Upstream Capital Flows, Intangible Investment, and Allocative Efficiency

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November 2019

Abstract

The recent decades have witnessed upstream financial flows and the surge of intangible capital in advanced economies, while the growth rate of intangible capital differs across countries. This paper explores a theoretical mechanism linking financial flows with the heterogeneous growth rate of intangible capital. In a model where only tangible capital serves as collateral for external funding, financial inflows have opposite short-run and long-run impacts on the within-firm, intangible-tangible investment ratio and the productivity. In particular, the elasticity of domestic investment is a key factor determining whether and how far financial inflows stimulate intangible investment and lead to productivity gains. Thus, country-specific market frictions and regulatory requirements that reduce the elasticity of domestic investment may help explain the heterogeneous growth rate of intangible capital in the recent era of financial globalization.

Keywords: financial frictions, upstream financial flows, heterogeneous pledgeability, intangible investment, investment elasticity, minimum investment requirements, wealth inequality

JEL Classification: E22, E25, F41

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I thank Debasis Bandyopadhyay, Andres Bellofatto, Shutao Cao, Qingyuan Du, Benedikt Heid, Simona Fabrizi, Chung Tran, Murat Ungor, Lawrence Uren, Dennis Wesselbaum, Fang Yao, Yao Yao, and the seminar participants at the University of Auckland, Victoria University of Wellington, the University of Otago for helpful comments and suggestions. Financial support from the University of Auckland Business School is sincerely acknowledged.
Since the late 1990s, intangible capital (e.g., computerized information, patents and brands, and organizational capital) has become highly important for production in advanced economies. In particular, intangible investments exceeded the tangibles in the United States, the United Kingdom, France, Finland, and Sweden between 2000-2013 (Corrado et al., 2018), while the growth rate of intangible investment differs across countries (Demmou et al., 2019). Featuring intangible capital in macroeconomic models becomes increasingly relevant for the analysis of investment and productivity dynamics in the knowledge-based economies.

As the intangibles are hard to liquidate, they usually do not serve as collateral for loans (Döttling et al., 2018; Falato et al., 2018), while the tangibles are commonly pledged as collateral (Eisfeldt and Rampini, 2009). Given heterogeneous pledgeability between the intangibles and the tangibles, a recent literature has explored the implications of rising intangible capital to allocative efficiency in the *closed-economy* setting (Dell’ariccia et al., 2017; Döttling and Perotti, 2017; Giglio and Severo, 2012; Lopez and Moppett, 2018; Wang, 2017).

By introducing heterogeneous pledgeability in an *open economy* setting, we identify within-firm, intangible-tangible investment composition as a novel channel through which financial flows have the opposite short-run and long-run effects on intangible investment and allocative efficiency. In particular, the elasticity of domestic investment becomes a key factor determining whether and how far financial inflows stimulate intangible investment and lead to productivity gains. Thus, country-specific market frictions and regulatory requirements that weaken the elasticity of domestic investment may help explain the heterogeneous growth rate of intangible capital in the recent era of financial globalization.

The recent financial globalization has a prominent feature, i.e., financial flows are “upstream” from poor to rich countries (Prasad et al., 2006). The literature has shown that upstream financial flows are an equilibrium outcome, if rich countries are more financially developed than poor countries (Caballero et al., 2008; Gourinchas and Rey, 2014; Ju and Wei, 2010; Mendoza et al., 2009; von Hagen and Zhang, 2014). Upstream financial flows are regarded as one of the causes for the declining interest rates in advanced economies\(^1\) (Bernanke, 2011; Caballero and Krishnamurthy, 2009). Consider a model where both tangible capital and intangible capital are essential for production, while only the tangibles can be pledged as collateral. By boosting up the collateral value of tangibles, the inflows of cheap foreign funds lower the unit cost of tangibles relative to that of intangibles,\(^2\) which induces entrepreneurs to shift the investment from the intangibles towards the tangibles. *This prediction is opposite to the empirical fact of the rising intangible-tangible investment ratio in advanced economies.*\(^3\)

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\(^1\)The recent literature has two major hypotheses for the global decline of interest rates. The global savings glut hypothesis (Bernanke, 2005) relies on the excessive supply of savings from fast-growing emerging markets, while the secular stagnation hypothesis (Summers, 2014) focuses on the paucity of investment opportunities in advanced economies. Rachel and Smith (2017) and Crews et al. (2016) compare the empirical plausibility of the two hypotheses. See CEA (2015) for a comprehensive literature survey on alternative causes.

\(^2\)As the intangibles are financed fully with internal funds, the unit cost of intangible is constant at one. As entrepreneurs can borrow up to a fraction of the present value of tangibles and use own funds to cover the rest, the unit cost of tangibles is less than one and negatively related to the collateral value of tangibles. Under autarky, the unit-cost differential induces entrepreneurs to invest inefficiently more (less) in tangibles (intangibles).

\(^3\)Changes in the intangible-tangible investment ratio can be a natural consequence of technological shifts. Taking it as given, we analyze the impacts of an exogenous change in the world interest rate on this ratio.
We offer a tractable, analytical framework to address this puzzle. Besides the interest rate effect, financial inflows also stimulate the domestic capital formation, while the resulting decline in the marginal product of capital (MPK, hereafter) reduces the collateral value of tangibles over time. Upon financial inflows, the interest rate effect dominates the MPK effect, which raises the collateral value and reduces the unit cost of tangibles. Thus, entrepreneurs shift the investment towards the tangibles, which worsens allocative efficiency in the short run. Along the convergence path, the rise in aggregate income stimulates domestic investment and attract more financial inflows. As the interest rate is constant at the world level, the MPK effect lowers the collateral value and raises the unit cost of tangibles. Thus, entrepreneurs shift the investment towards the intangibles, which improves allocative efficiency over time. In this paper, allocative efficiency is measured by the output-input ratio of the individual project for capital formation. For simplicity, we call it the within-project or within-firm productivity. By changing the unit-cost differential, financial inflows reduce the within-firm allocative efficiency and productivity in the short run and raise them over time. This is the first finding of our paper.

Can the productivity eventually exceed its initial level? For a given interest rate decline, the more elastic the domestic investment demand, the larger the financial inflows and the domestic investment expansion, the stronger the MPK effects, the smaller the initial worsening and the larger the subsequent improvements in allocative efficiency, the more likely the intangible-tangible investment ratio and the productivity exceed their initial levels in the long run.

Our paper features a particular barrier to entrepreneurial entry and characterize the elasticity of domestic investment analytically. Besides collateral constraints, we make two more assumptions. First, the individual project is subject to a minimum investment requirement (MIR, hereafter). Second, agents differ in net wealth. If the collateral constraints are binding, only those with sufficiently high net wealth can meet the MIR and run the project, and we call them entrepreneurs. The collateral constraints and the MIR jointly act as a barrier to entrepreneurial entry. A fall in the interest rate as well as a rise in aggregate income allow more agents to overcome the MIR and become entrepreneurs. The smaller the net wealth dispersion, the more elastic the mass of entrepreneurs with respect to the interest rate and the income changes, the more elastic the domestic investment along the extensive margin. If wealth dispersion is sufficiently low, the investment elasticity can be so high that the intangible-tangible capital ratio and the productivity exceed their initial levels. This is the second finding of our paper.

Income and wealth inequality has recently made a comeback after declining in the first half of the twentieth century (Piketty, 2014). Mendoza et al. (2007), Antunes and Cavalcanti (2013), and Jaumotte et al. (2013) show that financial globalization affects wealth inequality and welfare. However, the impacts of inequality on the consequences of financial globalization are not well studied. We contribute to this literature by offering a novel mechanism, i.e., by reducing the investment elasticity along the extensive margin, rising wealth inequality dampens financial inflows and weakens the intangible-driven productivity growth.

Using a panel of 32 OECD countries and 30 industries from 1990 to 2014, Demmou et al. (2019) find that financial frictions contribute to the cross-country differences in the growth rate of intangible assets at the sectoral level. Besides the barrier to entrepreneurial entry featured in this paper, other market frictions and regulatory requirements may also reduce the elasticity
of sectoral investment. Thus, the mechanism behind our findings may apply more broadly. For example, one may use the investment elasticity as an instrument and test its empirical relevance for the growth rate of intangible capital at the country and at the sectoral level.

The first two findings are obtained in the setting where financial inflows lower the interest rate marginally and the model economy converges to a new steady state in the neighborhood of the initial one. By moving from the marginal analysis to the global analysis, we study the productivity dynamics over the entire state spaces and obtain the third finding, i.e., the world interest rate change may have disproportional and asymmetric impacts on productivity, if it shifts the model economy from an equilibrium with multiple steady states to the one with a unique steady state. To be specific, the endogenous extensive margin amplifies the responses of domestic investment and the productivity under financial integration. Thus, multiple steady states may arise if the world interest rate is moderate, while there is a unique steady state if the world interest rate is either sufficiently high or low.

Our third finding offers a new perspective for understanding the impacts of the current U.S. interest rate hikes on intangible investment and productivity. Given that the world interest rate has stayed at the record low level for nearly a decade, if the U.S. interest rate hikes lead to a moderate rise in the world interest rate, it may have little impacts on income and allocative efficiency in small open economies. However, if the U.S. interest rate hikes are moderately large, the impacts on income and allocative efficiency can be disproportionately larger. This global analysis approach differs from the marginal analysis approach commonly used in the business cycles literature (Bernanke et al., 1999; Carlstrom and Fuerst, 1997; Kiyotaki and Moore, 1997) which takes the linear approximation around a deterministic steady state.

Related Literature The literature has identified various channels through which financial integration improves allocative efficiency (Acemoglu and Zilibotti, 1997; Alessandria and Qian, 2005; Obstfeld, 1994; Tressel and Verdier, 2011; Varela, 2018). Most research features cross-firm and/or cross-project resource reallocation as the channel through which financial integration has the positive effect on aggregate productivity. In contrast, we highlight within-firm investment reallocation as a novel channel through which financial flows may have opposite short-run versus long-run effects on firm-level productivity.

In Mendoza and Yue (2012), domestic and imported input varieties are imperfect substitutes in the production, while some imported input varieties require working capital financing from foreign creditors. Sovereign defaults keep firms away from world credit markets and the imported varieties have to be replaced by imperfect substitutes. The distortions on the within-firm composition of the imported and the domestic input varieties lower the firm’s productivity. In our model, the tangibles and the intangibles are imperfect substitutes in capital formation, while only the tangibles can serve as collateral. By affecting the unit-cost differential, financial inflows trigger the within-firm investment composition. Our paper shares the similar mecha-

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4In our model, the dynamic and steady-state patterns are fundamentally different across two ranges of the world interest rate. Brunnermeier and Sannikov (2014) also conduct the global analysis and find a bimodal stationary distribution over the entire state space, i.e., a stable normal regime and a volatile crisis regime.

5The recent trade literature pioneered by Melitz (2003) also assume the exogenous productivity distribution among firms and trade liberalization triggers the cross-firm reallocation effect.
nisms as Mendoza and Yue (2012) through which financial flows affect allocative efficiency. The decreasing MPK effect is key to the dynamics and the long-run level of productivity in our model, while it is absent in Mendoza and Yue (2012), due to the fixed capital stock.

Although the literature has proposed various channels through which financial integration may foster productivity growth, the empirical evidence is rather mixed (Kose et al., 2009b). Bonfiglioli (2008) and Bekaert et al. (2011) offer the evidence on the positive productivity effect of financial integration, while Kose et al. (2009a) find that external debt is negatively correlated with TFP growth and this negative relationship is partially attenuated in economies with better-developed financial markets and better institutional quality. Kose et al. (2011) point out that, in order to gain from financial integration, an economy needs to attain certain “threshold” levels of institutional quality and financial development. Our model predicts that debt flows may trigger the non-monotonic productivity responses, and the long-run productivity gains or losses depend critically on the elasticity of domestic investment. Besides financial frictions and the MIR, other market frictions and regulatory requirements may hamper entrepreneurial entry and cause incumbent firms to grow more slowly (Gutierrez et al., 2019; Klapper et al., 2006), which lowers the investment elasticity. Thus, we call for further empirical research on identifying the fundamental and institutional factors crucial for the investment elasticity as well as exploring the relevant threshold conditions for long-run productivity gains. In this sense, our theoretical findings support and broaden the insights of Kose et al. (2011, 2009a).

In the traditional theory of investment (Hayashi, 1982; Lucas and Prescott, 1971; Tobin, 1969), convex capital adjustment costs (CAC, hereafter) are a common assumption for preventing firms from changing their capital stock too quickly, while and the exogenous convexity of CAC determines the elasticity of aggregate investment. Although convex CAC are consistent with the empirical data on aggregate investment, the data on firm-level investment is rather lumpy and volatile. In a RBC model with idiosyncratic shocks to capital formation, Wang and Wen (2012) argue that collateralized borrowing can give rise to convex adjustment costs at the aggregate level yet at the same time generate lumpiness in plant-level investment, and the elasticity of aggregate investment depends on the distribution of idiosyncratic shocks. In this paper, we show analytically that, in the presence of financial frictions and the MIR, the elasticity of aggregate investment is inversely related to the degree of wealth inequality.

The MIR has been used in the literature to characterize investment indivisibility at the individual level, an important feature of business ideas, physical and human capital (Aghion and Bolton, 1997; Banerjee and Newman, 1993; Banerjee and Moll, 2010; Galor and Zeira, 1993; Piketty, 1997). Recently, the firm-level fixed costs or entry costs are widely introduced in the quantitative macroeconomic models (Barseghyan and DiCecio, 2011; Buera et al., 2011; Erosa and Hidalgo-Cabrillana, 2008; Midrigan and Xu, 2014), which ensures that the individual investment is above a minimum scale in equilibrium. Technically, either the MIR or fixed costs makes the individual production set non-convex so that a change in aggregate income affects the individual’s net wealth and the mass of investors becomes endogenous. For analytical tractability, we use the MIR to endogenize the extensive margin of investment in this model.

Although the intangibles do not serve as collateral for loans, a recent literature shows that they can be financed by equity or labor contracts (Döttling et al., 2018; Kiyotaki and Zhang, 2001).
As shown in appendix A.3, our model predictions still hold, as long as the tangibles have the external financing advantage over the intangibles.

The rest of the paper is structured as follows. Section 1 sets up the model and section 2 analyzes the autarkic equilibrium. Section 3 studies the productivity dynamics under financial integration. Section 4 analyzes the model dynamics over the entire state space and explores the implications of equilibrium shifts on productivity. Section 5 concludes with some remarks. Appendices include supporting materials and technical proofs.

1 The Model Setting

This model features three key assumptions in a two-period, overlapping-generation framework: (1) the heterogeneous pledgeability between tangible and intangible investments, (2) the MIR for the project of capital formation, and (3) the heterogeneity in individual net wealth.

Consider a small country N in the world economy. A continuum of agents indexed by $j \in [0, 1]$ are born every period and they live for two periods, young and old. The population size of each generation is constant and normalized at unity. Agents only consume when old.

When young, agent $j$ supplies its labor endowment $l_j = (1 - \theta)\varepsilon_j$ inelastically to the market, where $\varepsilon_j \in (1, \infty)$ follows the Pareto distribution, with the cumulative distribution function $G(\varepsilon_j) = 1 - \varepsilon_j^{-\theta}$ and $\theta \in (0, 1)$ denotes the inverse of the shape parameter. Thus, the aggregate labor supply is constant at $L = \int_1^\infty l_j dG(\varepsilon_j) = 1$.

A final good is internationally tradable and chosen as the numeraire. It can be consumed or converted into tangible and intangible capital, which are non-tradable and available in the next period. Capital and labor are combined in the Cobb-Douglas fashion for the production of the final good contemporaneously. Capital fully depreciates after the production. The markets for final goods, capital, and labor are perfectly competitive. There is no uncertainty in the model economy. $Y_t$ denotes aggregate output of final goods, $L = 1$ and $K_t$ denote the aggregate inputs of labor and capital, $w_t$ and $q_t$ denote the wage rate and the rental price of capital in period $t$.

$$Y_t = \left(\frac{K_t}{\alpha}\right)^\alpha \left(\frac{L}{1 - \alpha}\right)^{1 - \alpha}, \quad q_t K_t = \alpha Y_t \quad \text{and} \quad w_t L = (1 - \alpha)Y_t. \quad (1)$$

When young, agents have two options to save the labor income $n_{jt} = w_t l_j$ for future consumption: lending at the gross interest rate $r_t$ and running a project for capital formation. For

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6Given that agents are endowed with labor only when young and they consume only when old, domestic saving is interest-inelastic, which simplifies the credit supply dynamics and allows us to focus on the dynamics of entrepreneurial credit demand and investment. Appendix A.6 shows that, if these two assumptions are relaxed, young agents optimally choose consumption and saving so that domestic saving becomes interest-elastic; as long as domestic saving is not perfectly interest-elastic, our findings still hold qualitatively.

As mentioned in remark 2 in appendix A.3, we choose the overlapping-generation framework to ensure that some agents cannot overcome the borrowing constraints by accumulating their savings over time.

