge  $\mu_{t+1}$  as a function of  $\delta_t$ .

**Third, derive the condition for the unique steady state**, i.e., the steady state if exists is stable. For  $w_t \geq \bar{w}_A$ , the law of motion for wage in logarithm is linear with the slope less than unity, as shown by equation (22). If there exists a steady state with  $w_A \geq \bar{w}_A$ , it is stable.



For  $w_t < \bar{w}_A$ , sectoral investment is inefficient. Use equations (13), (23), (24) to get

$$\ln \delta_t = \frac{1-\theta}{\theta}(\ln w_t - \ln \bar{w}), \quad \frac{\partial \ln \delta_t}{\partial \ln w_t} = \frac{1-\theta}{\theta}, \quad (52)$$

$$\ln \mu_{t+1} = \ln \delta_t - \ln(1-\delta_t) + \ln \frac{1-\eta}{\eta}, \quad \frac{\partial \ln \mu_{t+1}}{\partial \ln \delta_t} = \frac{1}{1-\delta_t}, \quad (53)$$

$$\ln \Gamma_t = \eta \ln \mu_{t+1} - \ln(1-\eta + \eta \mu_{t+1}), \quad \frac{\partial \ln \Gamma_t}{\partial \ln \mu_{t+1}} = \eta(1-\mu_{t+1})(1-\delta_t). \quad (54)$$

Combine them with equation (23) to get (25). The condition for the stable steady state is.

$$\frac{\partial w_{t+1}}{\partial w_t} \Big|_{w_{t+1}=w_t} < 1, \Rightarrow \mu_A > 1 - \frac{\mathfrak{g}}{\eta}, \text{ where } \mathfrak{g} \equiv \frac{\frac{1}{\alpha} - 1}{\frac{1}{\theta} - 1}. \quad (55)$$

Given  $\mu_{t+1} \in (0, 1]$ , a sufficient condition for inequality (55) to hold is

$$1 - \frac{\mathfrak{g}}{\eta} \leq 0, \Rightarrow \theta \geq \underline{\theta} \equiv \frac{\eta\alpha}{\eta\alpha + (1-\alpha)}. \quad (56)$$

Consider first the case of  $\theta \geq \underline{\theta}$ , i.e.,  $\{\alpha, \theta\}$  in region U of the left panel of figure 4. Derive the condition under which the borrowing constraints are binding at the autarkic steady state, i.e.,  $\mu_A < 1$  or equivalently  $\tilde{\delta}_A < \eta$ . Combine  $w_{t+1} = w_t = w_A$  with equations (13) and (23)-(24) to get  $\mu_A < 1$  as a function of  $\lambda$ ,

$$w_A = \left( \frac{1}{\rho} \frac{\mu_A^\eta}{1-\eta + \eta\mu_A} \right)^\rho = \delta_A^{\frac{\theta}{1-\theta}} \bar{w} = \left[ \frac{\eta\mu_A}{1-\eta + \eta\mu_A} \right]^{\frac{\theta}{1-\theta}} \frac{\mathfrak{m}}{1-\theta} (1-\lambda)^{\frac{1}{1-\theta}}$$

$$\Rightarrow Z = \frac{\frac{1}{\rho}}{\left( \frac{\mathfrak{m}}{1-\theta} \eta^{\frac{\theta}{1-\theta}} \right)^{\frac{1}{\rho}}} = (1-\lambda)^{\frac{1}{\rho(1-\theta)}} \frac{(\mu_A)^{\mathfrak{g}-\eta}}{(1-\eta + \eta\mu_A)^{\mathfrak{g}-1}}, \quad (57)$$

$$\frac{\partial \ln \mu_A}{\partial \ln \lambda} = \frac{\frac{1}{\theta} \frac{\lambda}{1-\lambda}}{(1-\delta_A) \left[ 1 - \frac{(1-\mu_A)\eta}{\mathfrak{g}} \right]}. \quad (58)$$

Given  $\theta > \underline{\theta}$ ,  $\mathfrak{g} > \eta$  and hence,  $\frac{(1-\mu_A)\eta}{\mathfrak{g}} < 1$  and hence,  $\frac{\partial \ln \mu_A}{\partial \ln \lambda} > 0$ . Let  $\tilde{\lambda}_A$  denote the level of financial development that ensures  $\tilde{\delta}_A = \eta$  and then  $\mu_A = 1$ . Combine them with equation (57),

$$\tilde{\lambda}_A \equiv 1 - Z^{\rho(1-\theta)}. \quad (59)$$

Finally, in the case of  $\theta < \underline{\theta}$ , i.e.,  $\{\alpha, \theta\}$  in region M of the left panel of figure 4,  $\mathfrak{g} < \eta$ . Derive the threshold condition for multiple steady states, i.e., the existence of unstable steady state M. In the case of  $\lambda = \hat{\lambda}_A$ , the law of motion for wage is tangent with the 45° line at  $w_M \in (0, \bar{w}_A)$ ,

$$\frac{\partial w_{t+1}}{\partial w_t} \Big|_{w_M} = 1, \Rightarrow \mu_M = 1 - \frac{\mathfrak{g}}{\eta}. \quad (60)$$

Combine equations (60) and (57) to get the threshold condition,

$$\hat{\lambda}_A = 1 - \left[ Z \frac{\left( \frac{\eta-\mathfrak{g}}{\eta} \right)^{\eta-\mathfrak{g}}}{(1-\mathfrak{g})^{1-\mathfrak{g}}} \right]^{\rho(1-\theta)}, \quad (61)$$

In the case of  $\theta \in (0, \underline{\theta})$ , figure 19 shows two threshold values,  $\hat{\lambda}_A$  and  $\tilde{\lambda}_A$ , in the  $\{\lambda, Z\}$  space. The solid curves in the three panels of figure 20 show the law of motion for wage, given  $\{\lambda, Z\}$  in the three regions of figure 19, respectively, while the dashed curves show the laws of motion for wage in the benchmark case of  $\lambda = 1$ . The steady state properties in the three cases are summarized as follows.