7Pareto distribution is widely used in the literature to feature the income and wealth distribution (Atkinson et al., 2011; Gabaix, 2009; Jones, 2015). The top tail of income distribution is very well approximated by a Pareto distribution (Kuznets and Jenks, 1953; Piketty and Saez, 2003). Besides, assuming that the individual labor endowment follows the Pareto distribution allows us to get the analytical solution.
the second option, by investing \( m_{jt} \) units of final goods in period \( t \), agent \( j \) gets \( k_{j,T,t+1} \) units of tangible capital and \( k_{j,t,t+1} \) units of intangible capital in period \( t+1 \), which jointly yields 

\[
q_{t+1} = (k_{j,t+1})^\eta \left( \frac{k_{j,T,t+1}}{1-\eta} \right)^{1-\eta} \]

units of capital service for the final goods production.\(^8\) If the project meets the MIR, \(^9\) \( m_{jt} = k_{j,t,t+1} + k_{j,T,t+1} \geq m \); otherwise, the project has zero output. The MIR features a barrier to entrepreneurial entry, which endogenizes the mass of investors. As shown later, the endogenous extensive margin of investment is key to our major findings.\(^10\)

Suppose that agent \( j \) can meet the MIR.\(^11\) Let \( a_{jt} \equiv \frac{k_{j,t+1}}{m_{jt}} \) denote the intangible fraction of its investment. For each unit of investment in period \( t \), agent \( j \) puts \( a_{jt} \) as the intangibles and \( 1 - a_{jt} \) as the tangibles, which yields \( \Phi_{jt} \equiv \left( \frac{a_{jt}}{\eta} \right) \left( \frac{1-a_{jt}}{1-\eta} \right)^{1-\eta} \) units of capital service in period \( t+1 \). Thus, the project has a productivity \( \Phi_{jt} \) and a gross rate of return \( q_{t+1} \Phi_{jt} \). The individual capital formation function can be rewritten as 

\[
k_{j,t+1} = \Phi_{jt} m_{jt}. \]

If the project rate of return exceeds the interest rate,\(^12\) the agent prefers to borrow as much as possible and gain from leveraged investment. How much can it borrow? If the agent defaults, intangible capital is lost and the project has zero output.\(^13\) The best that lenders can do is to seize and liquidate tangible capital in period \( t+1 \). After deducting liquidation costs, the lenders get \( \lambda p_{t+1} k_{j,T,t+1} \), where \( p_{t+1} \) denotes the price of tangible capital and \( \lambda \in (0,1] \) measures the level of financial development. In the no-default equilibrium, agent \( j \) borrows up to the collateral value of tangible capital and cover the gap with own funds,\(^14\)

\[
b_{jt} = \frac{\lambda p_{t+1} k_{j,T,t+1}}{r_t}, \quad \text{and} \quad k_{j,t,t+1} + k_{j,T,t+1} - b_{jt} = n_{jt} = w_t l_j. \tag{2} \]

In the following, we explain the implications of the three key assumptions to individual choices and aggregate variables.

### Heterogeneous Pledgeability and Endogenous Intangible-Tangible Investment

As the revenues of intangible capital cannot be pledged for external financing, agent \( j \) has to finance its intangible investment with own funds. For agent \( j \), the unit cost of intangibles

\(^8\)Giglio and Severo (2012) and McGrattan (2017) also take the Cobb-Douglas form for capital formation, while Caggesse and Pérez-Olive (2018) assume that tangible and intangible capital are perfect complements. As shown in appendix A.1, our findings still hold qualitatively if capital formation takes a general functional form with the constant elasticity of substitution as in Falato et al. (2018).

\(^9\)One could assume alternatively that the MIR applies to tangible investment, \( k_{j,T,t+1} \geq m \). As shown in appendix A.2, it only complicates the technical analysis, without changing our key findings.

\(^10\)Appendix D analyzes an alternative model with an exogenous mass of investors. Due to the absence of the extensive margin effect, financial inflows strictly lower the productivity in the long run.

\(^11\)Equation (7) specifies the condition under which this is the case.

\(^12\)As shown in equation (3), the intangible fraction of investment in equilibrium is common for the agents who can meet the MIR and so is the project productivity. One can drop the subscript \( j \). In equilibrium, the project’s gross rate of return must be no less than the interest rate, \( q_{t+1} \Phi_t \geq r_t \); otherwise, nobody would run the project.

\(^13\)Excluding the intangibles from the loan contract facilitates an analytical exploration of the model mechanism. As shown in appendix A.3, our findings still hold in the setting where both the tangibles and the intangibles can serve as collateral, but the former has a higher degree of pledgeability than the latter.

\(^14\)Matsuyama (2004) assumes that an agent can borrow against its future project revenue, while we assume that an agent can borrow against the market value of tangible capital. As explained in remark 1 of appendix A.3, these two assumptions are analytically equivalent.
is \( u_{t,j} = 1 \) and the unit return is the marginal revenue of intangibles. In contrast, for each unit of tangible investment, the agent can borrow up to \( \frac{\lambda p_{t+1}}{r_t} \) and the unit cost of tangibles is \( u_{T,t} = 1 - \frac{\lambda p_{t+1}}{r_t} \), while the unit return is the marginal revenue minus the debt repayment per unit of tangibles. Agent \( j \) allocates its net wealth between the two types of investments optimally so that their internal rates of return equalize, 
\[
\frac{q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,T,t+1}}}{u_{t,j}} = \frac{q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,T,t+1}}}{u_{T,t}} \equiv \frac{a_{j,t}}{1 - a_{j,t}} \left( 1 - \frac{\lambda p_{t+1}}{q_{t+1}} \right) \left( 1 - \frac{\eta}{1 - \eta} \right)^{-\eta},
\]

or equivalently,\(^{15}\)
\[
u_{T,t} = 1 - \frac{\lambda p_{t+1}}{r_t} = \frac{a_{j,t}}{1 - a_{j,t}} \left( 1 - \frac{\lambda p_{t+1}}{q_{t+1}} \right) \left( 1 - \frac{\eta}{1 - \eta} \right)^{-\eta}.
\]

Thus, the optimal choice of \( a_{i,t} \) is a function of the parameters (\( \lambda \) and \( \eta \)) and the market prices (\( r_t, p_{t+1}, \) and \( q_{t+1} \)). In other words, the agents who meet the MIR optimally choose the same intangible fraction of investment. Hereafter, we drop off subscript \( j \) and use \( a_t \) to denote it.

Agent \( j \) equalizes its marginal revenue of tangible capital to the market price,
\[
q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,T,t+1}} = q_{t+1} \Phi_t \frac{1 - \eta}{1 - a_t} = p_{t+1}, \quad \text{where} \quad \Phi_t \equiv \left( \frac{a_t}{\eta} \right) \left( \frac{1 - a_t}{1 - \eta} \right)^{1-\eta}
\]

denotes the allocative efficiency and the productivity of capital formation in equilibrium. As tangible capital is an input for the production of capital service used in the final goods production, one should distinguish between the market price of tangible capital \( p_{t+1} \) and the rental price of capital service \( q_{t+1} \). For \( a_t = \eta \), the allocation is efficient and the productivity is maximized at unity, \( \Phi_t = 1 \); for \( a_t < \eta \), the lower the the \( a_t \), the less efficient the within-firm investment composition, the lower the productivity of capital formation, \( \frac{\partial \ln \Phi_t}{\partial \ln a_t} = \frac{\eta - a_t}{1 - a_t} > 0 \).

Plug equation (4) in (3) and substitute away \( p_{t+1} \),
\[
u_{T,t} = 1 - \frac{1 - \eta}{1 - a_t} \frac{\lambda}{\psi_t} = \frac{a_t}{1 - a_t} \frac{1 - \eta}{\eta} (1 - \lambda), \quad \text{where} \quad \psi_t \equiv \frac{r_t}{q_{t+1} \Phi_t} < 1
\]

denotes the normalized interest rate. For \( r_t < q_{t+1} \Phi_t \) or equivalently \( \psi_t < 1 \), the borrowing constraints are binding and, according to equation (5), \( a_t < \eta \) holds.\(^{16}\) Intuitively, by creating the unit-cost differential \( u_{T,t} - u_{T,t} = 1 - u_{T,t} > 0 \), the heterogeneous pledgeability induces agent \( j \) to invest inefficiently more (less) in the tangibles (intangibles), which distorts allocative efficiency and productivity. The lower the \( \psi_t \), the higher the pledgeable value per unit of tangibles, the lower the unit cost of tangibles, the larger the unit-cost differential, the more the agents prefer tangible investment, the lower the \( a_t \) and \( \Phi_t \). The positive comovement of \( a_t \) and \( \psi_t \) reflects the individual optimization of tangibles-intangibles composition.

**Remark 1:** heterogeneous pledgeability creates the unit-cost differential, which determines the within-firm composition of tangible and intangible investments.

As shown in section 3, the within-firm allocative efficiency and the productivity fall upon financial inflows, while they rise with aggregate income over time. One can use equation (5) to explain intuitively the logic behind this result. Suppose that country \( N \) witnesses financial inflows from period 0 on. In period 0, the interest rate falls to the world level, while the

\(^{15}\)See the proof of Proposition 1 for a formal technical analysis of agent \( j \)'s optimization.

\(^{16}\)If the borrowing constraints are weakly binding, \( \psi_t = 1 \) and the allocation is efficient with \( a_t = \eta \) and \( \Phi_t = 1 \).
domestic investment expansion lowers the MPK so that the rental price of capital also falls in period 1. The responses of $\psi$ and $a_0$ depend on the relative size of the two forces.

$$\frac{\partial \ln a_0}{\partial \ln r_0} = \frac{\partial \ln \psi_0}{\partial \ln r_0} - \frac{\partial \ln q_1 \Phi_0}{\partial \ln r_0}$$

If the interest rate effect dominates the MPK effect, $\psi_0$ falls and so do $a_0$ and $\Phi_0$.

From period $t = 1$ on, the rise in aggregate income stimulates the domestic investment demand and attract more financial inflows, which further reduces the MPK and the rental price of capital. Meanwhile, the interest rate is constant at the world level. Thus, $\psi_t$ strictly rises over time and so do $a_t$ and $\Phi_t$.

Endogenizing the Mass of Entrepreneurs with the Collateral Constraints and the MIR

If $q_{t+1} \Phi_t > r_t$, the agent puts its entire labor income in the project and borrows to the limit. Define the unit cost of investment as the weighted average of the two unit costs,

$$u_t \equiv a_t u_{I,t} + (1 - a_t) u_{T,t} = 1 - \frac{\lambda(1 - \eta)}{\psi_t} = \frac{a_t}{\eta} [1 - \lambda(1 - \eta)].$$

For each unit of investment, the agent puts down $u_t$ units of own funds. A cutoff value $\varepsilon_t$ is associated with the agents who just meet the MIR, 18

$$\frac{w_t (1 - \theta) \varepsilon_t}{u_t} = m, \Rightarrow \varepsilon_t = \frac{u_t}{w_t} \frac{m}{1 - \theta}.$$

17See appendix A.5 for an alternative way of endogenizing the investment elasticity.

18Given the constant population size in each generation, the cutoff value should be specified more precisely as $\varepsilon_t = \max \left\{ 1, \frac{w_t}{m} \frac{m}{1 - \theta} \right\}$, implying the existence of the mass-of-entrepreneurs (MoE, hereafter) constraint, $\tau_t \leq 1$. Under autarky, if the borrowing constraints are binding in equilibrium, there must be some agents who cannot overcome the MIR and hence, the MoE constraint is not binding; if the borrowing constraints are slack, those who can overcome the MIR do not have strong incentive to be entrepreneurs and hence, the MoE constraint is irrelevant. Thus, one does not need to consider explicitly the MoE constraint under autarky. The MoE constraint may become binding under financial integration, as shown in the proof of proposition 3. For simplicity, we focus on the case of $\varepsilon_t > 1$ in section 3, while the proof of proposition 3 covers the case where the MoE constraint is binding under financial integration.
The agents with \( \epsilon_j \geq \epsilon \) can meet the MIR and they are called *entrepreneurs*, with the mass of \( \tau_t = \epsilon^{-\frac{1}{\theta}} \); when young, they invest their entire labor income in the project and borrow to the limit; when old, they get the investment revenue, repay the debt, and consume the rest, \( c_t^{e, j+1} \). The agents with \( \epsilon_j \in [1, \epsilon) \) cannot meet the MIR and they are called *households*; when young, they lend out their labor income; when old, they consume the return to their savings, \( c_t^{h, j+1} \).

\[ \tau_t = \epsilon^{-\frac{1}{\theta}}, \quad n_{j,t} = w_t l_j, \quad c_t^{e, j+1} = n_{j,t} \left[ \frac{q_{t+1} \Phi_t}{u_t} + r_t \left( - \frac{1}{u_t} - 1 \right) \right], \quad c_t^{h, j+1} = n_{j,t} r_t. \quad (8) \]

**Remark 2:** the collateral constraints and the MIR jointly endogenize the mass of entrepreneurs.

### Net Wealth Heterogeneity and Partial Elasticities of Domestic Investment Demand

Given the inelastic labor supply, the wage rate is proportional to aggregate income and determined by capital installed one period earlier, \( w_t = \frac{(1-\alpha) Y_t}{L} = \left( \frac{1-\alpha}{\alpha} \right)^\theta \). Thus, we treat the wage rate as a predetermined variable\(^{19}\) and use it as a proxy for aggregate income.

In this model, the labor income is the only source of net wealth for young agents. A rise in aggregate income raises the wage rate and the average net wealth of individual agents, while a fall in the unit cost of investment raises the leverage multiplier for entrepreneurs. In either case, each entrepreneur can borrow and invest more in the project, while more agents can meet the MIR and become entrepreneurs. Thus, the domestic investment demand rises along the intensive and the extensive margins, respectively.

\[ M_t \equiv \int_{\epsilon_j}^{\infty} m_{j,t} dG(\epsilon_j) = \frac{\delta_t w_t L}{u_t}, \quad \text{where} \quad \delta_t \equiv \int_{\epsilon_j}^{\infty} w_t l_j dG(\epsilon_j) = \epsilon_j^{1-\theta} = \tau_t^{1-\theta} \quad (9) \]

denotes the entrepreneurial net wealth share\(^{20}\) and captures the extensive-margin effect. Use equations (7)-(9) to solve for the partial elasticities of \( M_t \) with respect to the two variables,

\[
\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{\partial \ln \delta_t}{\partial \ln w_t} + \frac{\partial \ln \tau_t}{\partial \ln w_t} = 1^{19} \]

Int. margin effect = 1

Ext. margin effect = \( \frac{1}{\theta} - 1 \)

\[
\frac{\partial \ln M_t}{\partial \ln u_t} = \frac{\partial \ln \frac{1}{u_t}}{\partial \ln u_t} + \frac{\partial \ln \delta_t}{\partial \ln u_t} = \frac{1}{\theta}. \quad (10)
\]

Int. margin effect = -1

Ext. margin effect = -\( (\frac{1}{\theta} - 1) \)

For \( \theta \in (0, 1) \), the two partial elasticities (in absolute value) exceed unity, due to the extensive margin effect. The smaller the \( \theta \), the less dispersed the net wealth of young agents, the more elastic the mass of entrepreneurs \( \frac{\partial \ln \tau_t}{\partial \ln w_t} = -\frac{\partial \ln \tau_t}{\partial \ln u_t} = \frac{1}{\theta} \), the stronger the extensive margin effect \( \frac{\partial \ln \delta_t}{\partial \ln w_t} = -\frac{\partial \ln \delta_t}{\partial \ln u_t} = \frac{1}{\theta} - 1 \), the more elastic the domestic investment demand.\(^{21}\)

---

\(^{19}\)Given the inelastic aggregate labor supply, endogenous variables can be characterized as the functions of the wage rate under autarky as well as under financial integration. If the labor supply is assumed to be elastic, the wage rate turns into a jump variable, which complicates the model analysis and invalidates the model tractability.

\(^{20}\)As young agents save their entire labor income, domestic savings in period \( t \) is \( w_t L \).

\(^{21}\)One needs to distinguish between two related but different concepts. \( M_t \) specified in equation (9) refers to
Remark 3: if the borrowing constraints are binding, the partial elasticities of the domestic investment demand (in absolute value) are inversely related to wealth dispersion, due to the extensive margin effect.

Market Clearing Conditions under Autarky versus under Financial Integration

Under autarky, the markets for credit and capital clear domestically,\(^22\)

\[
\int_{\xi}^{\infty} (m_{j,t} - n_{j,t})dG(\varepsilon_j) = \int_{1}^{\xi} n_{j,t}dG(\varepsilon_j), \quad \Rightarrow \quad M_t = w_tL_t, \quad (12)
\]

\[
K_{t+1} = \int_{\xi}^{\infty} \Phi_{j,t}m_{j,t}dG(\varepsilon_j) = \Phi_tM_t. \quad (13)
\]

So far, we have focused on the case of \(q_{t+1}\Phi_t > r_t\) where the borrowing constraints are binding. If \(q_{t+1}\Phi_t = r_t\), the borrowing constraints are slack and entrepreneurs choose \(a_{j,t}\) to maximize the productivity \(\Phi_{j,t}\). As a result, the intangible fraction of investment is equal to the factor share of intangibles in capital formation, \(a_t = \eta\).\(^23\) Those who can meet the MIR do not have strong incentive to put their labor income in the project or borrow to the limit. Despite the indeterminacy at the individual level, domestic investment is still fully financed by domestic saving, and the productivity is constant at \(\Phi_t = 1\).

Definition 1. Under autarky, a market equilibrium in country \(N\) is a set of choices of agents, \(\{n_{j,t}, m_{j,t}, a_t, \Phi_t, c_{j,t}^h, c_{j,t}^b, u_t\}\), the threshold value \(\{\xi\}\), aggregate quantities \(\{Y_t, K_t, M_t\}\), and the prices \(\{q_t, w_t, r_t\}\) satisfying equations (1), (4), (6)-(8), and (12)-(13).

Country \(N\) and the rest of the world are inherently identical, except that country \(N\) is more financially developed and its population share in the world economy is negligible. Under financial integration, agents are allowed to borrow and lend abroad,\(^24\) which aligns the interest rate to the world level,\(^25\) \(r_t = r^*.\) Financial flows cover the domestic investment-savings gap.

Definition 2. Under financial integration, a market equilibrium in country \(N\) is a set of choices of agents, \(\{n_{j,t}, m_{j,t}, a_t, \Phi_t, c_{j,t}^h, c_{j,t}^b, u_t\}\), the threshold value \(\{\xi\}\), aggregate quantities \(\{Y_t, K_t, M_t\}\), and the prices \(\{q_t, w_t, r_t\}\) satisfying equations (1), (4), (6)-(8), and (13), given \(r_t = r^*.\)

At the aggregate level, \(M_t\) units of final goods are invested in period \(t\), which yields \(K_{t+1} = \Phi_tM_t\) units of capital service in period \(t + 1\). Define the social rate of return as \(\frac{q_{t+1}K_{t+1}}{M_t} = q_{t+1}\Phi_t\).

---

\(^22\) As mentioned above, all entrepreneurial projects have the same productivity, \(\Phi_{j,t} = \Phi_t\).

\(^23\) Alternatively, one can consider the case where the borrowing constraints are weakly binding and \(q_{t+1}\Phi_t = r_t\). According to equation (6), \(\eta_r = 1\) implies \(\eta_t = \eta\).

\(^24\) Following Matsuyama (2004), we exclude FDI flows by assumption. von Hagen and Zhang (2014) analyze the joint determination of financial capital flows and FDI flows.

\(^25\) As country \(N\) has a negligible population share in the world economy, its financial opening does not affect the world interest rate and hence, we drop off the time subscript, \(r^* = r^*_t\).
2 The Autarkic Equilibrium

In this section, we use the law of motion for wage to analyze the dynamics of aggregate income and allocative efficiency under autarky. Combine equations (12)-(13) with (1) to get

\[ w_{t+1} = \left( \frac{K_{t+1}}{L^\rho} \right)^\alpha = \left( \frac{M_t \Phi_t}{L^\rho} \right)^\alpha, \]

where \( \rho = \frac{\alpha}{1-\alpha}. \)

\[ \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \left( \frac{\partial \ln M_t}{\partial \ln w_t} + \frac{\partial \ln \Phi_t}{\partial \ln w_t} \right) \left[ 1 - \left( \frac{1-\alpha}{\alpha} \right) \right]. \]

A rise in current income \( w_t \) affects capital formation \( K_{t+1} \) through two channels. First, it raises the size of domestic investment, \( M_t \). As domestic investment is fully financed by domestic saving, the investment size effect is constant at unity. Second, it may affect within-firm allocative efficiency and the productivity of capital formation, \( \Phi_t \). Given the neoclassical aggregate production function, capital formation affects aggregate income \( w_{t+1} \) less than proportionately.

In the following, we characterize the productivity effect in two cases before addressing the steady-state property of the model economy.

The Unconstrained Case

Let \( \bar{w}_A \equiv \left[ 1 - \lambda (1-\eta) \right]^{1-m} \frac{m}{1-\theta} \). For \( w_t \geq \bar{w}_A \), aggregate income is sufficiently large and so are the mass of entrepreneurs and the domestic credit demand, which aligns the interest rate with the social rate of return.\(^{26}\) In this case, the borrowing constraints are slack and the productivity is constant at \( \Phi_t = 1 \). As the productivity effect is mute and the investment size effect is constant at unity, the neoclassical effect is a convergence force that drives country N towards a steady state. The smaller the \( \alpha \), the stronger the neoclassical effect, the faster the convergence. Here, wealth inequality does not affect the aggregate dynamics if the borrowing constraints are slack.

The Constrained Case

For \( w_t < \bar{w}_A \), aggregate income is inefficiently low and so are the mass of entrepreneurs and the domestic credit demand. Thus, the interest rate is below the social rate of return so that entrepreneurs borrow to the limit. Due to the unit-cost differential, they invest inefficiently less (more) in the intangibles (tangibles) and the productivity is inefficiently low.

According to equations (4) and (6), the productivity is an increasing function of \( a_t \), while the latter is proportional to \( u_t \). According to equations (9) and (11), the domestic investment demand is a decreasing, log-linear function of \( u_t \). Thus, \( u_t \) is key to both the productivity effect and the investment size effect specified in equation (15). Next, we use the domestic investment-saving diagram to determine \( u_t \) and then characterize the two effects analytically.

Given \( w_t \), point \( E_t \) in figure 1 denotes the domestic investment-saving balance where the domestic investment demand \( M_t \) and the domestic saving \( S_t = w_t L \) intersect, with both axes in logarithms. Given a rise in current income \( w'_t > w_t \), the domestic saving shifts rightwards.

\(^{26}\)See the proof of Proposition 1 for the technical derivation of the autarkic equilibrium.
Investment Unit Cost of ward shift of the domestic investment demand \( \partial \theta \) to equation (10), the smaller the If the borrowing constraints are binding, \( a \). Meanwhile, given the MIR, the lower the functions of the predetermined variable \( u \) to solve for the endogenous variables \( \{ \tau, \delta, u, \psi, a, \Phi \} \) as the variables \( w \) of tangibles as well as the extensive margin effect intangibles, which improve allocative efficiency and productivity. Here, the extensive margin effect is key to the positive comovement of productivity and aggregate income. Combine equations (4), (6), (7), and (9)-(12) to solve for the endogenous variables \( \{ \tau, \delta, u, \psi, a, \Phi \} \) as the functions of the predetermined variable \( w \),

\[
\tau = \frac{w L (1 - \theta)}{m}, \quad u = \delta = \tau^{1-\theta}, \quad \psi = \frac{\lambda}{1 - \delta}, \quad \text{and} \quad a = \frac{u \eta}{1 - \lambda (1 - \eta)}.
\]

If the borrowing constraints are binding, \( a < \eta \) and the productivity effect is active. According to equation (10), the smaller the \( \theta \), the stronger the extensive margin effect, the large the rightward shift of the domestic investment demand \( \frac{\partial \ln M}{\partial \ln \psi} = 1 \), the larger the rises in the unit cost and the intangible fraction of investment \( \frac{\partial \ln a}{\partial \ln \psi} = 1 - \theta \), the stronger the productivity effect. Meanwhile, given the MIR, the lower the \( \lambda \), the larger the investment distortion, the lower the \( a_t \), the stronger the productivity effect.

27In equation (10), \( \frac{\partial \ln M}{\partial \ln \psi} = \frac{1}{\lambda} \) is the partial elasticity and reflects the direct impact of an income change on domestic investment demand, given \( u_t \). Under autarky, the income change also affects domestic investment demand indirectly via \( u_t \), as shown in figure 1; the domestic investment-savings balances \( M_t = \frac{\delta w L}{u_t} = S_t = w_t L \) gives \( \delta_t = u_t \), implying that the extensive margin effect is fully offset by the unit-cost-of-investment effect, \( \frac{\partial \ln a}{\partial \ln \psi} = \frac{\partial \ln a}{\partial \ln \psi} = 1 \); and hence, the investment size effect in equation (15) is constant at unity \( \frac{\partial \ln M}{\partial \ln \psi} = 1 \). Under financial integration, capital flows break the domestic investment-saving balance, \( M_t \neq S_t \) and \( \delta_t \neq u_t \); according to equation (54), the investment size effect is not constant at unity, \( \frac{\partial \ln M}{\partial \ln \psi} \neq 1 \).

28In appendix D, I develop an alternative model with a fixed mass of entrepreneurs. In the absence of the extensive margin effect, \( \frac{\partial \ln M}{\partial \ln \psi} = \frac{\partial \ln a}{\partial \ln \psi} = 1 \) so that \( u_t \) is independent of income change and so are \( \psi_t \) and \( a_t \).
The Steady-State Properties

Put $\frac{\partial \ln M_t}{\partial \ln w_t} = 1$ and equation (17) into (15). There is a unique, autarkic steady state in country N, iff the neoclassical effect always dominates the productivity effect at any steady state. The neoclassical effect is negatively related to the capital share ($\alpha$), while the productivity effect is negatively related to the degrees of wealth inequality ($\theta$) and financial frictions ($\lambda$). As shown in proposition 1, the steady-state property of the model economy depends on the relative size of these parameters. Let $X_A$ denote the steady-state value of variable $X_t$ under autarky.

**Proposition 1.** Let $\theta \equiv \max \{1 - \frac{1-\alpha}{\alpha\eta}, 0\}$, $Z \equiv \frac{1-\theta}{\rho^m}$ and $\tilde{\lambda}_A \equiv \frac{1-Z^{1-\theta}}{1-\eta}$.

For $\theta \in [0, 1)$, there exists a unique, stable steady state under autarky.$^{29}$

Given $\theta \in [0, 1)$ and $\lambda < \min\{\tilde{\lambda}_A, 1\}$, the borrowing constraints are binding, the interest rate is below the social rate of return, the intangible fraction of investment and the productivity are inefficiently low in the autarkic steady state, i.e., $\psi_A < 1$, $a_A < \eta$ and $\Phi_A < 1$.

Besides, $\frac{\partial \ln \Phi_A}{\partial \ln \kappa} = \frac{\eta-a_A}{1-a_A} \frac{\partial \ln a_A}{\partial \ln \kappa} > 0$, $\frac{\partial \ln w_A}{\partial \ln \kappa} = \rho \frac{\partial \ln \Phi_A}{\partial \ln \kappa} > 0$, and $\frac{\partial \ln r_A}{\partial \ln \kappa} = \frac{\partial \ln \psi_A}{\partial \ln \kappa} > 0$.

**Assumption 1.** $\theta \in [0, 1)$, $0 < \lambda^* < \lambda < \min\{\tilde{\lambda}_A, 1\}$, and $\frac{L}{L^*+L} \to 0$.\(^{30}\)

Under assumption 1, there exists a unique, autarkic steady state in country N as well as in the rest of the world, respectively; the borrowing constraints are binding in both regions; as country N is more financially developed, it has a higher interest rate, a higher productivity, and a higher income per capita than the rest of the world.

3 Productivity Dynamics under Financial Integration

Suppose that country N is initially in the autarkic steady state. From period 0 on, country N is financially integrated with the rest of the world in the sense that agents are allowed to borrow and lend abroad freely. Given the initial interest rate differential, financial flows are “upstream” to the rich country (country N) and the interest rate in country N falls to the world level.

Under financial integration, equation (14) still characterizes the law of motion for wage, except that $r_t = r^*$ and domestic investment is not constrained by domestic saving. Let $\tilde{w}_F \equiv [1 - \lambda(1 - \eta)](\frac{m}{1-\theta})^{1-\theta} \rho^\theta (r^*)^{1-\frac{\alpha}{\rho}}$. For $w_t \geq \tilde{w}_F$, $q_{t+1} \Phi_t = r^*$ so that the borrowing constraints are slack, $a_t = \eta$, the productivity is constant at $\Phi_t = 1$, and $w_{t+1} = (r^*)^{-\rho}$. For $w_t < \tilde{w}_F$, the borrowing constraints are binding with $a_t < \eta$ and $\Phi_t < 1$.

Appendix B analyzes the dynamic stability and the steady-state property under financial integration. Given $r^*$ marginally below $r_A$, financial integration allows country N to converge towards a stable steady state in the neighbourhood of the autarkic one, either if $\{\lambda, \theta\}$ in region U of the left panel of figure 7, or if $\{\lambda, \theta\}$ in region M of the left panel and $\{\lambda, Z\}$ in region A and AB of the right panel of figure 7. In the following, we focus on this case and explore the impacts of a marginal decline in the interest rate on productivity and income dynamics. Our findings also hold in an alternative case where country N is initially in the steady state under financial integration, while the world interest rate falls marginally from period 0 on.

$^{29}$See figure 11 in the proof of proposition 1 for a graphical illustration of the threshold values.

$^{30}$By definition, the composite parameter $Z$ is independent of the level of financial development and the population size. Thus, $Z$ takes the same value for both regions.
3.1 Immediate Responses upon Financial Inflows

In this subsection, we first use the domestic investment-saving diagram to show the response of $u_0$ to financial inflows and then characterize the productivity effect.

![Diagram](image)

Figure 2: Financial Integration and Domestic Investment-Saving Imbalance in Period 0

In figure 2, point $E_A$ denotes the domestic investment-saving balance in the autarkic steady state where the domestic saving $S_A$ intersects with the domestic investment demand $M_A$, given $\theta \in (0, 1)$. In period 0, as aggregate income is determined by the capital stock installed one period earlier, it is independent of financial inflows and $w_0 = w_A$. Thus, the two lines stay put. Given the downward-sloping $M_0$, the unit cost of investment must fall so as to accommodate financial inflows. According to equation (6) and (4), $u_0 < u_A$ implies the fall in the normalized interest rate, which then leads to the falls in the intangible fraction of investment and the productivity. The logic behind the change in the normalized interest rate is as follows.

Financial inflows directly reduce the interest rate in period 0 and indirectly reduce the MPK in period 1. The response of $\psi_0$ depends on the relative size of these two effects.

\[
\frac{\partial \ln \psi_0}{\partial \ln r_0} = \frac{\partial \ln r_0}{\partial \ln r_0} - \frac{\partial \ln q_1 \Phi_0}{\partial \ln r_0} = \frac{1}{1 + \frac{1-u_0}{u_0} \left( \frac{1-\alpha}{\theta} + \alpha \frac{\eta-a_0}{1-a_0} \right)} > 0. \tag{18}
\]

Given the interest rate decline $r_0 = r^* < r_A$, $\frac{\partial \ln \psi_0}{\partial \ln r_0} > 0$ implies a fall in variable $x_t$, while $\frac{\partial \ln M_0}{\partial \ln w_t} < 0$ implies a rise in variable $x_t$ in period 0. For a given decline in the interest rate, the smaller the financial inflows, the weaker the MPK effect, the larger the fall in $\psi_0$, the larger the worsening in allocative efficiency and productivity. What determines the size of financial inflows?

For a given decline in the interest rate, the larger the $\theta$, the weaker the extensive margin effect, the less elastic the domestic investment demand $|\frac{\partial \ln M_0}{\partial \ln w_t}| = 1/\theta$, the steeper the $M_0$ line in figure 2, the smaller the financial inflows and domestic investment expansion, the smaller the MPK effect, the larger the declines in $\psi_0$ and the productivity. Here, the endogenous extensive margin is the key channel through which wealth inequality dampens the expansion of the domestic investment demand and amplifies the productivity decline in period 0.

The inflows of cheaper foreign funds directly augment domestic investment and indirectly
3.2 Dynamic Responses along the Convergence Path

Financial inflows stimulate the domestic capital formation in period 0 and aggregate income rises in period 1. As shown in figure 1, the higher aggregate income shifts the domestic saving curve rightwards proportionately, while it shifts the domestic investment demand curve rightwards more than proportionately, $\frac{\partial \ln M_t}{\partial \ln w_t} = 1 > \frac{\partial \ln S_t}{\partial \ln w_t} = 1$, due to the extensive margin effect. In period 1, the widening domestic investment-saving gap amplifies financial inflows, which stimulates the domestic capital formation and lowers the MPK. As the interest rate is constant at the world level $r_t = r^*$, the fall in the MPK leads to the rise in $\psi_t$.

$$\frac{\partial \ln \psi_t}{\partial \ln w_t} = \frac{\partial \ln r_t}{\partial \ln w_t} - \frac{\partial \ln q_{t+1}}{\partial \ln w_t} \Phi_t = \frac{u_t}{1-u_t} \left( \frac{u_t}{1-u_t} + \alpha \eta - \alpha \eta \right) + 1 > 0. \quad (19)$$

The larger the $\theta$, the smaller the partial elasticity $\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{1}{\theta}$, the weaker the investment size effect and the productivity effect over time. The investment size effect and the productivity effect compete with the neoclassical effect, which drives the dynamics of aggregate income over time. The proof of proposition 2 shows that, given $r_t = r^*$, the endogenous variables $\{u_t, \psi_t, a_t, \Phi_t, M_t, w_t\}$ are the implicit functions of the predetermined variable $w_t$. See equations (54)-(56) for the analytical solutions to the investment size effect, the productivity effect, and the dynamics of aggregate income.

So far, we have shown that financial inflows lower the productivity in period 0 and rise it over time. Can the subsequent rises dominate the initial fall so that the productivity eventually exceeds its autarkic level?

According to equations (10)-(11), the larger the $\theta$, the weaker the extensive margin effect, the smaller the investment elasticities (in absolute value), the smaller the financial inflows and domestic investment expansions, the weaker the MPK effect, the larger the initial decline and the smaller the subsequent rises in productivity over time. Thus, in the presence of financial inflows, wealth inequality has a negative effect on the long-run level of productivity.

According to equation (15), the larger the $\alpha$, the weaker the neoclassical effect, the larger the contribution of capital formation to aggregate income $\frac{\partial \ln Y_t}{\partial \ln K_t} = \alpha$, the higher the income level and the mass of entrepreneurs in the long run, the larger the financial inflows, the larger the falls in the MPK and the rises in the the normalized interest rate. In the presence of financial inflows, the capital share has a positive effect on the long-run level of productivity.

\[31\]

In order to highlight the role of the extensive margin effect in determining the productivity effect, we analyze in appendix C a special case of $\theta \to 0$ where the labor endowment distribution degenerates into a unit mass at $l_j = 1$ and agents have the same labor income, $n_t = w_t$. 