- For  $\{\lambda, Z\}$  in region  $N$ , the economy converges to the steady state with the income level at zero,  $w_A = 0$ . In this case, there does **not** exist a non-trivial steady state.
- For  $\{\lambda, Z\}$  in region  $MB$ , there is an unstable steady state  $M$  and a stable steady state  $A$  with  $0 < w_M < w_A < w_B$ , and the borrowing constraints are **binding** in steady state  $A$ .
- For  $\{\lambda, Z\}$  in region  $MS$ , there is an unstable steady state  $M$  and a stable steady state  $A$  with  $0 < w_M < w_A = w_B$ , and the borrowing constraints are **slack** in steady state  $A$ .

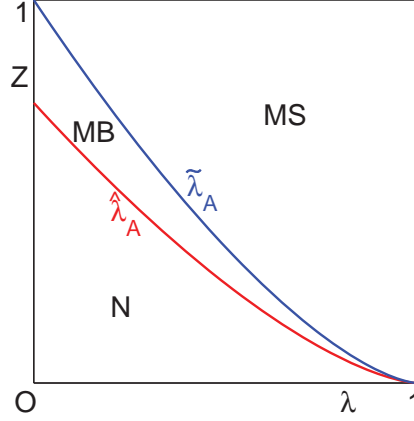


Figure 19: Steady-State Properties under Autarky

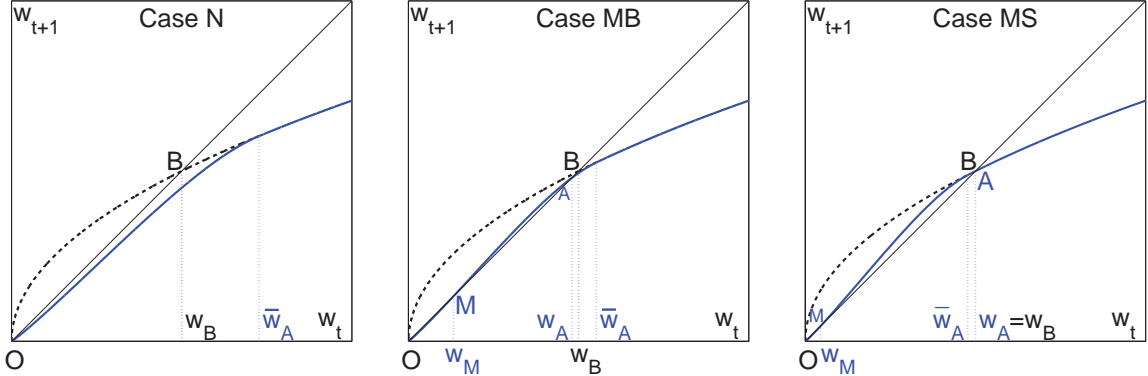


Figure 20: Laws of Motion for Wage under Autarky:  $\theta < \underline{\theta}$

□

### Proof of Lemma 2

*Proof.* Under autarky, the markets for sectoral outputs clear domestically,  $V_{s,t} = Y_{s,t}$ , implying  $K_{1,t+1} > 0$  and  $r_t = q_{0,t+1}$ . According to equations (1)-(2),  $w_{t+1}^{1-\alpha}(q_{0,t+1})^{\alpha(1-\eta)}(q_{1,t+1})^{\alpha\eta} = 1$ . Combine them to get  $w_{t+1} = \left(\frac{1}{r_t} \mu_{t+1}^\eta\right)^\rho$ . Combine it with equation (23) to get (26).

For  $\theta > \underline{\theta}$ , there exists a unique steady state under autarky with  $w_A = \left(\frac{\Gamma_A}{\rho}\right)^\rho$  where

$$\Gamma_A = \frac{\mu_A^\eta}{1 - \eta + \eta\mu_A} \quad \text{and} \quad \frac{\partial \ln \Gamma_A}{\partial \ln \mu_A} = \eta(1 - \delta_A) > 0, \quad \text{given } \lambda \in [0, \tilde{\lambda}_A]. \quad (62)$$

In the autarkic steady state,  $w_{t+1} = w_t$  implies that  $\Upsilon_A = \rho$  and  $r_A = \rho[1 - \eta + \eta\mu_A]$ . According to equation (58),  $\frac{\partial \mu_A}{\partial \lambda} > 0$  for  $\theta > \underline{\theta}$ . Thus,  $\frac{\partial r_A}{\partial \lambda} > 0$ ,  $\frac{\partial w_A}{\partial \lambda} > 0$ ,  $\frac{\partial \ln \chi_A}{\partial \ln \lambda} > 0$ . Given  $\lambda^* < \lambda < \tilde{\lambda}_A$ , it holds that  $Y_A^* < Y_A$ ,  $\chi_A^* < \chi_A < 1$ , and  $r_A^* < r_A < \rho$ . □

### Proof of Lemma 3

*Proof.* For  $w_t < \bar{w}$ , the maximum possible share of investment in sector 1 is less than unity. In this case, sector 0 is active  $K_{0,t+1} > 0$  and the interest rate coupling gives  $r_t = q_{0,t+1}$ . Thus, the borrowing constraints are binding in North,  $\mu_{t+1} = \frac{q_{0,t+1}}{q_{1,t+1}} = \frac{r_t}{q_{1,t+1}} = \mu_A^* < 1$  and  $\delta_t = \tilde{\delta}_t < 1$  and  $\zeta_{t+1} = \frac{1}{(\frac{1}{\delta_t} - 1)\mu^* + 1} \in (\delta_t, 1)$ .