15
Figure 3: Two Competing Factors Relevant for the Long-Run Productivity Effects

Figure 3 features the entrepreneurial wealth share $\delta_F$ as the key channel through which wealth inequality and the capital share have the opposite effects on the long-run productivity implications of financial inflows.

Let $X_F$ denotes the steady-state value of variable $X_t$ under financial integration.

**Proposition 2.** Given $\theta \in (\theta, 1)$ and $r^* < r_A < \rho$, the productivity falls in period 0, $\Phi_0 < \Phi_A$, and then rises over time, $\Phi_{t+1} > \Phi_t$.

For $\theta \in (\theta, \alpha)$, the productivity eventually exceeds its autarkic level, i.e., $\Phi_0 < \Phi_A < \Phi_F$.

Figure 4 shows the impulse responses of productivity in three cases, with the horizontal axis denoting the time period $t = 0, 1, 2, ...$

In our model, young agents save their labor income inelastically and hence, the distribution of individual net wealth $n_{jt} = w_t(1 - \theta)l_j$ is Pareto with the tail index $\frac{1}{\theta}$ and the Gini coefficient $\frac{1}{\theta - 1}$, while $\alpha$ denotes the capital share in the production function. Although the model is highly stylized and not suitable for calibration, one may use the Gini coefficients from the empirical literature to infer $\theta$ and then compare it with the conventional value for $\alpha$ to get a sense about the possibility that financial inflows lead to productivity gains in the long run.

As shown in figure 1 and 2, the dynamics and the long-run level of productivity under financial integration depend on the elasticities of domestic investment demand.

Besides financial frictions and the MIR, other frictions or regulatory requirements hampering entrepreneurial entry may also reduce the investment elasticities and hence, our mechanism applies, too.

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32 As the steady-state value of productivity is a function of wealth inequality, we scale the vertical axes of figure 4 purely for illustration purpose. One should not compare the productivity levels literally in the three cases.

33 The partial elasticity $\frac{\partial \ln M_0}{\partial \ln \theta_0}$ determines the magnitude of the initial fall in productivity, while the partial elasticity $\frac{\partial \ln M_1}{\partial \ln \theta_1}$ determines the magnitude of the subsequent rises in productivity.
Our findings imply that a country may obtain productivity gains from debt inflows, if the investment elasticities exceed certain “threshold” value (i.e., the inverse of the capital share $\frac{1}{\alpha}$ in our model). It supports and broadens the insights of Kose et al. (2011, 2009a) as well as calls for the empirical research identifying the institutional factors relevant for the elasticity of domestic investment demand and exploring the threshold conditions.

According to equations (10)-(11), the endogenous entrepreneurial wealth share $\delta_t$ is a key channel through which the domestic investment demand responds more than proportionately to the changes in $w_t$ and in $u_t$. The stronger the extensive margin effect, the more elastic the domestic investment demand, the more likely the productivity exceed its autarkic level in the long run. For comparison purpose, we set up an alternative model in appendix D where the mass of entrepreneurs is exogenous and hence, $\delta_t$ is constant. Given $r^* < r_A$, the productivity falls upon financial integration and then rises over time, similar as in the current model. However, in the absence of the extensive margin effect, the unit cost of investment in the long run is strictly lower than its initial level and so is the productivity.$^{34}$

For analytical simplicity, We have made various assumptions in the current model. Appendix A discusses the robustness of our findings under alternative assumptions.

In the current model, the distribution of individual labor endowment is exogenous and so is the net wealth distribution among young agents in the same generation. According to equations (8), entrepreneurs earn a higher rate of return on their savings than households, $\frac{q_t + \Phi_t - r_t}{u_t} + r_t > r_t$, which widens the consumption inequality between households and entrepreneurs when they become old. As shown in appendix A.8, by affecting the rate-of-return differential, financial inflows have the redistributional welfare implications among agents of the same generation; by stimulating domestic capital formation and income growth, financial inflows also have the redistributional welfare implications across generations.

So far, we have focused on the case where country $N$ is initially in the autarkic steady state with the binding borrowing constraints and the world interest rate is marginally below that in country $N$. Under financial integration, country $N$ converges to a stable steady state in the neighbourhood of the initial one. In this sense, we make the marginal analysis.

4 Equilibrium Shifts and Productivity Patterns

In this section, we conduct the global analysis and explore the model dynamics under financial integration over the entire state spaces, taking explicitly into account the possibility of multiple steady states. According to equations (51)-(56), the investment size and the productivity are the two channels through which financial inflows affect the domestic capital formation and the income dynamics. Iff the borrowing constraints are binding, $\alpha_t < \eta$ and the productivity channel is active. The tighter the borrowing constraints and the lower the wealth inequality, the more strongly the $\delta_t$ responds to financial inflows, the stronger the capital formation effect.

$^{34}$In the current model, for $\theta \rightarrow 1$, the extensive margin effect in equations (10)-(11) vanishes $\frac{1}{\theta} - 1 \rightarrow 0$ and hence, $\delta_t$ is independent of changes in $w_t$ or $u_t$. Thus, $\psi_t$ and $\Phi_t$ have the same dynamic patterns as those in the alternative model in appendix D. In other words, the responses of $\psi_t$ and $\Phi_t$ in the alternative model constitute the lower bound for those in the current model. See appendix A.5 for endogenizing $\delta_t$ in an alternative setting.
The capital formation effect is negatively related to $\lambda$ and $\theta$, while the neoclassical effect is negatively related to $\alpha$. If the former dominates the latter at any steady state, multiple steady states arise. Proposition 3 specifies the threshold conditions for multiple steady states in terms of $\{\lambda, \theta, \alpha\}$, while appendix B characterizes the dynamic stability and the steady-state property.

**Proposition 3.** For $\theta \in (\theta, \alpha)$ and $\lambda < \tilde{\lambda}_F \equiv \min\{\frac{\alpha - \theta}{(1-\theta)(1-\eta)}, 1\}$, financial integration may lead to multiple steady states in country $N$.  

**Corollary 1.** Given $\theta \in (\theta, \alpha)$, $\lambda < \tilde{\lambda}_F$, and $Z \in (Z_F, \tilde{Z}_F)$, there exist two threshold values, $\hat{r}^*$ and $\tilde{r}^*$, where $\hat{r}^* < r_A < \tilde{r}^*$. Under financial integration, multiple steady states arise if the world interest rate is moderate $r^* \in [\hat{r}^*, \tilde{r}^*]$; there is a unique steady state if $r^* < \hat{r}^*$ or $r^* > \tilde{r}^*$.

According to corollary 1, if $r^*$ either falls below $\hat{r}^*$ or rises above $\tilde{r}^*$, country $N$ shifts from the equilibrium with multiple steady states to the one with a unique steady state, which has substantial impacts on income and productivity. We clarify this mechanism with two examples.

### 4.1 From International Autarky to Financial Integration

In the first example, we revisit the long-run impacts of financial integration, taking into account the possibility of equilibrium shift. Given $\theta \in (\theta, \alpha)$, $\lambda < \tilde{\lambda}_F$, and $Z \in (Z_F, \tilde{Z}_F)$, the dashed curve in the left panels of figure 5 shows the law of motion for wage under autarky, while point $A$ denotes the autarkic steady state with the binding borrowing constraints and $\Phi_A < 1$. 

![Figure 5: Impacts of Financial Integration: the Case of $w_0 = w_A$](image)

Country $N$ is initially at the autarkic steady state $A$. If the world interest rate is slightly below the autarkic interest rate in country $N$ $r^* \in (\hat{r}^*, r_A)$, financial inflows are moderate and
and so are their impacts on domestic capital formation. The thin, solid curve in the upper-left panel of figure 5 shows the law of motion for wage under financial integration. From period 0 on, country N converges from the autarkic to a new steady state F where aggregate income is higher, $w_F > w_A$ and the borrowing constraints are still binding. The thin, solid curve in the upper-middle (lower-middle) panel shows the impulse responses of the productivity (income), with the time period $t = 0, 1, 2, ...$ on the horizontal axis. Given $\theta \in (\theta, \alpha)$, the productivity in the long run is higher than its initial level, despite its initial fall in period 0, $\psi_0 < \psi_A < \Phi_F < 1$.

Besides the stable steady state F, there are an unstable steady state (U) and another stable steady state (H), as shown in the upper-left panel of figure 5. Starting from the autarkic steady state A, country N converges along a unique path to the stable steady state F. Thus, in the presence of multiple steady states, the initial condition matters for the long-run allocation.

For $r^* < \hat{r}^*$, the initial interest rate difference $r_A - r^*$ is so large that financial inflows shift the law of motion for wage upwards, as shown by the thick, solid curve in the lower-left panel. From period 0 on, a rise in the world interest rate reduces financial inflows and undermines the capital formation in country N, which shifts the law of motion for wage downwards. In figure 6, the solid curves in the upper-left and the lower-left panels show respectively the laws of motion for wage in the cases of a moderate rate rise $r^* \in (\hat{r}^*, \tilde{r})$ vs. a large rate rise $r^* > \tilde{r}$.

• For the moderate interest rate rise, country N converges from point H’ to H'' where the income level is lower, $w_{H''} < w_{H'}$, but the productivity is unaffected, $\Phi_{H''} = \Phi_{H'} = 1$.

4.2 The World Interest Rate Hike under Financial Integration

In the second example, we analyze the impacts of a permanent rise in the world interest rate. Given $r^* < \hat{r}^*$, the dashed curve in the left panels of figure 6 shows the law of motion for wage under financial integration. Country N is initially in the steady state denoted by point H’ where the borrowing constraints are slack and $\Phi_{H'} = 1$.

From period 0 on, a rise in the world interest rate reduces financial inflows and undermines the capital formation in country N, which shifts the law of motion for wage downwards. In figure 6, the solid curves in the upper-left and the lower-left panels show respectively the laws of motion for wage in the cases of a moderate rate rise $r^* \in (\hat{r}^*, \tilde{r})$ vs. a large rate rise $r^* > \tilde{r}$.

35 Under financial integration, the law of motion for wage is piecewise: for $w_t < \bar{w}_F$, it is upward-sloping, the borrowing constraints are binding, and $\Phi_t < 1$; for $w_t \geq \bar{w}_F$, it is flat at $w_{t+1} = (r^*)^{-\rho}$, the borrowing constraints are slack, and $\Phi_t = 1$. See appendix B and the proof of proposition 3 for the technical analysis.

36 For $r^* = \hat{r}^*$, the law of motion for wage is tangent with the 45° line in the convex part and crosses the 45° line in the flat part. In this case, there are two steady states.
For the large interest rate rise, country N converges from point $H'$ to $F$ where the income level and the productivity are substantially lower, $w_F < w_{H'}$ and $\Phi_F < \Phi_{H'}$.

Given $w_0 = w_{H'}$, if the world interest rate rises marginally from below to above $\bar{r}^*$, it shifts country N from the equilibrium with multiple steady states to the one with a unique steady state. The equilibrium shift causes a discontinuous jump in the long-run pattern of productivity (income), as shown by the piecewise, solid curve in the upper-right (lower-right) panel of figure 6. The horizontal axis denotes the world interest rate $r^*$, while $r_H^*$, $\hat{r}$, and $\bar{r}$ denote respectively the initial world interest rate and the two threshold values.

4.3 Path Dependence and Equilibrium Shift as the Cause of Discontinuity

For a moderate world interest rate $r^* \in [\hat{r}, \bar{r}]$, multiple steady states arise under financial integration, which has two implications. First, the existence of multiple steady states implies path dependence, i.e., the initial condition matters for the long-run allocation. Second, if $r^*$ rises above $\hat{r}$ or falls below $\bar{r}$, country N witnesses the equilibrium shift from multiple steady states to a unique steady state. Both are crucial for the discontinuity in the long-run patterns.

In the first example, given $r^* < r_A$, the discontinuity occurs around $r^* = \tilde{r}$, if the initial income is sufficiently low, e.g., $w_0 = w_A$. However, if the initial income is sufficiently high, e.g., $w_0 > w_U$, country N converges to steady state $H$ in the case of $r^* \in [\tilde{r}, r_A)$ and to steady state $H'$ in the case of $r^* < \tilde{r}$. See the left panels of figure 5. In both cases, the long-run level of productivity is equal to unity and the long-run level of income takes the form of $(r^*)^{-\rho}$. In other
words, the long-run patterns of income and productivity are continuous for \( r^* < r_A \), despite the equilibrium shift around \( r^* = \bar{r} \). Hence, the initial condition is crucial for the discontinuity.

Only the shift from multiple steady states to a unique steady state may cause the discontinuity.\(^{37}\) In the first example, financial integration lowers the domestic interest rate and the equilibrium shift relevant for the discontinuity occurs when \( r^* \) falls marginally below the lower bound \( \bar{r} \). In the second example, the world interest rate hike raises the domestic interest rate and the equilibrium shift relevant for the discontinuity occurs when \( r^* \) rises marginally above the upper bound \( \bar{r} \). Hence, the direction of the equilibrium shift is crucial for the discontinuity.

The discontinuous patterns of endogenous variables has two implications. First, the long-run impact of a world interest rate change depends crucially on whether it crosses the relevant threshold value (\( \bar{r} \) or \( \tilde{r} \)). In other words, the initial level of the world interest rate matters. Second, a rise and a decline in the world interest rate may have asymmetric effects on endogenous variables. In the first example, country N is initially at the autarkic steady state and a marginal decline in the world interest rate in the interval of \( r^* \in (\bar{r}, r_A) \) raises the long-run level of productivity, as shown by the downward-sloping part of the solid curve in the upper-right panel of figure 5. In the second example, country N is initially at the steady state \( H' \) with \( \bar{r}_{H'} < \bar{r} \) and a marginal rise in the world interest rate in the interval of \( r^* \in (\bar{r}, \tilde{r}) \) does not affect the long-run level of productivity, as shown by the flat part of the solid curve in the upper-right panel of figure 6. The long-run patterns of productivity in the two examples differ in the interval of \( r^* \in (\bar{r}, r_A) \). For a direct comparison, we use the dashed curve in the upper-right panel of figure 6 to show the long-run pattern of productivity in the first example. Obviously, depending on the initial condition, the productivity may respond asymmetrically to a rise and a fall in the world interest rate within the same range.

The global analysis allows us to identify the equilibrium shift as an amplification mechanism through which a minor change in the world interest rate around the threshold values may bring country N far away from its initial allocation and cause the disproportionately large changes in endogenous variables in the long run. One can use this mechanism to investigate the implications of the ongoing U.S. interest rate hikes. Given that the world interest rate has stayed at the record low level for nearly a decade, if the U.S. interest rate hikes lead to a moderate rise in the world interest rate, it may have little impacts on income and allocative efficiency in a small open economy. However, the large U.S. interest rate hikes may have disproportionately large impacts on income and allocative efficiency in a small open economy.

\(^{37}\)The equilibrium shift from a unique steady state to multiple steady states does not cause the discontinuity. As shown in the upper-left panel of figure 6, given \( r^* < \bar{r} \), country N is initially in the equilibrium with a unique steady state \( H' \) under financial integration where \( w_{H'} = (r^*)^{-\rho} \) and \( \Phi_{H'} = 1 \). A mild world interest rate rise, i.e., \( r^* \in (\bar{r}, \tilde{r}) \), shifts the law of motion for wage downwards and the model economy moves from the equilibrium with a unique steady state to the one with multiple steady states. Then, country N converges from the initial steady state \( H' \) to the new steady state \( H'' \) where \( w_{H''} = (r^*)^{-\rho} \) and \( \Phi_{H''} = 1 \). Thus, for \( r^* < \bar{r} \), the long-run patterns of income and productivity are continuous.
5 Final Remarks

This paper proposes a particular channel, i.e., *within-firm intangible-tangible investment composition*, through which financial inflows may have opposite short-run and long-run effects on allocative efficiency and the productivity of capital formation. Whether the productivity exceeds its initial level depends critically on the elasticities of domestic investment demand.

Our model features the MIR and the borrowing constraints as the barrier to entrepreneurial entry, while the elasticities of domestic investment demand is inversely related to wealth inequality. By the same logic, other frictions and regulatory distortions hampering entrepreneurial entry may also reduce the investment elasticity and dampen the productivity dynamics. Our theoretical findings suggest that identifying the institutional factors relevant for the elasticity of domestic investment may improve the empirical estimates on the productivity effects of financial flows, consistent with the insights of Kose et al. (2011, 2009a). If so, ameliorating these frictions and distortions may raise the elasticity of domestic investment, which stimulates intangible investment and leads to productivity gains in the case of financial inflows.

Groth and Khan (2010) show that U.S. manufacturing sectors differ substantially in the investment elasticity with respect to the shadow value of capital. One can embed our mechanism into a multi-sector setting with sector-specific investment elasticity. Our mechanism predicts that financial inflows are more likely to stimulate intangible investment and lead to productivity gains in the sectors with less frictions and distortions.

One may embed our mechanism into a new Keynesian setting and revisit the impacts of a monetary policy shock on intangible investment and productivity. Intuitively, by lowering the cost of external credit, the expansionary monetary policy induces firms to shift the investment more towards the tangibles, which reduces allocative efficiency and productivity in the short run. Whether monetary expansion can stimulate intangible investment and improve allocative efficiency in the long run may depend on the elasticity of domestic investment. We leave it for future research.