For  $w_t \geq \bar{w}$ , North offshores sector 0,  $K_{0,t+1} = 0$ , implying the interest rate decoupling from the rate of return in sector 0. The credit market competition leads to the interest rate coupling with the rate of return in sector 1. Combine them with equations (20), (1) and (3)-(6) to get (27).  $\square$

### Proof of Lemma 4

*Proof.* Following the approach described in the proof of lemma 2 and taking into account equation (27), one can derive the interest rate as the piecewise function of national income characterized by (31)-(32).  $\square$

### Proof of Proposition 2

*Proof.* For  $\theta \in (\underline{\theta}, 1)$ , there exists a unique, autarkic steady state. As the static gains from trade raise North's income, the law of motion for wage under free trade lies strictly above that under autarky. According to equations (29) and (30), the law of motion for wage in logarithm has a slope less than unity for  $w_t > \bar{w}$ . Thus, there must exist at least one stable steady state under free trade. In the following, I derive the threshold conditions for the steady-state properties under free trade.

First, derive the condition under which free trade leads to multiple steady states in North. If there exists an unstable steady state M, it must hold that  $w_M \in (0, \bar{w})$  and

$$\frac{\partial w_{t+1}}{\partial w_t} \Big|_{w_M} = \alpha + \alpha \frac{\frac{1-\theta}{\theta}}{\frac{\mu^*}{(1-\mu^*)\delta_M} + 1} > 1, \quad \Rightarrow \quad \frac{\alpha}{1-\alpha} - \frac{\theta}{1-\theta} > \frac{\theta}{1-\theta} \frac{\mu^*}{(1-\mu^*)\delta_M}. \quad (63)$$

For  $\theta \geq \alpha$ , condition (63) does not hold. In this case, the law of motion for wage in logarithm has the slope less than unity so that there exists a unique, stable steady state under free trade.

In the following, I focus on the case of  $\theta \in (\underline{\theta}, \alpha)$ . Combine condition (63) with  $\delta_M = \left(\frac{w_M}{\bar{w}}\right)^{\frac{\theta}{1-\theta}} < 1$  to get an upper bound for  $\mu^*$ ,

$$\frac{\mu^*}{1-\mu^*} \frac{\mathfrak{g}}{1-\mathfrak{g}} < \delta_M < 1, \quad \Rightarrow \quad \mu^* \leq \hat{\mu}_T^* \equiv 1 - \mathfrak{g}, \quad \text{where } \mathfrak{g} \equiv \frac{\frac{1}{\alpha} - 1}{\frac{1}{\theta} - 1} \in (\eta, 1). \quad (64)$$

According to equations (57)-(58),  $\mu_A$  is an increasing function of  $\lambda$ . As South has the economic same structure as North, equations (57)-(58) also specify  $\mu_A^*$  as an increasing function of  $\lambda^* \in [0, \tilde{\lambda}_A)$ . Combine (64) and (57) to get the upper bound for  $\lambda^*$ ,

$$\lambda^* \leq \hat{\lambda}_T^* \equiv 1 - \left(\frac{Z}{\hat{Z}}\right)^{\rho(1-\theta)}, \quad \text{where } \hat{Z} \equiv (1-\mathfrak{g})^{\mathfrak{g}-\eta} (1-\eta\mathfrak{g})^{1-\mathfrak{g}} < 1. \quad (65)$$

In order to ensure  $\hat{\lambda}_T^* \geq 0$  and the existence of multiple steady states,  $Z \leq \hat{Z}$ .  $\hat{Z}$  is a function of  $\theta$ , as shown by the curve between region M and UF in the right panel of figure 16.

Given  $\{\theta, Z\}$  in region M of the right panel of figure 16, derive the condition in the  $\{\lambda^*, \lambda\}$  space under which the law of motion for wage is tangent with the 45° line at steady state M,

$$\frac{\partial w_{t+1}}{\partial w_t} \Big|_{w_M} = \alpha + \alpha \frac{\frac{1-\theta}{\theta}}{\frac{\mu^*}{(1-\mu^*)\delta_M} + 1} = 1, \Rightarrow \delta_M = \frac{\mathfrak{g}}{1 - \mathfrak{g}} \frac{\mu^*}{(1-\mu^*)}. \quad (66)$$

Combine equations (18), (27) with (66) and get a threshold value of  $\hat{\lambda}_T$  as a function of  $\mu^*$ ,

$$w_M = \left[ \frac{1}{\rho} (\mu^*)^\eta \left( 1 + \frac{1-\mu^*}{\mu^*} \delta_M \right) \right]^\rho = \bar{w} \delta_M^{\frac{\theta}{1-\theta}} = (1 - \hat{\lambda}_T)^{\frac{1}{1-\theta}} \frac{\mathfrak{m}}{1-\theta} \left[ \frac{\mathfrak{g}}{1 - \mathfrak{g}} \frac{\mu^*}{(1-\mu^*)} \right]^{\frac{\theta}{1-\theta}} \quad (67)$$

$$\hat{\lambda}_T = 1 - \left\{ \frac{Z}{\left( \frac{\mathfrak{g}}{(1-\mu^*)^\eta} \right)^\mathfrak{g} (\mu^*)^{\mathfrak{g}-\eta} (1-\mathfrak{g})^{1-\mathfrak{g}}} \right\}^{\rho(1-\theta)}. \quad (68)$$

For each  $\lambda^* \in [0, \hat{\lambda}_T^*]$ , use equation (57) to solve for  $\mu_A^*$  and then combine  $\mu^* = \mu_A^*$  with (68) to solve for  $\hat{\lambda}_T$ . The curve between region M and UF1 in the left panel of figure 17 shows  $\hat{\lambda}_T$  as a function of  $\lambda^* \in [0, \hat{\lambda}_T^*]$ .

Second, derive the condition under which North offshores sector 0 before reaching the stable steady state T. Given  $\mu^*$ , use equation (27) to get,

$$w_T = \left[ \frac{1}{\rho} (\mu^*)^{\eta-1} \right]^\rho \geq \bar{w} = \frac{\mathfrak{m}}{1-\theta} (1-\lambda)^{\frac{1}{1-\theta}}, \Rightarrow \lambda \geq \tilde{\lambda}_T = 1 - \left[ \frac{Z \eta^\mathfrak{g}}{(\mu^*)^{1-\eta}} \right]^{\rho(1-\theta)}. \quad (69)$$

Given  $\lambda \in [0, \tilde{\lambda}_A]$ , combine equation (69) with (59) to get an upper bound for  $\mu^*$ ,

$$\tilde{\lambda}_T \leq \tilde{\lambda}_A \Rightarrow \mu^* < \tilde{\mu}_T^* = \eta^{\frac{\mathfrak{g}}{1-\eta}}. \quad (70)$$

By the same logic as mentioned above, combine (70) and (57) to get an upper bound for  $\lambda^*$ ,

$$\lambda^* < \tilde{\lambda}_T^* \equiv 1 - \left( \frac{Z}{\tilde{Z}} \right)^{\rho(1-\theta)}, \text{ where } \tilde{Z} \equiv \eta^{\frac{\mathfrak{g}(\mathfrak{g}-\eta)}{1-\eta}} (1-\eta + \eta^{\frac{\mathfrak{g}}{1-\eta} + 1})^{1-\mathfrak{g}}. \quad (71)$$

In order to ensure  $\tilde{\lambda}_T^* \geq 0$ ,  $Z \leq \tilde{Z}$ .  $\tilde{Z}$  is a function of  $\theta$  and shown by the curve between region UF and UP in the right panel of figure 16.

Given  $\{\theta, Z\}$  in region UF and M in the right panel of figure 16, for each  $\lambda^* \in [0, \tilde{\lambda}_T^*]$ , use equation (57) to solve for  $\mu_A^*$  and then combine  $\mu^* = \mu_A^*$  with (69) to solve for  $\tilde{\lambda}_T$ . For  $\{\theta, Z\}$  in region UF of the right panel of figure 16, the curve between region UF1 and UP1 in the middle panel of figure 17 shows  $\tilde{\lambda}_T$  as a function of  $\lambda^* \in [0, \tilde{\lambda}_T^*]$ ; for  $\{\theta, Z\}$  in region M of the right panel of figure 16, the curve denoted by  $\tilde{\lambda}_T$  in the middle panel of figure 17 shows  $\tilde{\lambda}_T$  as a function of  $\lambda^* \in [0, \tilde{\lambda}_T^*]$ . □

## Proof of Proposition 2

*Proof.* Given the Cobb-Douglas production function at the aggregate level,

$$Y_{t+1} = \Pi_{s=0}^2 \left( \frac{V_{s,t+1}}{\eta_s} \right)^{\eta_s}, \text{ MRS}_{x,s} = \frac{\eta_x V_{s,t+1}}{\eta_s V_{x,t+1}}, \text{ where } x, s \in \{0, \dots, 2\}, s \neq x. \quad (72)$$

Following the proof of lemma 1, one can derive  $MRT_{x,s} = \frac{\zeta_{x,t+1} Y_{s,t+1}}{\zeta_{s,t+1} Y_{x,t+1}}$ . Under autarky, the sectoral output markets clear domestically,  $V_{s,t} = Y_{s,t}$ . Combine it with  $MRT_{x,s} = MRS_{x,s}$  to get the share of labor input in sector  $s$  at  $\zeta_{s,t+1} = \eta_s$ .

Given the Cobb-Douglas production function at the sectoral level,

$$\frac{q_{s,t+1}\delta_{s,t}w_tL}{\alpha} = p_{s,t+1}Y_{s,t+1} = \frac{w_{t+1}\zeta_{s,t+1}L}{1-\alpha}, \quad q_{s,t+1}\frac{\delta_{s,t}}{\zeta_{s,t+1}} = \frac{\rho w_{t+1}}{w_t} = q_{x,t+1}\frac{\delta_{x,t}}{\zeta_{x,t+1}}. \quad (73)$$

Thus, a sector's rate of return is inversely related to its capital-labor ratio.