References


Appendices for Online Publication

A Robustness and Extensions

A.1 Elasticity of Substitution between the Tangibles and the Intangibles

In the current model, tangible and intangible investments are combined for capital formation in a Cobb-Douglas fashion. Following Falato et al. (2018), one can assume alternatively that capital formation takes the CES (constant elasticity of substitution) form,

\[ k_{j,t+1} = \left[ \eta \left( \frac{k_{j,t+1}}{\eta} \right)^{\sigma_{j}} + (1 - \eta) \left( \frac{k_{j,t+1}}{1 - \eta} \right)^{\sigma_{j}} \right]^{\frac{1}{\sigma_{j}}} \].

(20)

In so doing, aggregate productivity and the unit cost of investment become,

\[ \Phi_t = \left[ \eta \left( \frac{a_t}{\eta} \right)^{\sigma_{a}} + (1 - \eta) \left( \frac{1 - a_t}{1 - \eta} \right)^{\sigma_{a}} \right]^{\frac{1}{\sigma_{a}}} \] and

(21)

\[ u_t = \frac{a_t}{\eta} \left[ \eta + (1 - \lambda)(1 - \eta)A_t^{1-\sigma} \right] \] , where \[ A_t \equiv \left( \frac{1 - \eta}{\eta} \frac{a_t}{1 - a_t} \right)^{\frac{1}{\sigma_{a}}} \].

(22)

\[ \frac{\partial u_t}{\partial a_t} = 1 - (1 - \lambda)A_t \left( 1 - \frac{1}{\sigma_{a,t}} \right) > 0 \] for \[ a_t \leq \eta, \ A_t < 1. \]

(23)
Equation (6) is a special case of equation (22) with \( \sigma = 1 \). Since \( \frac{\partial u_t}{\partial a_t} > 0 \) holds in this general setting, the mechanism of our current model still holds. The higher the \( \sigma \), the more substitutable the two types of investments in the project of capital formation, the larger the within-project investment reallocation triggered by financial inflows. Allowing \( \sigma \neq 1 \) only affects the magnitude of the productivity dynamics, while the opposite dynamic pattern still exists.

For \( a_t < \eta, A_t < 1 \). In the general setting, condition (58) becomes

\[
\frac{\partial \ln LHS}{\partial \ln a_t} = \frac{1 - \left(1 - \lambda \right) \left(1 - \frac{1}{\sigma a_t} \right) A_t}{1 + \left(1 - \lambda \right) \frac{1 - a_t}{a_t} A_t} - \theta = 1 - A_t > 0, \quad \text{and} \quad \frac{\partial \ln RHS}{\partial \ln w_s} = \frac{\alpha - \theta}{\theta}. \tag{24}
\]

In this case, \( \frac{\partial \ln LHS}{\partial \ln a_t} > 0 \) still holds and so does the proof of proposition 2 in appendix E. The relative size of \( \theta \) and \( \alpha \) still determines the long-run productivity effect of financial inflows, the same as in our current model.

\section*{A.2 The MIR on the Tangibles}

In the current model, the MIR and financial frictions jointly endogenize the mass of entrepreneurs, while the resulting endogenous extensive margin of domestic investment demand is key to the productivity dynamics under financial integration. Instead of assuming the MIR for the total project investment, one can assume that the MIR applies to tangible investment, \( k_{j,t,t+1} \geq m \). Accordingly, the cutoff value specified by equation (7) becomes

\[
\frac{w_t(1 - \theta)\varepsilon_t}{u_t} (1 - a_t) = m, \Rightarrow \varepsilon_t = \frac{u_t}{(1 - a_t)w_t} \frac{m}{1 - \theta}, \tag{25}
\]

while other conditions in section 1 are unaffected. In particular, the two partial elasticities of domestic investment demand are still specified by equations (10)-(11). Thus, the core mechanism of our current model still holds and so does the opposite productivity dynamics.

In the proof of proposition 2, \( \delta_s \) and equation (57)-(58) become

\[
\delta_s = \varepsilon_t^{\frac{1 - \sigma}{\sigma}} = \left(\frac{w_s(1 - a_s) 1 - \theta}{u_s m}\right)^{\frac{1}{\sigma}}, \tag{26}
\]

\[
\Phi_s = \left(\frac{m}{1 - \theta}\right)^{1 - \theta} \rho^\theta = \frac{w_s^{\theta}}{u_s^{\theta}}, \tag{27}
\]

\[
\Rightarrow \frac{\partial \ln LHS}{\partial \ln a_s} = 1 + (1 - \theta) \frac{a_t}{1 - a_t} - 0 = 0 \quad \text{and} \quad \frac{\partial \ln RHS}{\partial \ln w_s} = \frac{\alpha - \theta}{\alpha}. \tag{28}
\]

In this case, \( \frac{\partial \ln LHS}{\partial \ln a_s} > 0 \) still holds and so does the proof of proposition 2. The relative size of \( \theta \) and \( \alpha \) still determines the long-run productivity effect, the same as in our current model.

Assuming the MIR for the tangibles does not change the findings of our current in terms of the dynamic pattern of productivity and the conditions for the long-run productivity effect. However, it does make the analytical solution more cumbersome. For example, the productivity effect specified in equation (17) becomes

\[
\frac{\partial \ln \Phi_t}{\partial \ln w_t} = \left(1 - \frac{1 - \eta}{1 - a_t}\right) \frac{1}{1 - \theta + \frac{a_t}{1 - a_t}}. \tag{29}
\]
A.3 Collateral Constraints Revisited

In the current model, only the tangibles can serve as collateral for loans, while the intangibles have to be fully financed by the entrepreneur’s own funds. Using a large sample of syndicated loans to US corporations, Falato et al. (2018) have verified that only 3% of secured syndicated loans have patents or brands used as collateral. Here, we check the robustness of our findings in a generalized setting (Caggese and Pérez-Orive, 2018) where the intangibles can also serve as collateral for loans but they have a lower degree of pledgeability than the tangibles.

To be specific, if agent $j$ defaults, lenders can seize and liquidate not only the tangibles but also the intangibles; after deducting the liquidation costs, the lenders get $p_{T,j}$, while the lenders get $p_{I,j}$, where $p_{T,j}$ and $p_{I,j}$ denote respectively the market price and the pledgeability of intangibles. Let $\kappa \equiv \frac{\kappa}{\lambda} < 1$ denote the intangible-tangible pledgeability ratio. If the borrowing constraints are binding, the unit cost of intangibles is $u_{I,j} = 1 - \frac{\kappa \lambda p_{I,j+1}}{r}$, while the unit return is $q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,t+1}} - \lambda p_{I,j+1}$. The agent equalizes the internal rates of return.

\[
\begin{align*}
q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,t+1}} - \kappa \lambda p_{I,j+1} & = q_{t+1} \frac{\partial k_{I,j+1}}{\partial k_{I,j+1}} - \lambda p_{I,j+1} \\
\Rightarrow \quad 1 - \frac{\lambda p_{I,j+1}}{r} & = q_{t+1} \Phi_{j,t} \frac{1 - \eta}{1 - \eta} - \lambda p_{I,j+1} - q_{t+1} \Phi_{j,t} \frac{\eta}{a_{j,t}} - \kappa \lambda p_{I,j+1}.
\end{align*}
\]

According to equation (30), the optimal choice of $a_{j,t}$ is a function of the parameters ($\lambda, \kappa, \eta$) and the market prices ($p_{t+1,j}, p_{I,t+1}, r, q_{t+1}$). Thus, the agents who meet the MIR optimally choose the same intangible fraction of investment. Hereafter, we drop off subscript $j$ and use $a_t$ to denote it. Agent $j$ equalizes its marginal revenue of intangibles to the market price,

\[
q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,t+1}} = a_{t} \Phi_{t} \frac{\eta}{a_{t}} = p_{I,t+1},
\]

Combine equations (4) and (30)-(31) to solve for the unit cost of investment,

\[
\begin{align*}
u_{t} & \equiv a_{t} u_{I,t} + (1 - a_{t}) u_{T,t} = 1 - \left[\eta \kappa + (1 - \eta)\right] \frac{\lambda}{\Psi_{t}} \quad (32) \\
u_{I,t} & = 1 - \frac{\eta}{a_{t}} \frac{\lambda}{\Psi_{t}} = \frac{\eta}{a_{t}} (1 - \kappa \lambda) \frac{1}{1 - \left[\kappa \eta + (1 - \eta)\right] \lambda} \quad (33) \\
u_{T,t} & = 1 - \frac{1 - \eta}{a_{t}} \frac{\lambda}{\Psi_{t}} = \frac{1 - \eta}{a_{t} \Psi_{t}} (1 - \lambda) \frac{1}{1 - \left[\kappa \eta + (1 - \eta)\right] \lambda} \quad (34) \\
a_{t} \eta \left(1 - \left[\kappa \eta + (1 - \eta)\right] \lambda\right) & = 1 - \kappa \lambda - \frac{(1 - \eta)(1 - \kappa) \lambda}{\Psi_{t}} \left(1 - \frac{\lambda}{\eta} \frac{1}{1 - \frac{\lambda}{\eta}} \right) = \frac{(1 - \kappa)}{\Psi_{t}} \frac{1}{1 - \frac{\lambda}{\eta}} \left(1 - \frac{\lambda}{\eta}ight) \quad (35)
\end{align*}
\]

Our current model is a special case of this generalized setting. One can put $\kappa = 0$ into equations (32)-(35) to get $u_{I,t} = 1$ and equations (5)-(6) in our current model. Except them, other equilibrium conditions are identical as in our current model.

In our current model, $\lambda$ measures the level of financial development. In the generalized setting, $\kappa$ is another measure of financial development, reflecting the heterogeneous pledgeability across different asset classes. If the intangibles and the tangibles have the same pledgeability $\kappa = 1$, use equation (33)-(35) to get $a_{t} = \eta$ and $u_{I,t} = u_{T,t} = u_{t}$. In this case, the unit costs of tangibles and intangibles equalize, the within-project investment composition is efficient, and the productivity is constant at $\Phi_{t} = 1$, despite the binding borrowing constraints $\kappa < \tilde{\lambda}_{\kappa}$. If the intangibles has a lower pledgeability than the tangibles, $\kappa < 1$ creates the unit-cost differential $u_{I,t} > u_{T,t}$ and hence, entrepreneurs invest less (more) in the intangibles (tangibles) $a_{t} < \eta$, which distorts allocative efficiency and undermines the productivity $\Phi_{t} < 1$. Thus, it is $\kappa$ rather than $\lambda$ that distorts allocative efficiency in the general setting.
Besides, in the case of $\kappa \in [0,1)$, the unit-cost differential $u_{t,j} > u_{T,j}$ implies that the unit cost of tangibles is more elastic to the change in $\psi_t$ than that of intangibles,

$$\frac{\partial \ln u_{T,j}}{\partial \ln \psi_t} = \frac{1}{u_{T,j}} - 1 > \frac{\partial \ln u_{t,j}}{\partial \ln \psi_t} = \frac{1}{u_{t,j}} - 1 > 0.$$  \hspace{1cm} (36)

In period 0, financial inflows lower $\psi_0$, which causes $u_{T,0}$ to fall by a larger proportion than the change in $u_{t,0}$. It widens the unit-cost differential and induces entrepreneurs to shift investment further towards the tangibles. From period $t = 1$ on, the rise in $\psi_t$ also causes $u_{T,j}$ to rise by a larger proportion than the change in $u_{t,j}$, and the narrowing unit-cost differential induces entrepreneurs to shift investment towards the intangibles. Besides, the relative size of $\theta$ and $\alpha$ still matters for the long-run productivity effect in the same way as specified in proposition 2. In comparison with our current model, allowing intangibles to serve as collateral with $\kappa \in [0,1)$ narrows the unit-cost differential and weakens the magnitude of the productivity dynamics, while our findings still hold.

**Remark 1**: in Matsuyama (2004) and Zhang (2017), the productivity of capital formation is constant at $R$ and entrepreneurs can borrow up to a fraction of the future project revenue, i.e. $b_{j,t} = \frac{\lambda_{h,t+1} R_{m,j,t}}{\gamma_t}$. In the current model, we assume that entrepreneurs can borrow up to a fraction of the future market value of tangibles, as shown in equation (2). Although the borrowing constraints take different forms in the two specifications, they are technically equivalent. According to equation (33), if the intangibles have the same degree of pledgeability $\kappa = 1$, the unit cost of investment is $u_t = 1 - \frac{\lambda_{h,t+1} \Phi_h}{\gamma_t}$, the same as in Matsuyama (2004) and Zhang (2017). Introducing heterogeneous pledgeability essentially reduces the pledgeable value of the project revenue. For $\kappa = 0$, the unit cost of investment is higher, $u_t = 1 - \frac{(1-\eta) \lambda_{h,t+1} \Phi_h}{\gamma_t}$.

**Remark 2**: in order to ensure the binding borrowing constraints in the steady state, we adopt the two-period, overlapping-generation model and exclude the possibility that agents overcome the entry barrier by accumulating net wealth over time. Besides finite lifetime (Bernanke et al., 1999), researchers may also assume impatient entrepreneurs (Kiyotaki and Moore, 1997) to ensure the binding borrowing constraints in the long run. Moll (2014) develops a model with idiosyncratic productivity shocks and shows that, if shocks are transitory, self-financing cannot fully mitigate the borrowing constraints at the aggregate level so that steady-state capital misallocation is large; if shocks are relatively persistent, self-financing undoes capital misallocation from financial frictions in the long run, but transitions to this steady state take a very long time.

### A.4 Within-Project vs. Cross-Project Investment Reallocation

In the current model, agents are endowed with the same project and entrepreneurs choose the same intangible fraction of investment in equilibrium. Thus, the productivity at the aggregate level coincides with that at the individual level. By triggering within-project investment reallocation along the tangibles-intangibles margin, financial inflows affect allocative efficiency.

Financial inflows may affect allocative efficiency by triggering investment reallocation across projects with heterogeneous productivity. Consider a model with two projects indexed by $h \in \{1,2\}$. In order to feature explicitly the cross-project margin, we turn off the within-project margin by assuming that both projects are linear with the exogenous productivity, $k_{h,j,t+1} = \phi_{h} m_{h,j,t}$. The borrowing constraints are project-specific, $b_{h,j,t} \leq \lambda_{h,t} \frac{\phi_{h} m_{h,j,t}}{\gamma_t}$. Project 2 is more productive than project 1, $\phi_2 > \phi_1$, but it is subject to the tighter borrowing constraint, $\lambda_2 < \lambda_1 \frac{\phi_1}{\phi_2}$.\footnote{Matsuyama (2007) introduces such a model with exogenous heterogeneity in productivity and pledgeability.} The two projects are subject to the same MIR.
Agents prefer to invest in the more productive project if they can meet the MIR. In the case where the borrowing constraints are binding for both projects, there are two cutoff values, $\varepsilon_{1,t} > \varepsilon_{2,t} > 1$, that split agents into three groups. Those with $\varepsilon_j \in (1, \varepsilon_{1,t})$ lend out the labor income and are called households; those with $\varepsilon_j \in [\varepsilon_{1,t}, \varepsilon_{2,t})$ invest in project 1 and are called group-1 entrepreneurs; those with $\varepsilon_j \geq \varepsilon_{2,t}$ invest in project 2 and are called group-2 entrepreneurs. The aggregate productivity is the weighted average of the project productivity, $\Phi_t = \chi_2 \phi_2 + (1 - \chi_2) \phi_1$, where $\chi_2 = \frac{M_j}{M_{1,t} + M_j}$ denotes the fraction of domestic investment allocated in project 2.

Our preliminary analysis shows that, given $r^* < r_A$, domestic investment first shifts disproportionately towards project 1 upon financial inflows and then towards project 2 over time. Thus, aggregate productivity falls in period 0 and then rises over time. The higher the wealth inequality, the less elastic the mass of entrepreneurs in each group, the smaller the elasticities of domestic investment for each type of project, the larger the initial fall and the smaller the subsequent rises in aggregate productivity. This way, by reducing the elasticities of domestic investment demand, wealth inequality dampens the productivity dynamics along the within-project and the cross-project margins in the same way.

A.5 Alternative Way of Endogenizing the Entrepreneurial Wealth Share

As shown in subsections 3.1-3.2 and appendix D, and endogenous entrepreneurial wealth share $\delta_t$ and the extensive margin effect are the key channel through which wealth inequality affects the elasticity of domestic investment demand and the productivity dynamics. In this two-period, OLG framework, agents save their entire labor income when young and consume their entire investment return when old. Thus, the labor income is the only source of individual net wealth that matters for domestic investment, $n_{j,t} = l_j \omega_t$, and hence, the entrepreneurial wealth share is driven purely by the changes in the mass of entrepreneurs, $\delta_t = \eta_t^{1-\theta}$.

Besides, one can endogenize $\delta_t$ by allowing for individual wealth accumulation over a longer time horizon. Keeping the mass of entrepreneurs exogenous, one can embed the core elements of the model specified in appendix D into a continuous-time, perpetual youth framework (Blanchard, 1985). For $q_{t+1} \Phi_t > r_t$, the borrowing constraints are binding and, due to the leverage effect, entrepreneurs earn a higher rate of return on their net wealth than households. The rate-of-return differential allows entrepreneurs to accumulate wealth at a faster rate than households, which endogenizes the entrepreneurial wealth share.