Let us solve for the sectoral investment shares  $\delta_{s,t}$  in three cases. **Case 1:** if the borrowing constraints are binding in sector 1 and 2, the sectoral investment shares are specified by equations (35)-(37). Given  $\zeta_{s,t+1} = \eta_s$ , assumption 2 ensures  $\frac{\delta_{1,t}}{\eta_1} > \frac{\delta_{2,t}}{\eta_2}$ . Combine it with equation (73) to get  $q_{1,t+1} < q_{2,t+1}$ . Thus, the agents with  $\epsilon_j \geq \underline{\epsilon}_2$  borrow to the limit and invest in sector 2. Besides,  $w_t < \bar{w}_{1,A}$  ensures  $\frac{\delta_{0,t}}{\eta_0} > \frac{\delta_{1,t}}{\eta_1}$ . Combine it with equation (73) to get  $q_{0,t+1} < q_{1,t+1}$ . Thus, the agents with  $\epsilon_j \in [\underline{\epsilon}_1, \underline{\epsilon}_2)$  borrow to the limit and invest in sector 1. Thus, assumption 2 and  $w_t < \bar{w}_{1,A}$  are the necessary and sufficient condition for this case to arise. **Case 2:** use the same logic to prove that  $w_t \in [\bar{w}_{1,A}, \bar{w}_{2,A})$  ensure the binding (slack) borrowing constraints in sector 2 (1). In this case,  $\delta_{2,t}$  is still specified by equation (35), while the capital-labor ratio equalizes in sector 0 and 1, i.e.,  $\delta_{s,t} = \frac{\eta_s}{\eta_0 + \eta_1}(1 - \delta_{2,t})$  for  $s \in \{0, 1\}$ . **Case 3:** use the same logic to prove that  $w_t > \bar{w}_{2,A}$  ensure the efficient sectoral investment shares, i.e.,  $\delta_{s,t} = \eta_s$ .

Next, derive the conditions for the unique, autarkic steady state. Under autarky,  $V_{s,t} = Y_{s,t}$ . Combine it with  $\zeta_{s,t+1} = \eta_s$  and equations (35)-(37) and (72) to get,

$$w_{t+1} = \left(\frac{w_t}{\rho}\Gamma_t\right)^\alpha, \quad \text{where } \Gamma_t = \begin{cases} \prod_{s=0}^2 \left(\frac{\delta_{s,t}}{\eta_s}\right)^{\eta_s}, & \text{if } w_t < \bar{w}_1; \\ \left(\frac{1-\delta_{2,t}}{\eta_0+\eta_1}\right)^{\eta_0+\eta_1} \left(\frac{\delta_{2,t}}{\eta_2}\right)^{\eta_2}, & \text{if } w_t \in [\bar{w}_1, \bar{w}_2); \\ 1, & \text{if } w_t > \bar{w}_2. \end{cases} \quad (74)$$

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \alpha + \alpha \frac{\partial \ln \Gamma_t}{\partial \ln w_t}, \quad \text{where } \frac{\partial \ln \Gamma_t}{\partial \ln w_t} = \begin{cases} \frac{1-\theta}{\theta} \left(1 - \frac{\eta_0}{\delta_{0,t}}\right), & \text{if } w_t < \bar{w}_1; \\ \frac{1-\theta}{\theta} \left(1 - \frac{\eta_0+\eta_1}{1-\delta_{2,t}}\right), & \text{if } w_t \in [\bar{w}_1, \bar{w}_2); \\ 1, & \text{if } w_t > \bar{w}_2. \end{cases} \quad (75)$$

Given  $\theta > \underline{\theta}_3 \equiv \frac{\alpha}{\alpha + \frac{1-\alpha}{\eta_1 + \eta_2}}$ ,  $\frac{\partial \ln w_{t+1}}{\partial \ln w_t} < 1$  holds strictly. As the slope of the law of motion for wage at any steady state is less than unity, the autarkic steady state is unique. In the following, I focus on the case of  $\theta > \underline{\theta}_3$ .

Finally, derive the threshold value  $\tilde{\lambda}_{1,A}$  such that for  $\lambda < \tilde{\lambda}_{1,A}$ , the borrowing constraints are binding in sector 1 and 2 at the autarkic steady state. As the law of motion for wage is a piecewise function with two kinks at  $w_t = \bar{w}_{1,A}$  and  $w_t = \bar{w}_{2,A}$ , one can solve  $\tilde{\lambda}_{1,A}$  by making the first kink point a steady state, i.e.,  $w_A = \bar{w}_{1,A}$  and  $\frac{\delta_{0,A}}{\eta_0} = \frac{\delta_{1,A}}{\eta_1} = \frac{1-\delta_{2,A}}{\eta_0+\eta_1}$ . Let  $\mathbb{D}_1 \equiv \frac{(1-\tilde{\lambda}_{1,A})^{\frac{1}{1-\theta}}}{1-\theta}$ . Thus,  $\bar{w}_2 = \mathbf{m}_2\mathbb{D}_1$  and  $\bar{w}_1 = \gamma\bar{w}_2$ . Combine them with  $w_A = \bar{w}_{1,A}$ , the definition of  $\bar{w}_{1,A}$ , and

equations (35)-(37) and (74),

$$\delta_{2,A} = \left( \frac{w_A}{w_2} \right)^{\frac{1-\theta}{\theta}} = \left( \frac{\bar{w}_{1,A}}{\mathbb{D}_1} \right)^{\frac{1-\theta}{\theta}} \mathbf{m}_2^{-\frac{1-\theta}{\theta}}, \quad (76)$$

$$\delta_{0,A} + \delta_{1,A} = 1 - \delta_{2,A} = \left( \frac{\bar{w}_{1,A}}{\mathbb{D}_1} \right)^{\frac{1-\theta}{\theta}} \mathbf{m}_2^{-\frac{1-\theta}{\theta}} \frac{(\eta_0 + \eta_1)}{\eta_1} \left( \gamma^{-\frac{1-\theta}{\theta}} - 1 \right), \quad (77)$$

$$\rho w_A^{\frac{1}{\rho}} = \Gamma_A = \left( \frac{\delta_{0,A} + \delta_{1,A}}{\eta_0 + \eta_1} \right)^{\eta_0 + \eta_1} \left( \frac{\delta_{2,A}}{\eta_2} \right)^{\eta_2}, \quad (78)$$

$$\rho \left( \frac{\bar{w}_{1,A}}{\mathbb{D}_1} \right)^{\frac{1}{\rho}} \mathbb{D}_1^{\frac{1}{\rho}} = \left( \frac{\bar{w}_{1,A}}{\mathbb{D}_1} \right)^{\frac{1-\theta}{\theta}} \mathbf{m}_2^{-\frac{1-\theta}{\theta}} \left( \frac{\gamma^{-\frac{1-\theta}{\theta}} - 1}{\eta_1} \right)^{1-\eta_2} \left( \frac{1}{\eta_2} \right)^{\eta_2} \quad (79)$$

$$\frac{\bar{w}_{1,A}}{\mathbb{D}_1} = \frac{\bar{w}_{1,A} \gamma \mathbf{m}_2}{\bar{w}_1} = \frac{\gamma \mathbf{m}_2}{\left[ 1 + \frac{\eta_0}{\eta_1} \left( 1 - \gamma^{\frac{1-\theta}{\theta}} \right) \right]^{\frac{\theta}{1-\theta}}} \quad (80)$$

$$\mathbb{D}_1^{\frac{1}{\rho}} = \left( \frac{\bar{w}_{1,A}}{\mathbb{D}_1} \right)^{\frac{1-\theta}{\theta} - \frac{1}{\rho}} \frac{1}{\rho} \mathbf{m}_2^{-\frac{1-\theta}{\theta}} \left( \frac{\gamma^{-\frac{1-\theta}{\theta}} - 1}{\eta_1} \right)^{1-\eta_2} \left( \frac{1}{\eta_2} \right)^{\eta_2} \quad (81)$$

$$(1 - \tilde{\lambda}_{1,A})^{\frac{1}{\rho(1-\theta)}} = \mathbb{K} Z_3, \Rightarrow \tilde{\lambda}_{1,A} = 1 - (\mathbb{K} Z_3)^{\rho(1-\theta)}, \quad (82)$$

where  $Z_3 \equiv \frac{\frac{1}{\rho}}{\left( \frac{\mathbf{m}_2}{1-\theta} \eta_2^{\frac{\theta}{1-\theta}} \right)^{\frac{1}{\rho}}}$  and  $\mathbb{K} \equiv \left( \frac{\gamma^{\frac{1-\theta}{\theta}}}{\eta_2 \left[ 1 + \frac{\eta_0}{\eta_1} \left( 1 - \gamma^{\frac{1-\theta}{\theta}} \right) \right]} \right)^{1 - \frac{\theta}{\rho(1-\theta)}} \left( \frac{\eta_2 (\gamma^{-\frac{1-\theta}{\theta}} - 1)}{\eta_1} \right)^{1-\eta_2} > 1$ .

By the same logic, one can derive another threshold value  $\tilde{\lambda}_{2,A} \equiv 1 - Z_3^{\rho(1-\theta)}$  such that, for  $\lambda = \tilde{\lambda}_{2,A}$ , the autarkic steady state is at the other kink point  $w_A = \bar{w}_{2,A}$ . For  $\lambda \in (\tilde{\lambda}_{1,A}, \tilde{\lambda}_{2,A})$ , the borrowing constraints are binding (slack) in sector 2 (1) at the autarkic steady state; for  $\lambda > \tilde{\lambda}_{2,A}$ , the borrowing constraints are slack in all sectors at the autarkic steady state.  $\square$

### Proof of Lemma 6

*Proof.* According to proposition 4, there is a unique steady state where  $w_A < \bar{w}_{1,A}$  and the borrowing constraints are binding in sector 1 and 2. Let  $\mathbb{H} \equiv \frac{\delta_{0,A}}{\delta_{2,A}} = \delta_{2,A}^{-1} - \gamma^{-\frac{1-\theta}{\theta}}$ . At the autarkic steady state, combine  $w_{t+1} = w_t = w_A$  with equations (35)-(37) and (74),

$$\rho w_A^{\frac{1}{\rho}} = \Gamma_A, \quad \rho \delta_{2,A}^{\frac{\theta}{\rho(1-\theta)}} \bar{w}_2^{\frac{1}{\rho}} = \delta_{2,A} \left( \frac{\mathbb{H}}{\eta_0} \right)^{\eta_0} \left( \frac{\gamma^{-\frac{1-\theta}{\theta}} - 1}{\eta_1} \right)^{\eta_1} \left( \frac{1}{\eta_2} \right)^{\eta_2} \quad (83)$$

$$\bar{w}_2^{\frac{1}{\rho}} = [\mathbb{H} + \gamma^{-\frac{1-\theta}{\theta}}]^{\frac{\theta}{\rho(1-\theta)} - 1} \mathbb{H}^{\eta_0} \frac{(\gamma^{-\frac{1-\theta}{\theta}} - 1)^{\eta_1}}{\rho \eta_0^{\eta_0} \eta_1^{\eta_1} \eta_2^{\eta_2}} \quad (84)$$