One can endogenize the cross-project differences in pledgeability as follows. Suppose that entrepreneurs can choose between a traditional project indexed by $h = 1$ and a modern project indexed by $h = 2$ for capital formation. The traditional project is linear $k_{1,j,t+1} = \phi_1 m_{1,j,t}$, with the input of tangibles $m_{1,j,t}$ only; the modern project takes the Leontief form, $k_{2,j,t+1} = \phi_2 \min \left\{ \frac{m_{2,j,t,T}}{\eta_t}, \frac{m_{2,j,t,T}}{1-\eta_t} \right\}$, with the inputs of tangibles $m_{2,j,t,T}$ and intangibles $m_{2,j,t,T}$. Let $m_{2,j,t} = m_{2,j,t,T} + m_{2,j,t,T}$ denote agent-$j$’s total investment in the modern project. In equilibrium, agent $j$ chooses $\frac{m_{2,j,t,T}}{\eta_t} = \frac{m_{2,j,t,T}}{1-\eta_t} = m_{2,j,t}$ and the modern project is linear, $k_{2,j,t+1} = \phi_2 m_{2,j,t}$. By assumption, intangible investment improves the project productivity $\phi_2 > \phi_1$, while only tangibles can be used as the collateral for loans. When running the traditional project, agents face the borrowing constraints $b_{1,j,t} \leq \lambda \frac{q_{t+1} \phi_1 m_{1,j,t}}{r_t}$. When running the modern project, agents face the borrowing constraints $b_{2,j,t} \leq \frac{\phi_2 m_{2,j,t,T}}{r_t} \equiv \lambda (1 - \eta) \frac{q_{t+1} \phi_2 m_{2,j,t}}{r_t}$. Thus, the cross-project difference in pledgeability reflects the cross-project difference in tangibility, $\frac{\phi_2}{\phi_1} \equiv 1 - \eta$.

For each unit of investment in period $t$, an entrepreneur has to put down $u_t = 1 - (1 - \eta) \phi_2 m_{2,j,t,\Phi_t}$ units of own fund. In period $t + 1$, it uses the project revenue $q_{t+1} \Phi_t$ to pay off the debt $(1 - \eta) \phi_2 m_{2,j,t,\Phi_t}$ and consumes the rest, $[1 - (1 - \eta) \lambda] q_{t+1} \Phi_t$. The gross rate of return on entrepreneurial wealth is $\Gamma_t \equiv \frac{1 - (1 - \eta) \lambda \phi_2 m_{2,j,t}}{u_t}$, where $\phi_2 m_{2,j,t,\Phi_t}$ features the leverage effect. In contrast, households lend out their own funds for the gross interest rate $r_t$. Given $q_{t+1} \Phi_t > r_t$, the leverage effect is positive, which ensures $\Gamma_t > r_t.$
By reducing the interest rate, financial inflows widens the rate-of-return differential, which raises $\delta_t$ along the intensive margin in country N. The mechanism of the current model still applies. A complete analysis of the productivity implications in that model is beyond the scope of the current paper and we leave it for future research.

**A.6 Inelastic vs. Elastic Aggregate Saving**

For analytical tractability, we keep the aggregate saving interest-inelastic in the current model by assuming that agents are endowed with labor only when young and they consume only when old. Following von Hagen and Zhang (2014), we can relax these two assumptions so that individual agents take the intertemporal consumption-saving optimization. According to the Euler equation, the individual saving becomes elastic with respect to the interest rate and the future income. Overall, the domestic saving becomes elastic, while the domestic investment demand becomes even more elastic, due to the extensive margin effect. As long as domestic saving is not perfectly interest elastic, the $S_t$ line is upward-sloping in figure 2. Upon financial integration, a fall in the interest rate induces households to save less, while it induces entrepreneurs to save and invest more. In the net term, the domestic saving falls, while the domestic investment demand rises. One can still use figure 2 to illustrate the mechanism in this setting and our findings still hold qualitatively.

**A.7 Exogenous vs. Endogenous Wealth Inequality**

In the current model, the distribution of labor endowment is exogenous and so is the wealth distribution of young agents. A larger wealth inequality weakens the responses of $\delta_t$ with respect to the changes in the interest rate and aggregate income, which then dampens capital inflows and productivity dynamics.

Wealth distribution in the model mentioned in appendix A.5 is endogenous. By widening the rate-of-return differential, financial integration raises the entrepreneurial wealth share, which amplifies capital inflows as well as widens wealth inequality. In contrast to the findings of the current model, the endogenous wealth inequality moves ex post positively with the size of capital inflows in that model.\(^{40}\) The dynamic, two-way interactions between wealth inequality and capital inflows deserves further research.

**A.8 Intra- vs. Inter-generational Welfare Effects of Financial Inflows**

By assumption, agents consume the return to their savings when old. According to equations (8), the individual consumption depends on two components, i.e., the individual labor income and the relevant rate of return. The rate of return on household saving is simply the interest rate $r_t$. Let the equity rate denote the rate of return on entrepreneurial own funds in the project,

$$\Upsilon_t \equiv \frac{q_t + \Phi_t - r_t}{u_t} + r_t \frac{1 - \lambda(1 - \eta)}{(\psi_t - \lambda(1 - \eta))}. \quad (37)$$

Country N is initially in the autarkic steady state and financial integration does not affect the welfare of the agents born before period 0.

Consider first the generation born in period 0. Upon financial integration, the interest rate in country N falls to the world level $r_0 = r^* < r_A$, which raises the leverage multiplier $\frac{1}{w_0} > \frac{1}{w_A}$ and the equity rate

\(^{40}\)The exogenous wealth inequality in the current model can be regarded as an ex ante measure of inequality, while the endogenous wealth inequality in that model can be regarded as an ex post measure of inequality.
$\Upsilon_0 > \Upsilon_A$. Meanwhile, financial inflows lower the cutoff value $\varepsilon_0 < \varepsilon_A$, which splits young agents into three groups.

- The first group consists of those with $\varepsilon_j \in (1, \varepsilon_0)$ who would become households if they were born before period 0. When they are born in period 0, they still become households but they earn a lower interest rate $r_0 < r_A$ on their savings than otherwise in the autarkic steady state. Given $w_0 = w_A$, the interest rate effect makes them worse off.

- The second group consists of those with $\varepsilon_j \geq \varepsilon_A$ who would become entrepreneurs if they were born before period 0. When they are born in period 0, they still become entrepreneurs but they earn a higher equity rate $\Upsilon_0 > \Upsilon_A$ on their savings than otherwise in the autarkic steady state. Given $w_0 = w_A$, the equity rate effect makes them better off.

- The third group consists of those with $\varepsilon_j \in [\varepsilon_0, \varepsilon_A)$ who would become households if they were born before period 0. When they are born in period 0, they become entrepreneurs and earn a higher rate of return $\Upsilon_0 > \Upsilon_A > r_A$ on their savings than otherwise in the autarkic steady state. Given $w_0 = w_A$, the rate-of-return effect makes them better off.

Next, consider the generation born in period $t \geq 1$. As mentioned above, aggregate income rises over time, which affects individual welfare in three ways. First, by raising the wage rate, it benefits all agents in equal proportions via the individual labor income channel. In particular, given the interest rate constant at the world level, the labor income effect makes the agents with $\varepsilon_j \in (1, \varepsilon_t)$ better off than otherwise born one period earlier. Second, by stimulating financial inflows and domestic investment, it reduces the project rate of return $q_{t+1}\Phi_t$ and lowers the equity rate. If the labor income effect dominates the equity rate effect, the agents with $\varepsilon_j \geq \varepsilon_t - 1$ are better off than otherwise born one period earlier. Third, by further reducing the cutoff value $\Upsilon_t > \Upsilon_{t-1}$, it allows the agents with $\varepsilon_j \in [\varepsilon_t, \varepsilon_{t-1})$ to become entrepreneurs and earn the equity rate rather than the interest rate on their savings. The labor income effect and the rate-of-return effect jointly make them better off than otherwise born one period earlier.

Besides comparing with the generation born one period earlier, we can evaluate the welfare implications of financial inflows by compare the agents born in period $t$ against those born before period 0. Compared to the allocation in the autarkic steady state, financial inflows reduces the interest rate and raises aggregate income.

- The agents with $\varepsilon_j > \varepsilon_A$ are better off, due to the relative size of the labor income effect and the equity rate effect.

- The agents with $\varepsilon_j \in (\varepsilon_t, \varepsilon_A)$ are better off, due to the labor income effect $w_t > w_A$ and the rate-of-return effect $\Upsilon_t > r_A$.

- Whether the agents with $\varepsilon_j \in (1, \varepsilon_t)$ are better off depends on the relative size of the labor income effect and the interest rate effect.

To sum up, by affecting the interest rate and the equity rate in the opposite direction, financial inflows have redistributional effects on the generation born in period 0; by raising aggregate income over time, financial inflows have redistributional effects across generations.

### B Steady-State Property under Financial Integration

This section analyzes the steady-state property of the current model under financial integration.
Figure 7: Steady-State Property under Financial Integration: the Case of \( r^* = r_A \)

Let \( \tilde{\lambda}_F \equiv \min\{\frac{\alpha - \theta}{(1-\theta)(1-\eta)}, 1\} \). In figure 7, the downward-sloping curve in the left panel shows \( \tilde{\lambda}_F \) as a function of \( \theta \); for \((\lambda, \theta)\) in region U, the autarkic steady state is still the unique steady state under financial integration; for \((\lambda, \theta)\) in region M, the right panel shows the parameter constellations for five cases in the \((\lambda, Z)\) spaces, while the solid (dashed) curves in figure 8 show the laws of motion for wage under financial integration (under autarky) in these five cases, respectively.\(^{41}\) The proof of proposition 3 characterizes the law of motion for wage and specifies the threshold values. Given \( r^* = r_A \),\(^{42}\) multiple steady states arise in three cases.

- In case B, financial integration destabilizes the autarkic steady state (point A), which leads to two stable steady states (point H and point L), with \( w_L < w_A < w_H \).
- In case AB, the autarkic steady state (point A) is still stable under financial integration, while an unstable steady state (point U) and another stable steady state (point H) arise, with \( w_A < w_U < w_H \).
- In case BC, the autarkic steady state (point A) is still stable under financial integration, while an unstable steady state (point U) and another stable steady state (point L) arise, with \( w_L < w_U < w_A \).

\(^{41}\)Under financial integration, the law of motion for wage consists of two or three parts, depending on the bindingness of the borrowing constraints and the MoE constraint. See footnote 18 for the definition of the MoE constraint. The dashed curve in the right panel of figure 7 shows a threshold value \( \tilde{\lambda}_F \) in the \((\lambda, Z)\) space.

- For \((\lambda, Z)\) to the right of the dashed curve, \( \lambda > \tilde{\lambda}_F \) and the MoE constraint is always slack as long as the borrowing constraints are binding; when the borrowing constraints are slack, agents who can overcome the MIR do not have strong incentive to run the project and hence, the MoE constraint is irrelevant. Let \( \tilde{w}_F \equiv [1 - \lambda(1 - \eta)] \left( \frac{m}{\rho} \right)^{1-\theta} \rho \theta (r^*)^{\frac{1}{\gamma}} \). For \( w_t > \tilde{w}_F \), the borrowing constraints are slack; for \( w_t < \tilde{w}_F \), the borrowing constraints are binding. Hence, the law of motion for wage consists of two parts and the solid (dashed) curves in figure 8 show the laws of motion for wage under financial integration (autarky).

- For \((\lambda, Z)\) to the left of the dashed curve, \( \lambda < \tilde{\lambda}_F \). There are two threshold values \( \tilde{w}_F < \tilde{w}_F \) such that, for \( w_t < \tilde{w}_F \), the borrowing constraints are binding and the MoE constraint is slack; for \( w_t \in [\tilde{w}_F, \tilde{w}_F] \), both the borrowing constraints and the MoE constraint are binding; for \( w_t > \tilde{w}_F \), the borrowing constraints are slack and the MoE constraint is irrelevant. Hence, the law of motion for wage consists of three parts and the solid (dashed) curves in figure 13 show the laws of motion for wage under financial integration (autarky).

\(^{42}\)One can use the solution approach described in the proof of proposition 3 to analyze the case of \( r^* \neq r_A \).
C The Case of Homogeneous Wealth Distribution: $\theta \to 0$

In order to further highlight the critical role of the extensive margin effect in determining the productivity effect, we analyze a special case of $\theta \to 0$ where the distribution of labor endowment degenerates into a unit mass at $l_j = 1$ and agents have the same labor income, $n_t = w_t$. If $w_t < m$, an agent has to borrow at least $m - w_t$ to run the project. As argued in Matsuyama (2004), the equilibrium allocation involves credit rationing, i.e., the credit is allocated randomly to a fraction of agents who become entrepreneurs, while the rest are denied credit and become households. Due to competition on the credit market, each entrepreneur only demands for the credit of $m - w_t$ and invests at the level of the MIR, $m_t = n_t u_t = m$.\(^{\text{43}}\)

In this case, domestic investment demand is endogenous along the extensive margin only $M_t = m \tau_t L$.

In figure 9, point $E_A$ denotes the domestic investment-saving balance in the autarkic steady state. According to equation (11), $\theta \to 0$ makes domestic investment demand perfectly elastic with respect to $u_t$ and the line of $M_A$ is flat. In period 0, given $w_0 = w_A$ and $m_0 = m_A = m$, the credit demand of each individual entrepreneur is the same as before $m_0 - w_0 = m_A - w_A$ and so is the unit cost of investment $u_0 = \frac{w_0}{m_0} = \frac{w_A}{m_A} = u_A$. Thus, financial capital inflows stimulate domestic investment demand only along the extensive margin. As a result, the domestic investment expansion is so large that the social rate of return falls by the same proportion as the change in the interest rate $\frac{\partial \ln q_1}{\partial \ln r_0} = 1$. Thus, $\psi_0 = \psi_A$ holds and so does $\Phi_0 = \Phi_A$. One can confirm these findings by putting $\theta \to 0$ into equations (18) and (52).

\(^{\text{43}}\)According to equations (16), for $\theta \to 0$, $u_t = \tau_t = \frac{w_t}{m}$ holds under autarky and so does $m_t = \frac{n_t}{u_t} = \frac{w_t}{m} = m$. According to equation (63), this result also holds under financial integration.
Figure 9: Financial Inflows and Domestic Investment-Saving Imbalance in Period 0: θ → 0

D A Model with the Exogenous Mass of Entrepreneurs

The endogenous entrepreneur wealth share is key to our findings in the current model. For comparison, we set up a model which differs from the current model in two aspects. First, there is no MIR. Second, only a constant fraction τ of agents in each generation are endowed with the investment project and they are called entrepreneurs, while the others do not have the project and are called households. Agents are equally endowed with one unit of labor when young and their labor income is homogeneous at \( w_t \). Due to the exogenous mass of entrepreneurs, domestic investment adjusts only along the intensive margin. Thus, we call it model IM.

Assumption 2. \( \tau < \eta \) and \( \lambda \in (0, 1] \).

Under assumption 2, the entrepreneurial wealth share is less than the efficient share of intangible investment \( \delta_t = \frac{\tau w_t L}{u_t} = \tau < \eta \). Thus, the borrowing constraints are binding under autarky and the individual optimization is the same as shown in section 1. A rise in current income raises the net wealth of all agents in equal proportions. Due to the fixed masses of entrepreneurs and households, the aggregate credit demand and the aggregate credit supply rise along the intensive margin in equal proportions so that the normalized interest rate stays put and so do the unit cost of investment, the intangible fraction of investment, and the productivity.

\[
M_t = \frac{\tau w_t L}{u_t} = w_t L, \quad u_t = u_A = \tau, \quad \psi_t = \psi_A = \frac{(1 - \eta)\lambda}{1 - u_t} = \frac{(1 - \eta)\lambda}{1 - \tau} < 1,
\]

\[
a_t = a_A = \eta \frac{u_t}{1 - \lambda(1 - \eta)} = \eta \frac{\tau}{1 - \lambda(1 - \eta)} < \eta, \quad \Phi_t = \Phi_A < 1.
\]

Combine them with equations (1) to get the law of motion for wage,

\[
w_{t+1} = \left( \frac{\Phi_t w_t}{\rho} \right)^{\alpha} \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \left( 1 - \alpha \right) < 1.
\]

The dynamics of aggregate income are purely driven by the neoclassical effect.

Proposition 4. Under autarky, the borrowing constraints are binding, the normalized interest rate is constant at \( \psi_t = \psi_A < 1 \), the intangible fraction of investment is constant at \( a_t = a_A < \eta \), and the productivity is constant at \( \Phi_t = \Phi_A < 1 \). Besides, \( \frac{\partial \ln \Phi_t}{\partial \ln \lambda} = \frac{\eta - a}{1 - a} \frac{\partial \ln a_t}{\partial \ln \lambda} > 0 \).

44As wealth distribution does not matter for our findings in this model, we assume it away for simplicity.
There is a unique, autarkic steady state where the social rate of return is \( q_A \Phi_A = \rho \) and the interest rate is \( r_A = \psi_A \rho \), while \( \frac{\partial r_A}{\partial \lambda} > 0 \) and \( \frac{\partial w_A}{\partial \lambda} > 0 \).

Figure 10 shows the impacts of financial integration on domestic investment-saving imbalances. Point \( E_A \) denotes domestic investment-saving balance in the autarkic steady state where the domestic investment demand \( M_A \) and the domestic saving \( S_A \) intersect. Due to the exogenous mass of entrepreneurs and the homogeneous wealth distribution, the entrepreneurial wealth share is constant at \( \delta_t \equiv \tau \frac{w_t L}{w_t L} = \tau \). Given \( w_0 = w_A \) and \( \delta_t = \tau \), the two lines stay put in period 0. Thus, the normalized interest rate must fall \( \psi_0 < \psi_A \) so as to create the excess domestic credit demand and absorb financial inflows. The equilibrium moves downwards from point \( E_A \) to \( E_0 \). From period \( t = 1 \) on, the rise in aggregate income shifts the two lines rightwards in equal proportions, while domestic investment expansion reduces the social rate of return; given the interest rate constant at the world level \( r_t = r^* \), \( \psi_t \) rises over time; the dashed arrow shows the path along which country N converges to the new steady state \( F \). Given \( \delta_t = \tau \), \( u_F < u_A \) must hold so as to justify capital inflows with the excess domestic credit demand in the new steady state. Proposition 5 summarizes the productivity implications of financial integration, which follows closely the dynamics of \( \psi_t \).

Proposition 5. In model IM, given \( r^* < r_A \), the productivity falls upon financial integration, \( \Phi_0 < \Phi_A \) and then rises over time, \( \Phi_t > \Phi_{t-1} \); in the long run, the productivity is strictly lower than its initial level, \( \Phi_0 < \Phi_F < \Phi_A \).

In model IM, the entrepreneurial wealth share is constant; upon financial integration, the domestic investment demand and the domestic saving stay put, while the subsequent rises in aggregate income raise them in equal proportions. In the current model, due to the endogenous entrepreneurial wealth share, the domestic investment demand responds to income rises by a larger proportion than the change in domestic saving and hence, financial inflows are larger than in model IM. This way, the endogeneity of the entrepreneurial wealth share is key to the different patterns of the normalized interest rate and the productivity between the two models.

E Proofs

Proof of Proposition 1
Proof. The proof consists of four steps.

**Step 1: Solve the Individual Optimization Problem and Derive the Unit Costs**

Agent $j$ chooses tangible and intangible investments as well as loans to maximize its net investment revenue, subject to the budget constraint and the borrowing constraints.

\[
\Pi_{j,t+1} \equiv \max_{k_{j,t+1},b_{j,t},b_{j,t+1}} q_{t+1} k_{j,t+1} - r_t b_{j,t} - \xi_{j,t} (k_{j,t+1} + k_{j,T_t+1} - n_{j,t} - b_{j,t}) \]

\[
- \xi_{j,t} (b_{j,t} - \gamma \frac{p_{t+1}}{r_t} k_{j,T_t+1}) .
\]

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\[
- \xi_{j,t} (b_{j,t} - \gamma \frac{p_{t+1}}{r_t} k_{j,T_t+1}) .
\]

Let $\lambda_{j,t} \equiv \frac{a_{j,t}}{1 - a_{j,t}} \frac{1 - \eta}{\eta}$. The marginal revenues of intangibles and tangibles are respectively,

\[
MR_{j,t,t+1} \equiv q_{t+1} \frac{\partial k_{t+1}}{\partial k_{j,t,t+1}} = q_{t+1} \lambda_{j,t} \frac{1 - \eta}{\eta} \],

and $MR_{j,T_t+1} \equiv q_{t+1} \frac{\partial k_{t+1}}{\partial k_{j,T_t+1}} = q_{t+1} \lambda_{j,t} \frac{1 - \eta}{\eta}$.

If the borrowing constraint is slack for agent $j$, $\xi_{j,t} = 0$ and, according to equations (40)-(42),

\[
MR_{j,j,t+1} = MR_{j,T_t+1} = \xi_{j,t} = r_t, \Rightarrow a_{j,t} = \eta, \; \Phi_{j,t} = 1, \; \text{and} \; q_{t+1} = r_t .
\]

As the private and the social rates of return coincide, those who can meet the MIR do not have strong incentive to invest their entire net wealth in the project or borrow to the limit. Nevertheless, those who run the project choose $a_{j,t} = \eta$. Thus, $a_t = \eta$ and $\Phi_t = 1$ hold at the aggregate level.

If the borrowing constraint is binding for agent $j$, $\xi_{j,t} > 0$ and equations (40)-(42) imply

\[
MR_{j,j,t+1} = \xi_{j,t} = \frac{MR_{j,T_t+1} - \lambda p_{t+1}}{1 - \lambda p_{t+1}} ,
\]

which is equivalent to equation (3). As the right hand side of equation (3) increases in $\lambda_{j,t}$, there exists a unique $\lambda_{j,t}$ that solves this equation. Since all agents face the same market prices, they choose the same value for $\lambda_{j,t}$ in equilibrium and subscript $j$ can be left out, i.e., $\lambda_{j,t} = \lambda_t$ and $a_{j,t} = a_t$. Besides, as the price of tangibles is equal to the marginal revenue of tangibles at the aggregate level, we simplify the unit cost of tangibles as equation (5). Use it to derive equation (6) specifying the unit cost of total investment.

The binding borrowing constraints imply that $\xi_{j,t} > 0$. Combine it with equations (40)-(42) to get $MR_{j,j,t} = \xi_{j,t} > r_t$ and $MR_{j,T_t} = (\xi_{j,t} - r_t) a_{T,t} + r_t > r_t$. Thus, the social rate of return exceeds the interest rate in equilibrium,

\[
q_{t+1} \Phi_t = a_t MR_{j,t,t} + (1 - a_t) MR_{j,T,t} > r_t .
\]

**Step 2: Derive the Condition for the Binding Borrowing Constraints under Autarky**

In the case of the slack borrowing constraints, $r_t = \Phi_t q_{t+1}$, $a_t = \eta$, and $\Phi_t = 1$. Combine equations (13)-(12) with (1) to get equation (14) specifying the law of motion for wage.
In the case of the binding borrowing constraints, \( r_1 < \Phi_1 q_{t+1} \) and equations (16) specify the major endogenous variables in the autarkic equilibrium.

When the borrowing constraints are weakly binding, \( \Phi_1 = 1 \) and \( q_{t+1} = r_1 \). Use equation (6) to get \( u_t = 1 - \lambda(1 - \eta) \). Combine it with equation (16) to get the threshold value \( w_A = [1 - \lambda(1 - \eta)]^{1/\lambda} \frac{m}{1 - \eta} > 0 \).

When the borrowing constraints are strictly binding, use equations (16), (6), and (4) to get

\[
\frac{\partial \ln a_t}{\partial \ln w_t} = \frac{\partial \ln u_t}{\partial \ln w_t} = 1 - \theta < 1, \quad \text{and} \quad \frac{\partial \ln \Phi_t}{\partial \ln a_t} = \frac{\eta - a_t}{1 - a_t} > 0, \text{if } a_t < \eta. \tag{44}
\]

For \( w_t < \bar{w}_A \), the borrowing constraints are binding, with \( u_t < 1 - \lambda(1 - \eta) \), \( a_t < \eta \), and \( r_t < q_{t+1}\Phi_t \).

Combine equations (13)-(12) with (1) to get (14) for the law of motion for wage.

**Step 3: Derive the Condition for the Unique, Stable Steady State under Autarky**

Under autarky, there is a unique, stable steady state, if the slope of the law of motion for wage at any steady state is less than unity, \( \frac{\partial w_{t+1}}{\partial w_t} \bigg|_{w_A} < 1 \).

- According to equation (14), this condition holds, \( \frac{\partial \ln a_t}{\partial \ln w_t} \bigg|_{w_A} = \alpha < 1 \), if there is an autarkic steady state with \( w_A > \bar{w}_A \).

- If there is an autarkic steady state with \( w_A < \bar{w}_A \), use equation (14) to get

\[
\frac{\partial w_{t+1}}{\partial w_t} \bigg|_{w_A < \bar{w}_A} = (1 - 2\alpha) \left[ 1 - \rho(1 - \theta) \frac{1 - a_t}{1 - \eta} \right]. \tag{45}
\]

Given \( w_A < \bar{w}_A \), \( a_A < \eta \) and hence, \( \frac{1 - a_t}{1 - \eta} < 1 \). A sufficient condition for \( \frac{\partial w_{t+1}}{\partial w_t} \bigg|_{w_A < \bar{w}_A} < 1 \) is \( \rho(1 - \theta) \leq 1 \) or equivalently, \( \theta \geq \theta \equiv \max\{0, 1 - \frac{1}{\rho \eta}\} \).

**Step 4: Derive the Autarkic Steady-State Patterns of Endogenous Variables**

Given \( \theta \geq \theta \equiv \max\{0, 1 - \frac{1}{\rho \eta}\} \), there exists a unique, stable steady state under autarky. Let \( \bar{\lambda}_A \) denote a threshold value such that, for \( \lambda = \bar{\lambda}_A \), the borrowing constraints are weakly binding at the autarkic steady state, with \( w_A = \bar{w}_A \), \( a_A = \eta \). In this case, \( \Phi_A = 1 \) and

\[
w_A = \rho^{-\theta} = \bar{w}_A = (1 - \bar{\lambda}_A)^{1/\lambda} \frac{m}{1 - \theta}, \quad \Rightarrow \bar{\lambda}_A = 1 - Z^{1/\theta}, \quad \text{where } Z = \frac{1 - \theta}{\rho \theta \eta}. \tag{46}
\]

For \( \lambda > \bar{\lambda}_A \), the borrowing constraints are slack at the autarkic steady state where the endogenous variables \( \{a_A, u_A, r_A, \Psi_A, w_A\} \) are constant and independent of \( \lambda \). For \( \lambda < \bar{\lambda}_A \), the borrowing constraints are binding at the autarkic steady state. Combine equations (4), (14) and (16) to get

\[
w_A = u_A^{1/\lambda} \frac{m}{1 - \theta}, \quad \Rightarrow \rho \ln \Phi_A = \frac{1}{1 - \theta} \left( \ln a_A + \ln[1 - \lambda(1 - \eta)] - \ln \eta \right) - \ln Z \quad \text{and} \quad \frac{\lambda(1 - \eta)}{1 - \lambda(1 - \eta)} = \frac{\frac{\lambda(1 - \eta)}{1 - \lambda(1 - \eta)}}{1 - \rho(1 - \theta) \eta \left( 1 - \frac{1 - \theta}{\rho \theta} \right)}.
\]

Given \( \theta > \theta \) and \( \lambda < \bar{\lambda}_A \), \( a_A < \eta \) and \( \frac{\partial \ln a_A}{\partial \ln \lambda} > 0 \). Use equations (4) and (6) to get

\[
\frac{\partial \ln u_A}{\partial \ln \lambda} = \frac{\partial \ln a_A}{\partial \ln \lambda} > 0, \quad \frac{\partial \ln \Phi_A}{\partial \ln \lambda} = \frac{\eta - a_A}{1 - a_A} \frac{\partial \ln a_A}{\partial \ln \lambda} > 0, \quad \frac{\partial \ln w_A}{\partial \ln \lambda} = \rho \frac{\partial \ln \Phi_A}{\partial \ln \lambda} > 0.
\]

38
Under autarky, domestic investment in period $t$ is $M_t = w_t L$, while the revenue of capital goods in period $t + 1$ is $q_{t+1} K_{t+1} = \frac{n}{1-\alpha} w_{t+1} L$. Then, the social rate of return is

$$\frac{q_{t+1} K_{t+1}}{M_t} = q_{t+1} \Phi_t = \rho \frac{w_{t+1}}{w_t}. \tag{47}$$

In the steady state, $w_{t+1} = w_t$ implies that the social rate of return is constant at $\rho$ and the interest rate $r_A = \psi_A \rho$ rises strictly with $\lambda$.

Let $\psi_t \equiv \frac{n}{q_{t+1} w_t}$ denote the normalized interest rate. According to equation (47), the social rate of return in the steady state is independent of $\lambda$, i.e., $q_A \Phi_A = \rho$. Use equation (6) to get

$$r_A = \rho \psi_A, \quad \psi_A = \frac{\lambda}{1 - u_A}, \quad \Rightarrow \quad \frac{\partial \ln r_A}{\partial \ln \lambda} = \frac{\partial \ln \psi_A}{\partial \ln \lambda} = 1 + \frac{u_A}{1 - u_A} \frac{\partial \ln a_A}{\partial \ln \lambda} > 0. \tag{48}$$

![Figure 11: Threshold Values for the Autarkic Steady State](image)

In figure 11, the left panel shows the threshold value $\theta$ in the $\{\alpha, \theta\}$ space; given $\{\alpha, \theta\}$ in region U, the right panel shows the threshold value $\hat{\lambda}_A$ in the $(\lambda, Z)$ space. For $(\lambda, Z)$ in region UB (US) of the right panel, there is a unique, autarkic steady state where the borrowing constraints are binding (slack) with $w_A < \bar{w}_A$ ($w_A > \bar{w}_A$).\(^{45}\)

Given $\{\alpha, \theta\}$ in region U of the left panel and $(\lambda, Z)$ in region UB (US) of the right panel in figure 11, the solid curve in the left (right) panel of figure 12 show the law of motion for wage. For comparison purpose, the dashed curves in figure 12 show the laws of motion for wage in the absence of financial friction.\(^{46}\) For $w_t \in (0, \bar{w}_A)$, the solid curve lies below the dashed curve and the gap reflects the efficiency losses, $(1 - \Phi_t^\alpha) \left( \frac{w_t}{\rho} \right)^\alpha$.

\[^{45}\]See figure 12 in the proof of proposition 1 for the laws of motion for wage in these two cases.

\[^{46}\]In the absence of financial frictions, the intangible fraction of investment is always equal to its factor share in capital formation $a_t = \eta$ and the productivity is efficient at $\Phi_t = 1$. Then, the entire law of motion for wage under autarky is specified by equation (14) and there is a unique, autarkic steady state with the wage rate $w_B = \rho^{-p}$.

Proof of Proposition 2

**Proof.** This proof focuses on the case where $r^*$ is marginally below $r_A$ and financial integration allows country N to converge towards a stable steady state in the neighbourhood of the autarkic one. See appendix B and the proof of proposition 3 for the stability analysis over the entire state space.
Figure 12: Laws of Motion for Wage under Autarky: $\theta \geq 0$

Under financial integration, the law of motion for wage is still characterized by equation (14), except that $r_t = r^*$ and $M_t \neq w_t L$. Combine $r_t = r^*$ with equations (1) and (7)-(9) to get

$$w^{1-\eta}_{t+1} = q^{-1}_{t+1} = \frac{\lambda (1-\eta)p_0}{r^*(1-u_t)} \rho$$
$$\Rightarrow \frac{\lambda (1-\eta)p_0}{r^*(1-u_t)} \rho = \frac{M_t}{L} = \frac{w_t}{u_t} \delta_t = \left( \frac{w_t}{u_t} \right)^{1-\alpha} \left( 1 - \frac{\theta}{m} \right)^{\frac{1-\theta}{m}} < 0. \quad (50)$$

In the following, we solve analytically the impacts of financial integration on domestic investment, productivity and aggregate income in three scenarios.

Scenario 1: the Impacts of Financial Inflows in Period 0

In period 0, $w_0 = w_A$ and one can use equations (4), (6), and (49)-(50) to prove that the endogenous variables $\{u_0, \psi_0, a_0, \Phi_0, M_0, w_1\}$ are the implicit functions of $r_0$. Equations (18) and (51)-(53) characterize analytically the responses of $\psi_0, M_0, \Phi_0, w_1$ with respect to the interest rate decline $r_0 = r^* < r_A$.

$$\frac{\partial \ln M_0}{\partial \ln r_0} = -\frac{1}{(1-\alpha) + \theta \left( \frac{u_0}{1-a_0} + \alpha \frac{\eta-a_0}{1-a_0} \right)} < 0, \quad (51)$$

$$\frac{\partial \ln \Phi_0}{\partial \ln r_0} = \frac{\partial \ln \Phi_0}{\partial \ln u_0} \frac{\partial \ln u_0}{\partial \ln r_0} = \frac{\partial \ln \psi_0}{\partial \ln u_0} \frac{\partial \ln u_0}{\partial \ln r_0} = \frac{\eta-a_0}{1-a_0} + \left( \frac{u_0}{1-a_0} + \alpha \frac{\eta-a_0}{1-a_0} \right) > 0, \quad (52)$$

$$\frac{\partial \ln w_1}{\partial \ln r_0} = \left[ \frac{\partial \ln M_0}{\partial \ln r_0} \right] \frac{\partial \ln \Phi_0}{\partial \ln r_0} \left[ \frac{\partial \ln w_1}{\partial \ln \Phi_0} \right] \left[ 1 - (1-\alpha) \right] = -\frac{\alpha}{1-\alpha} \left( 1 - \frac{\theta \eta-a_0}{1-a_0} \right) > 0. \quad (53)$$

Step 2: the Impacts of Financial Inflows in Period $t > 0$

From period $t \geq 1$ on, $r_t = r^*$ and one can use equations (4), (6), and (49)-(50) to prove that the endogenous variables $\{u_t, \psi_t, a_t, \Phi_t, M_t, w_{t+1}\}$ are the implicit functions of $w_t$. Equations (19) and
Step 1: Derive the Law of Motion for Wage under Financial Integration

Combine equations (1) to get the factor price equation,
\[ q_t^\alpha w_t^{1-\alpha} = 1. \]  \hspace{1cm} (59)

Proof of Proposition 3

Proof. we first derive the law of motion for wage under financial integration. Then, we use it to analyze the dynamic and the steady-state properties of the model economy.