$$\frac{1}{\rho} = \left\{ \eta_0 + \left[ \frac{1}{\rho \left( \frac{1}{\theta} - 1 \right)} - 1 \right] \delta_{0,A} \right\} \frac{\partial \ln \mathbb{H}}{\partial \ln \bar{w}_2}, \quad (85)$$

$$\text{for } \theta > \underline{\theta}_3, \quad \eta_0 + \left[ \frac{1}{\rho \left( \frac{1}{\theta} - 1 \right)} - 1 \right] \delta_{0,A} > \eta_0 (1 - \delta_{0,A}), \Rightarrow \frac{\partial \ln \mathbb{H}}{\partial \ln \bar{w}_2} > 0; \quad (86)$$

$$\frac{\partial \ln \bar{w}_2}{\partial \ln \lambda} = - \frac{\lambda}{(1-\theta)(1-\lambda)} < 0, \Rightarrow \frac{\partial \ln \mathbb{H}}{\partial \ln \lambda} = \frac{\partial \ln \mathbb{H}}{\partial \ln \bar{w}_2} \frac{\partial \ln \bar{w}_2}{\partial \ln \lambda} < 0. \quad (87)$$

At the autarkic steady state, combine  $\zeta_{s,t+1} = \eta_s$  with equations (35)-(37) and (73),

$$\mu_{0,A} \equiv \frac{q_{0,A}}{q_{2,A}} = \frac{\frac{\delta_{2,A}}{\eta_2}}{\frac{\delta_{0,A}}{\eta_0}} = \frac{\eta_0}{\eta_2 \mathbb{H}}, \quad \frac{\partial \ln \mu_{0,A}}{\partial \ln \lambda} = -\frac{\partial \ln \mathbb{H}}{\partial \ln \lambda} > 0, \quad (88)$$

$$\mu_{1,A} \equiv \frac{q_{1,A}}{q_{2,A}} = \frac{\frac{\delta_{2,A}}{\eta_2}}{\frac{\delta_{1,A}}{\eta_1}} = \frac{\eta_1}{\eta_2 (\gamma^{-\frac{1-\theta}{\theta}} - 1)}, \quad \frac{\partial \ln \mu_{1,A}}{\partial \ln \lambda} = 0. \quad (89)$$

Thus, given  $\lambda^* < \lambda < \tilde{\lambda}_{1,A}$ ,  $\mu_{0,A}^* < \mu_{0,A}$  and  $\mu_{1,A}^* = \mu_{1,A}$  hold. According to equation (38), the ascending sectoral rate of return in each country implies,

$$\mu_{0,A}^* < \mu_{0,A} < \mu_{1,A}^* = \mu_{1,A} < \mu_{2,A}^* = \mu_{2,A} = 1. \quad (90)$$

For  $s \in \{0, 1, 2\}$ , use equation (73) to get

$$q_{s,t+1} = \alpha p_{s,t+1} \left( \frac{\delta_{s,t}}{\zeta_{s,t+1}} \right)^{\alpha-1} \mathbb{Y}_{t+1}, \quad \mu_{s,t+1} \equiv \frac{q_{s,t+1}}{q_{2,t+1}} = \frac{\frac{\delta_{2,t}}{\zeta_{2,t+1}}}{\frac{\delta_{s,t}}{\zeta_{s,t+1}}}, \quad (91)$$

$$\mu_{s,t+1} = \chi_{s,t+1} \left( \frac{\frac{\delta_{s,t}}{\zeta_{s,t+1}}}{\frac{\delta_{2,t}}{\zeta_{2,t+1}}} \right)^{\alpha-1} = \chi_{s,t+1} \mu_{s,t+1}^{1-\alpha}, \quad \Rightarrow \quad \chi_{s,t+1} = \mu_{s,t+1}^\alpha, \quad (92)$$

which holds under autarky as well as under free trade. Combine it with inequality (90) to get the cross-country patterns of the relative sectoral prices,

$$\chi_{0,A}^* < \chi_{0,A} < \chi_{1,A}^* = \chi_{1,A} < \chi_{2,A}^* = \chi_{2,A} = 1. \quad (93)$$

Combine  $r_t = q_{0,t+1}$  with equations (73) and (87) to get the steady-state interest rate,

$$r_A = \frac{\rho}{\frac{\delta_{0,A}}{\eta_0}} = \rho \eta_0 \left( 1 + \frac{1}{\mathbb{H} \gamma^{\frac{1-\theta}{\theta}}} \right), \quad \frac{\partial r_A}{\partial \mathbb{H}} < 0, \quad \Rightarrow \quad \frac{\partial r_A}{\partial \lambda} = \frac{\partial r_A}{\partial \mathbb{H}} \frac{\partial \mathbb{H}}{\partial \lambda} > 0, \quad \Rightarrow \quad r_A^* < r_A. \quad (94)$$

□

## Proof of Proposition 5

*Proof.* Under free trade, the relative sectoral prices in North are aligned to the world levels,  $\chi_{s,t} = \chi_s^*$ , and so are the relative sectoral rental prices of capital,  $\mu_{s,t} = \chi_{s,t}^{\frac{1}{\alpha}} = (\chi_s^*)^{\frac{1}{\alpha}} = \mu_s^*$ , according to equation (92).

For expositional convenience, define  $\bar{w}_0 \equiv 0$  and  $\bar{w}_3 \equiv \infty$ . As explained in subsection 5.3.2, trade integration may allow North to offshore the low-return sectors. Let  $\mathfrak{a} \equiv \min\{z : \delta_{z,t} > 0\}$  denote the lowest sector index among all **active** sectors in North. For  $w_t \in (\bar{w}_{\mathfrak{a}}, \bar{w}_{\mathfrak{a}+1})$ , North offshores sector  $u < \mathfrak{a}$  and specializes in sector  $s \geq \mathfrak{a}$ .