Step 1: Derive the Law of Motion for Wage under Financial Integration

Combine equations (1) to get the factor price equation,
\[ q_t^\alpha w_t^{1-\alpha} = 1. \]  \hspace{1cm} (59)
• If \( q_t + 1 \Phi_t = r^* \), the borrowing constraints are slack and \( \Phi_t = 1 \). Combine them with equation (59) to get

\[
wt+1 = (r^*)^{-\rho},
\]

which is constant and independent of \( w_t \). Thus, when the borrowing constraints are slack, the law of motion for wage is flat.

• If \( q_t + 1 \Phi_t > r^* \), the borrowing constraints are binding and the model dynamics are determined jointly by five equations. First, use \( r_t = r^* \) and equation (59) to rewrite the binding borrowing constraints (2) as

\[
r_t = \frac{\lambda(1 - \eta)q_t + 1 \Phi_t}{1 - ut} = r^*,
\]

\[
\Rightarrow \frac{1}{\rho} w_{t+1}^{-1} = q_t^{-1} = \frac{\lambda(1 - \eta)\rho \Phi_t}{r^*(1 - ut)\rho}.
\]

Second, combine equations (1) with (13) to get equation (14) specifying the law of motion for wage. Third, the mass of entrepreneurs cannot exceed the total population in each generation, \( \tau_t \leq 1 \). Combine this constraint with equations (7) and (9) to get

\[
\delta_t = \tau_t^{1-\theta} = \begin{cases} 
\frac{w_t^{1-\theta} m}{\theta} & \text{if } w_t < u_t \frac{m}{1-\theta}; \\
1 & \text{if } w_t \geq u_t \frac{m}{1-\theta}.
\end{cases}
\]

Under financial integration, when the borrowing constraints are binding, equations (4), (6), (9), (14), (61), (62) jointly determine \( \{wt+1, at, ut, \Phi_t, \delta_t\} \) as the functions of \( w_t \), which characterizes the law of motion for wage.

In the special case of \( \theta = 0 \) and \( \delta_t < 1 \), combine equations (9), (14), and (61)-(62) to get

\[
\frac{w_t}{m} = \frac{w_t}{m},
\]

implying that each entrepreneur borrows \( m - w_t \) and invests at the level of the MIR in equilibrium, \( m_{jt} = m \).

**Step 2: Derive the Conditions under which the Two Constraints are Binding**

For a sufficiently low level of aggregate income, the mass of entrepreneurs is inefficiently low so that the borrowing constraints are binding and the MoE constraint is slack. The rise in aggregate income allows more agents to become entrepreneurs, which may trigger two events: (1) the borrowing constraints become slack; (2) the MoE constraint becomes binding. The law of motion for wage is piecewise and its characterization depends on which event comes first. The MoE constraint matters only when the borrowing constraints are binding.\(^{48}\)

In the following analysis, we first derive the condition for the MoE constraint to be binding, given that the borrowing constraints are binding. Then, we specify the law of motion for wage in two scenarios.

Define two threshold values,

\[
\bar{w}_F \equiv \left[ \frac{Zp^{(1-\eta)}}{r^*} \right]^{\theta} \frac{1 - \lambda(1 - \eta)}{\eta Z \rho^\eta}, \quad \text{and} \quad \tilde{w}_F \equiv \frac{\tilde{a}_F}{\eta} \left[ \frac{1 - \lambda(1 - \eta)}{\eta Z \rho^\eta} \right]^{\theta},
\]

where \( \tilde{a}_F \) is a solution to

\[
\frac{\lambda(1 - \eta)\rho}{r^* \{1 - \tilde{a}_F / \eta [1 - \lambda(1 - \eta)]\}} \left[ \frac{\tilde{a}_F \eta}{\eta} \left( \frac{1 - \tilde{a}_F}{1 - \eta} \right)^{1-\eta} \right]^\rho Z = 1.
\]

\(^{48}\)When the borrowing constraints are slack, the agents who can overcome the MIR do not have strong incentive to be entrepreneurs and hence, the MoE constraint is irrelevant.
Step 3: Derive the Threshold Conditions for Multiple Steady States

Under financial integration, multiple steady states arise if there exists one unstable steady state, i.e., the law of motion for wage has a slope more than unity at a steady state. Let \( X_U \) denote the value of variable \( X \) at the unstable steady state. As shown in equation (60), for \( w_t \geq \bar{w}_F \), the borrowing constraints are slack and the law of motion for wage is flat at \( w_{t+1} = (r^*)^{-\rho} \). Thus, if there exists an unstable steady state, \( w_U < \bar{w}_F \) must hold and the borrowing constraints must be binding there. How about the bindingness of the MoE constraint at the unstable steady state?

In the case where both the MoE constraint and the borrowing constraints are binding, combine equations (4), (6), (9), (14), (61) with \( \delta_t = 1 \),

\[
\frac{\partial w_{t+1}}{\partial w_t} \mid_{w_{t+1}} = \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \frac{1}{1 + \rho \left( \frac{1 - \eta}{1 - \alpha_t} + \frac{\eta - \alpha_t}{1 - \alpha_t} \right)} < 1. \quad (65)
\]
Thus, if there exists a steady state where the MoE constraint is binding, the slope of the law of motion for wage is strictly less than unity there and hence, this steady state must be stable.

To sum up, if there exists an unstable steady state, the borrowing constraints must be binding and the MoE constraint must be slack there. Combine equations (4), (6), (9), (14), (61) with $\delta_i = \left(\frac{w_i 1-\theta}{\eta} \right)^{\frac{1-\eta}{\theta}}$ to get equation (56) describing the slope of the law of motion for wage in logarithm. In a threshold case where the law of motion for wage has a slope equal to unity at a steady state, combine $\frac{\partial \ln w_{t+1}}{\partial \ln w_t} |_{w_{t+1}=w_t=\bar{w}_U} = 1$ with (56) to get $a_U$ as a function of $\lambda$ and other parameters,

$$1 - \frac{\theta}{\alpha} = \frac{1 - \frac{\theta}{\alpha}}{1 - \lambda(1 - \eta)} = \frac{1 - \eta}{1 - a_U} = \frac{1 - \alpha}{\alpha}. \tag{66}$$

Next, we specify the conditions for the existence of multiple steady states, given $r^* = r_A$.

### 3.1. Parameter Constellation for Multiple Steady States in the $(\lambda, \theta)$ Space

Combine equation (66) with $a_U \leq \eta$ and $\lambda \leq 1$ to get a threshold value $\bar{\lambda}_F = \min\{\frac{\alpha - \theta}{\eta(1-\eta)}, 1\}$, as shown by the downward-sloping curve in the left panel of figure 7. For $(\lambda, \theta)$ in region U, equation (66) does not have a solution with $a_U \in (0, \eta]$ and hence, the autarkic steady state is still the unique, stable steady state under financial integration; for $(\lambda, \theta)$ in region M, multiple steady states may arise as proved below.

### 3.2. Parameter Constellation for Multiple Steady States in the $(\lambda, Z)$ Space

Given $(\lambda, \theta)$ in region M of the left panel of figure 7, multiple steady states arise in three scenarios with $(\lambda, Z)$ in region BC, B, and AB of the right panel in figure 7, respectively.

**Scenario BC:** For $(\lambda, Z)$ in region BC, $\lambda > \bar{\lambda}_A$ and the borrowing constraints are slack in the autarkic steady state, with $r_A = \rho$, as shown in the right panel of figure 11. Under financial integration, although the autarkic steady state is still stable, an unstable steady state (U) and another stable steady state (L) arise, $w_L < w_U < \bar{w}_F$, as shown in the upper-right panel of figure 8. In the threshold case, the law of motion for wage is tangent with the 45° line at the unstable steady state and accordingly, two conditions hold. First, equation (66) specifies $a_U$ as a function of $\{\lambda, \alpha, \theta, \eta\}$. Second, combine equations (61), (9), (14), (4), (6) with $\delta_i = \left(\frac{w_i 1-\theta}{\eta} \right)^{\frac{1-\eta}{\theta}}$, $r_{t+1} = w_t = \bar{w}_U$, and $r^* = r_A = \rho$ to characterize the unstable steady state,

$$\left[\frac{\lambda(1-\eta)}{1 - \frac{\theta}{\alpha} \lambda(1-\eta)}\right]^{\frac{\alpha - \theta}{\eta}} \frac{a_U}{\eta} [1 - \lambda(1 - \eta)] = \left\{\left(\frac{a_U}{\eta} \right) \left(1 - \frac{a_U}{\eta} \right)^{1-\eta} \right\}^{\frac{1-\theta}{\eta}}. \tag{67}$$
which specifies $a_U$ as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. Then, equations (66) and (67) jointly specify $Z$ as a function of $\lambda$, featuring the border between region BC and C in the right panel of figure 7.

**Scenario B:** for $(\lambda, Z)$ in region B, $\lambda < \hat{\lambda}_A$ and the borrowing constraints are binding in the autarkic steady state, with $r_A < \rho$, as shown in the right panel of figure 11. Financial integration destabilizes the autarkic steady state, while two stable steady states (L and H) arise, as shown in the upper-left panel of figure 8 and in the left panel of figure 13. In the threshold case, the law of motion for wage is tangent with the 45° line at the autarkic steady state and accordingly, two conditions hold. First, $a_A = a_U$ is a function of $\{\lambda, Z, \alpha, \theta, \eta\}$, according to equation (66). Second, combine $w_{t+1} = w_t = w_A$ with equations (4), (6), (16), (14) to characterize the autarkic steady state,

$$\frac{1}{1 - \frac{\theta}{\rho}} = w_A \frac{1 - \theta}{m} = Z \Phi^\rho_A, \quad \Rightarrow \quad \left(\frac{a_A}{\eta} \left[1 - \lambda(1 - \eta)\right]\right)^{\frac{1}{\eta}} = Z \left[\left(\frac{a_A}{\eta} \left(1 - a_A\right)\right)^{\frac{1}{\rho}}\right]. \quad (68)$$

which specifies $a_A$ as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. Then, equations (66) and (68) jointly specify $Z$ as a function of $\lambda \in (0, \hat{\lambda}_F)$, featuring the border between region B and AB in the right panel of figure 7.

**Scenario AB:** for $(\lambda, Z)$ in region AB, $\lambda < \hat{\lambda}_A$ and the borrowing constraints are binding in the autarkic steady state, with $r_A < \rho$, as shown in the right panel of figure 11. Under financial integration, although the autarkic steady state is still stable, an unstable steady state (U) and another stable steady state (H) arise, as shown in the upper-middle panel of figure 8 and in the middle panel of figure 13; as explained in step 2, the law of motion for wage is piecewise and consists of two or three parts; multiple steady states arise if at least one kink point in the law of motion for wage is above the 45° line.

- **Scenario AB.R:** for $(\lambda, Z)$ in region AB and to the right of the dashed curve, $\lambda \geq \hat{\lambda}_F$ and the law of motion for wage has one kink point at $w_t = \tilde{w}_F$, where $\tilde{w}_F$ is specified in equation (64). For $w_t \geq \tilde{w}_F$, the borrowing constraints are slack, $\Phi_t = 1$, and the law of motion for wage is flat at $w_{t+1} = (r^*)^{-\rho}$. In the threshold case, the kink point is on the 45° line and accordingly, two conditions hold. First, combine $r^* = r_A$ and $w_{t+1} = (r^*)^{-\rho} = w_t = \tilde{w}_F$ with equations (64) and (68) to characterize the kink point

$$\frac{1}{\lambda(1 - \eta)} + \frac{a_A}{\eta} = \left(\frac{a_A}{\eta} \left[1 - \lambda(1 - \eta)\right]\right)^{\frac{1}{\eta}} \Phi^\rho_A, \quad \text{where} \quad \Phi_A = \left(\frac{a_A}{\eta} \left(1 - a_A\right)\right)^{\frac{1}{\rho}}. \quad (69)$$

It specifies $a_A$ as a function of $\{\lambda, \alpha, \theta, \eta\}$. Second, equation (68) specifies $a_A$ as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. Then, equations (68) and (69) jointly specify $Z$ as a function of $\lambda$. It is shown as the part of the border between region AB and A, which is to the right of the dashed curve in the right panel of figure 7.

- **Scenario AB.L:** for $(\lambda, Z)$ in region AB and to the left of the dashed curve, $\lambda < \hat{\lambda}_F$ and the law of motion for wage has two kinks at $w_t = \tilde{w}_F$ and $w_t = \tilde{w}_F$, respectively. In the threshold case, the kink point at $w_t = \tilde{w}_F$ is on the 45° line, where $\tilde{w}_F$ is specified in equation (64). In this case, three conditions hold. Let $\tilde{X}_F$ denote the value of variable $X_t$ at that kink point. First, combine equations (64) and (68) with $r^* = r_A$ to characterize the kink point

$$\frac{1}{\lambda(1 - \eta)} + \frac{a_A}{\eta} \left[\frac{1 - \lambda(1 - \eta)}{1 - \lambda(1 - \eta)}\right] \left[\frac{a_A}{\eta} \left(1 - a_A\right)\right]^\rho = 1, \quad (70)$$
which specifies $\tilde{a}_F$ as a function of $a_A$ and $\{\lambda, \alpha, \eta, \theta\}$. Second, combine equations (4), (6), (9), (14), (61) with $\delta_t = 1$ and $w_{t+1} = w_t = \tilde{w}_F$ to characterize that kink point as a steady state

$$
\left(\frac{a_A}{\tilde{a}_F}\right)^{\eta} \left(1 - \frac{a_A}{\tilde{a}_F}\right)^{1-\eta} = \left\{ \frac{a_A}{\eta} \left[1 - \lambda(1 - \eta)\right] \right\}^{1-\eta} \left\{ 2 - \frac{a_A}{\eta} \left[1 - \lambda(1 - \eta)\right] \right\}^{1-\eta},
$$

which specifies $\tilde{a}_F$ as a function of $a_A$ and $\{\lambda, Z, \alpha, \eta, \theta\}$. Third, equation (68) specifies $a_A$ as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. Then, equations (68), (70), and (71) jointly determine $\{Z, \tilde{a}_F, a_A\}$ as the functions of $\lambda$. The relationship between $Z$ and $\lambda$ is shown as the part of the border between region AB and A in the right panel of figure 7, which is to the left of the dashed curve.

\[ \square \]

**Proof of Corollary 1**

**Proof.** $Z_F$ and $\tilde{Z}_F$ are characterized in the proof of proposition 3 as the functions of $\lambda$. They are shown as the upper and lower borders of region AB in the right panel of figure 7.

- Define $\tilde{r}^*$ as a threshold value such that, for $r^* = \tilde{r}^*$, the law of motion for wage under financial integration is tangent with the 45° line where $a_U$ denote the steady-state value of $a_t$. Use equation (66) to solve $a_U$ as a function of $\lambda$ and other parameters. Combine $w_{t+1} = w_t = w_U$ with equations (4), (6), (9), (14), (61) and $\delta_t = \left( \frac{w_t}{a_t} \frac{1-\theta}{m} \right)^{\frac{1}{1-\theta}}$ to get

$$
\tilde{r}^* \equiv \frac{\lambda(1 - \eta) \rho}{1 - \frac{a_U}{\eta} \left[1 - \lambda(1 - \eta)\right]} \left[ \frac{(\Phi_U^0 Z)^{1-\theta}}{a_U \eta \left[1 - \lambda(1 - \eta)\right]} \right]^{\frac{1-\eta}{1-\theta}}, \text{ where } \Phi_U = \left( \frac{a_U}{\eta} \right)^{\eta} \left(1 - \frac{a_U}{\eta} \right)^{1-\eta}.
$$

- Define $\tilde{r}^*$ as a threshold value such that, for $r^* = \tilde{r}^*$, the law of motion for wage under financial integration has the kink point on the 45° line. At the kink point, $a_k = \eta$, $\Phi_k = 1$, $u_k = 1 - \lambda(1 - \eta)$, and $w_{t+1} = w_T = w_k = \tilde{w}_F$. Combine them with equations (4), (6), (9), (14), (61) and $\delta_T = \left( \frac{w_T}{a_T} \frac{1-\theta}{m} \right)^{\frac{1}{1-\theta}}$ to get

$$
\tilde{r}^* = \rho \left[ \frac{Z^{1-\theta}}{1 - \lambda(1 - \eta)} \right]^{\frac{1-\eta}{1-\theta}}.
$$

\[ \square \]

**Proof of Proposition 4**

**Proof.** Consider first the case of the binding borrowing constraints under autarky. As domestic investment is financed by domestic saving, the unit cost of investment is constant and so are other major endogenous variables, as specified by equations (38). Under assumption 2, $\psi_A < 1$ implies that the borrowing constraints are binding. According to equations (38),

$$
\frac{\partial \psi_A}{\partial \lambda} = \frac{1-\eta}{1-\tau} > 0, \quad \frac{\partial \ln a_A}{\partial \ln \lambda} = \frac{\lambda(1-\eta)}{1-\lambda(1-\eta)} > 0, \quad \frac{\partial \ln \Phi_A}{\partial \ln \lambda} = \frac{\eta - a_A}{1 - \eta} \frac{\partial \ln a_A}{\partial \ln \lambda} > 0.
$$

As the project productivity is constant, the law of motion for wage is log-linear with the slope less than unity, according to equations (39). Thus, there exists a unique, autarkic steady state. The social rate of return is defined by equation (47) and, in the autarky steady state

$$
w_{t+1} = w_t = w_A, \Rightarrow q_A \Phi_A = \rho, \text{ and } r_A = \psi_A q_A \Phi_A = \psi_A \rho < \rho.
$$

\[ \square \]