Combine  $\mu_{s,t+1} = \mu_s^*$  with equation (91) and the labor market clearing condition,

$$\zeta_{s,t+1} = \frac{\mu_s^*}{\frac{\delta_{2,t}}{\zeta_{2,t+1}}} \delta_{s,t}, \quad \sum_{v=\mathfrak{a}}^2 \zeta_{v,t+1} = 1, \quad \frac{\delta_{2,t}}{\zeta_{2,t+1}} = \sum_{v=\mathfrak{a}}^2 \mu_v^* \delta_{v,t}, \quad \frac{\delta_{s,t}}{\zeta_{s,t+1}} = \frac{\sum_{v=\mathfrak{a}}^2 \mu_v^* \delta_{v,t}}{\mu_s^*} \quad (95)$$

Let  $\Gamma_t \equiv \frac{\sum_{v=a}^2 \mu_v^* \delta_{v,t}}{\prod_{v=0}^2 (\mu_v^*)^{\eta_v}}$ . Derive the law of motion for wage under free from the aggregate production function and the national income-expenditure identity,

$$Y_{t+1} = \prod_{v=0}^2 \left( \frac{V_{v,t+1}}{\eta_v} \right)^{\eta_v}, \quad p_{s,t+1} \frac{V_{s,t+1}}{\eta_s} = Y_{t+1}, \quad \prod_{v=0}^2 p_{v,t+1}^{\eta_v} = 1, \quad p_{s,t+1} = p_{2,t+1} \chi_s^*, \quad (96)$$

$$p_{2,t+1} = \frac{1}{\prod_{v=0}^2 (\chi_v^*)^{\eta_v}}, \quad p_{s,t+1} = \frac{\chi_s^*}{\prod_{v=0}^2 (\chi_v^*)^{\eta_v}} = \left[ \frac{\mu_s^*}{\prod_{v=0}^2 (\mu_v^*)^{\eta_v}} \right]^\alpha \quad (97)$$

$$p_{s,t+1} Y_{s,t+1} = \left[ \frac{\mu_s^*}{\prod_{v=0}^2 (\mu_v^*)^{\eta_v}} \right]^\alpha \left( \frac{\delta_{s,t}}{\zeta_{s,t+1}} \right)^\alpha \zeta_{s,t+1} \mathbb{Y}_{t+1} = \zeta_{s,t+1} \Gamma_t^\alpha \mathbb{Y}_{t+1}, \quad (98)$$

$$w_{t+1} = \frac{(1-\alpha)Y_{t+1}}{L} = \frac{(1-\alpha) \sum_{v=a}^2 p_{v,t+1} Y_{v,t+1}}{L} = \left( \frac{w_t}{\rho} \Gamma_t \right)^\alpha. \quad (99)$$

According to lemma 6, all sectors are active in the autarkic steady state,  $a = 0$ , with  $\mu_{0,A}^* < \mu_{0,A} < \mu_{1,A} = \mu_{1,A}^* < 1$ . Given  $w_0 = w_A$ , the static gains from trade are measured by the response of the aggregate efficiency index.

$$\frac{\partial \ln w_1}{\partial \ln \chi_1} = \frac{\partial \ln \Gamma_1}{\partial \ln \mu_1} = \frac{\mu_0^* - \mu_{0,A}}{\frac{\mu_{0,A}^*}{\eta_1 + \eta_2} + \frac{\mu_{0,A}}{1 - (\eta_1 + \eta_2)}}, \quad (100)$$

which is equivalent to equation (28) in the two-sector setting. Given  $\lambda^* < \lambda < \tilde{\lambda}_{1,A}$ , the lower  $\lambda^*$ , the larger the heterogeneity in financial development  $\lambda - \lambda^*$ , the larger the international relative price differential  $\chi_{0,A} - \chi_0^*$  and the international relative rate-of-return differential in sector 0,  $\mu_{0,A} - \mu_0^*$ , the larger the trade flows and the static gains.

From period  $t = 1$  on, the rise in national income triggers the sectoral investment reallocation towards the high-MIR, high-return sectors along the extensive margin, which improves aggregate efficiency indicator  $\Gamma_t$  and raises national income in period  $t + 1$ .

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - (1-\alpha) + \alpha \sum_{s=0}^2 \frac{\partial \ln w_{t+1}}{\partial \ln \delta_{s,t}} \frac{\partial \ln \delta_{s,t}}{\partial \ln w_t} = \alpha + \alpha \frac{\frac{1-\theta}{\theta}}{1 + \frac{1}{(\frac{\mu_1^*}{\mu_0^*} - 1)\delta_{1,t} + (\frac{1}{\mu_0^*} - 1)\delta_{2,t}}}, \quad (101)$$

which is similar as equation (29) in the two sector-setting. The dynamics of national income are driven by the competition of the decreasing MRK effect and the investment reallocation effect in the case of  $w_t < \bar{w}_1$ . Given  $\lambda$ , the lower the  $\lambda^*$ , the lower the  $\mu_0^*$ , the stronger the investment reallocation effect, the larger the dynamic gains from trade.

If  $\lambda - \lambda^*$  is sufficiently large, the static and the dynamic gains from trade can be so strong that North fully offshores the low-MIR, low-return sector(s).  $\square$